THE EFFECT OF THE $\pi\pi$ INTERACTION ON THE ELECTROMAGNETIC FORM FACTOR OF THE NUCLEON

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The isovector electromagnetic form factors of the nucleon are evaluated in the two-meson approximation including the $\pi\pi$ interaction. Use is made of the amplitude of the process $\pi + \pi \rightarrow N + \overline{N}$ obtained earlier by Galanin and Grishov[8] and a new expression for the electromagnetic form factor of the π meson which disagrees with the previously known expression[1,4] for the case of the Breit-Wigner resonance of the $\pi\pi$ amplitude. It is shown that the case of the Breit-Wigner resonance does not lead to agreement of the nucleon form factors with experiment and that results obtained earlier for this case[1,2] are in error. An agreement with experiment is, however, obtained for the case of a kinematical resonance in the amplitude around 750 MeV (the effect of the ρ meson).

RECENTLY attempts have been made to evaluate with dispersion techniques the contributions due to the $\pi\pi$ resonances to the isovector nucleon form factor.[1,2] In contrast to similar attempts omitting the $\pi\pi$ interaction,[3,4] the obtained electric and magnetic form factors F_1^V and F_2^V agree with the experiment in the momentum transfer range of $-4m\mu \lesssim t \leq 0$ (μ and m are the mass of the π meson and nucleon). We believe, however, that the expression for the electromagnetic form factor of the π meson* used in these calculations, namely

$$F_\pi(t) = \varphi_1(t)/\varphi_1(0), \tag{1}$$

where the dependence on the P-phase of the $\pi\pi$ scattering, $\delta_1(t)$, is given by

$$\varphi_1(t) = \exp\left[\frac{t-t_0}{\pi}\int_{4\mu^2}^{\infty}\frac{\delta_1(t')\,dt'}{(t'-t)(t'-t_0)}\right], \tag{2}$$

is in error, since it yields a nonvanishing effect

$$F_\pi(t) = t_r/(t_r - t) \tag{3}$$

for the experimentally unobservable $\pi\pi$ interaction (infinitely sharp resonance),† where $\delta_1(t)$ is of the form

$$\delta_1(t) = \begin{cases} 0 & t < t_r \\ \pi/2 & t = t_r \\ \pi & t > t_r \end{cases} \tag{4}$$

*In the following we consider the form factors without the factors e, e/2 and 1.85 e/2m, i.e., $F_\pi(0) = F_1^V(0) = F_2^V(0) = 1$.

†The necessity that the effect vanish for the case (4) has been pointed out to the authors by Ya. B. Zel'dovich (see also [5]).

A further drawback of the calculations in [1,2] is that the annihilation amplitude $f_\pm^1(t)$ was obtained there from the πN amplitude either by means of a numerical integration of divergent expressions or by using the following unproved approximation (see also[6,7])

$$f_\pm^1(t) = F_\pi(t)\,[f_\pm^1(t)]_0 \approx F_\pi(t)\cdot\text{cosnt}, \tag{5}$$

which together with (1) gives the absorptive parts of the nucleon form factors

$$\operatorname{Im} F^V(t) = |F_\pi(t)|^2\,[\operatorname{Im} F^V(t)]_0 \approx |\varphi_1(t)|^2\cdot\text{const}. \tag{6}$$

Here $[f_\pm^1(t)]_0$ and $[\operatorname{Im} F^V(t)]_0$ are the corresponding quantities for the case where the $\pi\pi$ interaction is not taken into account.

The method proposed by Galanin and Grashin[8] for the evaluation of the $\pi\pi$ interaction leads for the Breit-Wigner model

$$\delta_1(t) = \operatorname{arctg}\left[\frac{\gamma\sqrt{x}}{x_r - x}\right], \quad x = \frac{t}{4\mu^2} - 1 \tag{7*}$$

to the expression

$$F_\pi(t) = (x_r - x + \gamma)/(x_r - x + \gamma\sqrt{-x}), \tag{8}$$

which differs from (2) by having as an additional factor a first order polynomial with a zero close to the resonance. In the limit (4) the position of this zero approaches the resonance point t_r. As a result of this the effect of the $\pi\pi$ interaction then vanishes, i.e., $F_\pi \rightarrow 1$ for $\gamma \rightarrow 0$. The same result follows from the general formulae of Gell-

*arctg = arctan

Mann and Zachariasen[9] which have been obtained on the basis of field theory considering an intermediate vector meson, the ρ meson, if one assumes that for the case (4), which is equivalent to the vanishing of the interaction of the ρ meson with all other particles, its mass renormalization must vanish ($\delta m^2 = m^2_{\rho_0} - m^2_{\rho} \to 0$ for $\gamma \to 0$). The particular case which they actually have investigated, namely $\delta m^2_{\rho} = \infty$, leads to (3) and is incompatible with a narrow resonance γ and with a small ρ-meson interaction constant $g_{\rho\pi\pi} \sim \sqrt{\gamma}$.

The method which we are going to apply leads for the πN amplitude[8] to an analogous difference from the approximation (5). In both cases the reason for the difference turns out to be the nonuniqueness of the solution of an integral equation, obtained by inserting into the dispersion relations the unitarity condition in the two-meson approximation. In our method it is essential to retain only that particular solution of the inhomogeneous equation, which is stable against perturbations at infinity and does not depend on the conditions at infinity at low energies. On the other hand, the expressions (1) and (5) are particular solutions of the corresponding homogeneous equation and their existence is wholly due to the introduction of certain conditions at infinity.* We emphasize that the criterion of the stability of the solutions is a necessary condition for the solutions of our problem (evaluation of the effects of the near singularities). Only in this case is it possible to obtain the amplitudes for small energies without having to utilize the unknown behavior of the amplitudes at high energies.

The more general model of the $\pi\pi$ interaction

$$\delta_1(t) = \operatorname{arctg}\left[\frac{\sqrt{x}Q(x)}{X(x)}\right], \quad \varphi_1(t) = \frac{\prod_{k=1}^n (x - x_k)}{X(x) + Q(x)\sqrt{-x}}, \tag{9}$$

where X(x) and Q(x) are arbitrary polynomials and x_k ($k = 1, 2 \ldots n$) are the roots of the equation $X(x) + Q(x)\sqrt{-x} = 0$, $\operatorname{Re}(\sqrt{-x}) \geq 0$, leads to the following expression for the absorptive parts of the nucleon form factors:

$$\operatorname{Im} F_1^V(t) = \frac{1}{2}\varepsilon^2 g^2 |\varphi_1(t)|^2 \frac{x\sqrt{x/(1+x)}}{\prod_{k=1}^n |x - x_k|^2} \times \{X(x) - L_n(x)\}\left\{X(x)\frac{1+2x}{x} - L_{n+1}(x)\right\}; \tag{10}$$

$$\operatorname{Im} F_2^V(t) = \frac{1}{8}\frac{\varepsilon g^2}{1.85}|\varphi_1(t)|^2 \frac{x\sqrt{x/(1+x)}}{\prod_{k=1}^n |x - x_k|^2} \times \{X(x) - L_n(x)\}\left\{X(x)\frac{\pi}{\sqrt{x}} - M_{n+1}(x) - Q(x)\ln x\right\}. \tag{11}$$

Here $g^2 = 14.5$, $\epsilon = \mu/m = 0.15$ and the polynomials of degree n, $L_n(x)$, and of degree n+1, $L_{n+1}(x)$ and $M_{n+1}(x)$, are determined by the condition that they lead to the vanishing of the braces in (10) and (11) at the points $x = -1$ and x_k, and to the vanishing at $x = -1$ of the first derivative of the expressions which one obtains if one changes $X(x) \to -Q(x)\sqrt{-x}$ in the second braces.

The expressions (10) and (11) were obtained using the amplitudes $\pi + \pi \to N + \bar{N}$ evaluated by Galanin and Grashin.[8] Corrections of the order $\lesssim \epsilon(1+2x)/2\sqrt{x}$ were dropped. We note that the evaluation in [8] was performed for the special case $Q(x) = x^l$. However, the obtained formulae are valid if one replaces x^l by Q(x). We also point out that the contribution due to virtual scattering of the π meson on the nucleon (the rescattering corrections) are contained in $f^1_{\pm}$ and in Im F^V only in the discarded corrections $\lesssim \epsilon\sqrt{x}$.

The expressions (10) and (11) are correct in the region $\epsilon x \lesssim 1$; ($t \lesssim 4m\mu$). We think, however, that the evaluation of the contributions in the region $t > 4m\mu$ can be performed only if one takes into account in the unitarity conditions also heavier intermediate states (besides the considered two-meson states). This transcends the framework of the two-meson approximation.* Some restrictions exist also which limit the choice of the polynomials Q(x) and X(x) in the model (9), since it is necessary that the essential contribution to the integrals arising in the solution of the equation [which lead to (10) and (11)] be limited to the interval $4\mu^2 < t_{eff} \lesssim 16\mu^2$. For this it is necessary that

$$\psi(t) = Q(x)/\Pi_k(x - x_k) \lesssim \psi(16\mu^2) \quad \text{for } t \gtrsim 16\mu^2.$$

The factors (the first and second respectively) in the braces in (10) and (11) are responsible for the deviations of F_π and $f^1_{\pm}$ from (1) and (5) which, as has been noted above, vanish in the vicinity of the resonance. This leads to a sharp decrease in the effect as compared to the earlier expressions.

In Fig. 1 is shown the absorptive part of the electric form factor Im F_1^V (Im F_2^V has a similar appearance) for the model with scattering length $Q(x)/X(x) = 0.2x$ and $0.5x$ (variants 1 and 2)

*It is interesting that Federbush et al[4] based the choice of the solution (1) on its agreement with the iterative solution of the equation. However, the iterations give a vanishing effect for the limiting case (4), which contradicts the result (3) and agrees with (8). It is clear that in the iterative solution for the resonance model one has to keep the exact expression $\sin\delta\exp(-i\delta)$ in the kernel of the equation and one should not expand it in powers of the $\pi\pi$ phase shift δ.

*We emphasize that we consider the two-meson approximation to be a consistent description only in the interval $t \lesssim 16\mu^2$ (the closest singularities). This principle is, in particular, the basis of our criterion for the choice of the unique solution.

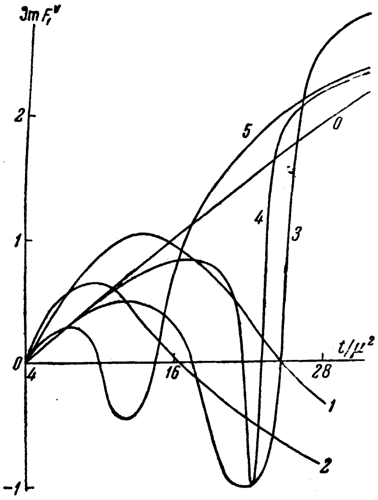

FIG. 1. Absorptive part of the electric form factor in the two-meson approximation for the following cases of the ππ interactions: 0 – absence of ππ interactions; 1 and 2 – scattering length model with $a = (0.2)^{1/3}$ and $(0.5)^{1/3}$; 3 and 4 – Breit-Wigner resonance at $t_r = 22.4\mu^2$ and with a width $\gamma = 0.5$ and 0.1; 5 – resonance at $t_r = 12\mu^2$ and with a width $\gamma = 0.4$.

and for model (7) with the parameters $t_r = 22.4\mu^2$; $\gamma = 0.5$ and 0.1 (variants 3 and 4) and $t_r = 12\mu^2$, $\gamma = 0.4$ (variant 5). Variant 0 corresponds to absence of ππ interactions and is obtained by putting $X(x) = \infty$ in (9), (10), and (11). Variant 3 corresponds to the ππ interaction discussed in [2] and variant 5 to the one discussed in [1,7]. Besides, variants 1 and 3 agree with the experimental ππ cross section obtained by Anderson et al [10] by the extrapolation method. One sees that account of the ππ interaction practically does not change the contribution to the form factor for the case of a narrow Breit-Wigner line. Models with a negative scattering length and effective interaction radius also do not lead to a significant change from variant 0. Thus all these cases lead to small contributions to the form factors which do not agree with experiment.

The above-considered models of the ππ interaction have the common characteristic that the phase $\delta(t) < \pi$ for small energies. The case of the kinematical resonance (Fig. 2) leads to a completely different result. There the phase exceeds the resonance value $\delta(t_0) = \pi$ (the ρ-meson effect). The simplest model of such a type is obtained by putting in (9), (10), and (11)

$$Q(x) = x_0 - x, \qquad X(x) = (x_r - x)(x_0 + x)/a, \qquad (12)$$

where x_0 is the position of the zero of the scattering amplitude and $\gamma = a(x_0 - x_r)/(x_0 + x_r)$ is the analog of the width in the model (7).

For the model (12) the absorptive parts (10) and (11) have a δ-function like form and independently of the position of the resonance

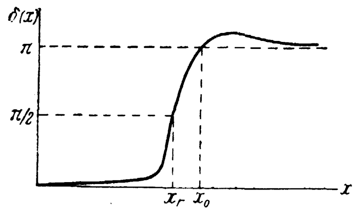

FIG. 2

$$\mathrm{Im}\, F_1^V(t_r) \approx 1.4\, \mathrm{Im}\, F_2^V(t_r).$$

This leads to a definite ratio of the two-meson contributions to the electric and magnetic form factors and in fact leads to the ratio of the mean square radii $\langle r_1^2\rangle_V / \langle r_2^2\rangle_V = 1.4$. We remark that in the earlier phenomenological analysis of the experimental data (see, e.g., [12]) δ-function like absorptive parts have been used with

$$\mathrm{Im}\, F_1^V(t_r) = \mathrm{Im}\, F_2^V(t_r),$$

but the latest data [11] give a different curvature for the electric and the magnetic form factors, which can be obtained only in the case

$$\mathrm{Im}\, F_1^V(t_r) \gtrsim 1.5\, \mathrm{Im}\, F_2^V(t_r).$$

For $t_r = 28\mu^2$ (mass of the ρ meson $m_0 = \sqrt{t_r} = 750$ MeV) and for various full widths $\Gamma \lesssim 100$ MeV, model (12) leads to the form factors

$$F_{1,2}^V(t) = 1 + \alpha_{1,2}\frac{t}{t_r - t} + \frac{\beta_{1,2}t}{4\mu^2},$$

$$\alpha_1 = 1.8, \quad \alpha_2 = 1.28 \quad (\alpha_1/\alpha_2 = 1.4),$$
$$\beta_1 = -0.025, \quad \beta_2 = 0.003, \qquad (13)$$

which virtually coincide with the experimental curves of Hofstadter et al [11] in the interval $-40\mu \le t \le 0$. It turns out that the parameter a and the distance $\Delta = x_0 - x_r$ between the zero of the scattering amplitude and the resonance point are functions of the width Γ as shown in Fig. 3. The expression (13) has been obtained by integrating the absorptive parts with two subtractions. However, the additional contribution from the more distant singularities ($\beta_{1,2}$) to the first derivative at t = 0 is ≲ 10% of the two-meson contribution. We note that model (12), like model (7) for the case of an infinitely sharp resonance, goes over into the limiting form (4) for which the effect of the ππ interaction must vanish. For this to be true we must have $a/\Delta \to 0$ if $\Gamma \to 0$, while in order to main-

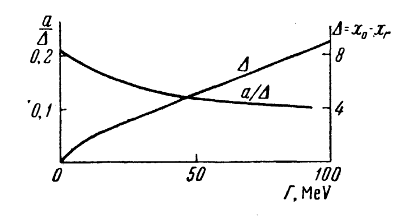

FIG. 3. Parameters of the model of a kinematical resonance at 750 MeV which agree with experiment [11] as a function of the total width Γ of the ρ meson.

tain the needed effect it is necessary that $a/\Delta \to 0.2$ (see Fig. 3). However, in contrast to the Breit-Wigner model, there exists here a sufficiently wide region of finite widths in which one can obtain agreement with experiment.

The results obtained in the present paper allow us to draw the following conclusions.

1) The isovector electromagnetic form factors of the nucleon evaluated in the two-meson approximation with account of the $\pi\pi$ interaction do not agree with experiment for a Breit-Wigner (dynamic) resonance. Furthermore, here the effect of the $\pi\pi$ interaction practically does not change the results as compared to the case of absence of this interaction. Analogous results are obtained with other models of the $\pi\pi$ interaction for which the $\pi\pi$ phase for small energies is $\delta(t) < \pi$.

2) One can obtain agreement with experiment[11] in the case of a kinematical resonance at the energy $\sqrt{t_r} = 750$ MeV (ρ-meson effect) for an arbitrary width $\Gamma \lesssim 100$ MeV. The considered position of the resonance is, however, already on the boundary of the applicability of the two-meson approximation. Therefore the results obtained for this case have mainly a qualitative character.

3) In the papers[1,2,4] an incorrect method was used to evaluate the contributions due to the $\pi\pi$ interaction to the amplitudes of the processes $\pi + \pi \to N + \bar{N}$ and $\pi + \pi \to \gamma$. This is equivalent to a wrong choice of a solution of the previously discussed integral equations. The question of a correct formulation of the problem which will lead to a unique solution will be discussed in more detail separately.

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