# Letters to the Editor

## *ON THE THEORY OF NONRADIATIVE TRANSITIONS IN LUMINESCENT IONIC CRYSTALS*

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IN solid-state lasers and irasers (infrared masers) nonradiative transitions occur between the energy levels of impurity ions of the activator with a partly filled electron shell. [1]

We have assumed that a nonradiative transition is stimulated by an alternating Coulomb electric field (in the point model approximation) of the nearest ions vibrating about their equilibrium positions. This mechanism whereby the excitation energy of the ions is transferred to the crystal lattice is in many respects similar to the theory of spin-lattice relaxation of Kronig and Van Vleck. [2]

Calculations were carried out for an impurity ion at the center of a complex with symmetry $O_h$ and with the nearest neighbors at the vertices of an octahedron or a cube (as for the ruby and fluorite crystals used in lasers). The Hamiltonian of the interaction of the impurity ion with the lattice vibrations has the form

$$H = \sum_{\alpha} V_\alpha Q_\alpha. \tag{1}$$

Direct calculations have shown that in most cases it is sufficient to take only the linear terms of the expansion, of the potential energy of the impurity ion in the crystal electric field in symmetrical vibrations of the complex $Q_\alpha$.

For elements of the iron group it is necessary to allow for the spin-orbit interaction (regarding it as a perturbation) in calculations of the probabilities of nonradiative transitions between levels of various multiplicity.

In the expression (1), $V_\alpha$ denotes functions of the coordinates of electrons in the unfilled shell, transformable by means of the same irreducible representations of the $O_h$ group as for $Q_\alpha$. Only the symmetrical vibrations, transformable by odd representations, are expanded in the optical waves of the crystal. The matrix elements of those $V_\alpha$ which are transformable by means of odd representations are equal to zero between states of definite electron configuration; consequently optical waves play no role in our approximation. In the expression (1) it is necessary to include only the symmetrical vibrations transformable by even representations $\Gamma_{3g}$ and $\Gamma_{5g}$, which can be expanded in acoustic waves of the crystal with a frequency spectrum given by the Debye distribution $\rho(\omega)$.

The probability of a nonradiative transition between the impurity-ion levels A and B, separated by the energy $\Delta = E_A - E_B > 0$, in a crystal with a Debye temperature $\Theta$, is obtained by a generalization of Eq. (37) of [2]:

$$w = \frac{2\pi}{\hbar^2 g(A)} \sum_i^{g(A)} \sum_j^{g(B)} \int \langle | H_{ij}^l (\psi_i^A,\ n_1 \ldots n_l;\ \psi_j^B,\ n_1 + 1 \ldots n_l + 1) |^2 \rangle \Big| \prod_{\beta=1}^{l} \frac{\rho(\omega_\beta)\, d\omega_\beta}{d\omega_1}, \tag{2}$$

where g(A) and g(B) are the degrees of degeneracy of the two levels; $\langle \ldots \rangle$ means averaging over all directions of polarization and propagation of the elastic transverse waves (longitudinal waves can be disregarded because of their much higher velocity of propagation in the crystal) and over the numbers $n_\beta$ of the harmonic lattice oscillators; $H_{ij}^l$ is the matrix element of the perturbation operator in the $l$-th approximation; $l$ is the minimum number of acoustic phonons (with a maximum energy $k\Theta$) which should appear on transfer of the energy $\Delta$ to the lattice. The order of approximation corresponding to the most probable mechanism of energy transfer between an impurity ion and the lattice is given by the relationship $\Delta/k\Theta = l - \gamma$, where k is the Boltzmann constant and $0 < \gamma \leq 1$. Integration is carried out over the region

$$\sum_{\beta=1}^{l} \omega_\beta = \Delta/\hbar, \qquad 0 \leqslant \omega_\beta \leqslant k\Theta/\hbar, \qquad \beta = 1, \ldots, l. \tag{3}$$

At temperatures considerably lower than $\Theta$, the probability w is independent of temperature.

The suggested method was used to calculate the probability of a nonradiative transition between the levels $^4\Gamma_5$ and $^2\Gamma_3$ of the $Cr^{3+}$ ion in a ruby laser working on the $R_1$ line. The calculated value is $10^6$ sec$^{-1}$ for $T \ll \Theta$, compared with the experimental value of $2 \times 10^7$ sec$^{-1}$. [3]

The calculated value of the probability of a nonradiative transition between the levels $^7F_1$ and $^7F_0$ in a laser based on $Sm^{2+}$ in $CaF_2$ [4] is $10^8$ sec$^{-1}$.

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[2] J. H. Van Vleck, Phys. Rev. **57**, 426 (1940).
[3] T. H. Maiman, Phys. Rev. Letters **4**, 564 (1960).
[4] Kaiser, Garrett, and Wood, Phys. Rev. **123**, 766 (1961).

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## *VIOLATION OF THE $\Delta Q = \Delta S$ RULE IN LEPTONIC DECAYS OF K MESONS AND THE HIGH-ENERGY BEHAVIOR OF WEAK INTERACTIONS*

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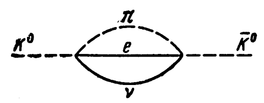
THE recently published experimental data [1] indicate that the $\Delta Q = \Delta S$ rule is violated in the leptonic decays of $K^0$ mesons. Namely, in addition to the decay mode $K^0 \rightarrow \pi^- + e^+ + \nu$, which is allowed by the $\Delta Q = \Delta S$ rule, one also has the mode $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}$, with the probabilities for the two types of decay approximately equal. It follows from the existence of both types of decay for the $K^0$ that also the $\bar{K}^0$ can decay in two ways: $\bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}$ and $\bar{K}^0 \rightarrow \pi^- + e^+ + \nu$, with the consequence that the transition $K^0 \rightarrow \bar{K}^0$ can proceed via the chain of interactions $K^0 \rightarrow \pi^- + e^+ + \nu \rightarrow \bar{K}^0$ or $K^0 \rightarrow \pi^+ + e^- + \bar{\nu} \rightarrow \bar{K}^0$.

Let us estimate the matrix element for the transition $K^0 \rightarrow \bar{K}^0$ due to these interactions by considering the diagram pictured. In this estimate we assume [2] that the weak leptonic interaction preserves its form up to momenta of the order of $\Lambda$, i.e., that the integration over the lepton momenta is to be cut off at $\Lambda$. (If it is assumed that the form of the weak leptonic interactions changes when energies are reached such that the weak interaction becomes effectively strong, then $\Lambda \sim G^{-1/2} \sim 300$ BeV, where G




$= 10^{-5}/m^2$ is the weak-interaction coupling constant.)

We further assume that, owing to the presence of a form factor arising from strong interactions, the integration over the pion momentum may be cut off at M (where M is of the order of the nucleon mass m). In view of the presence in the diagram of a quadratic divergence in the integration over the lepton momenta, it is obvious that the matrix element $\mathfrak{M}$ for the transition $K^0 \rightarrow \bar{K}^0$ will be of the order $\mathfrak{M} \sim G^2\Lambda^2M^3$, i.e., for $\Lambda \sim G^{-1/2}$ we have $\mathfrak{M} \sim GM^3$. On the other hand this matrix element is proportional to the difference $\Delta m_K$ in the masses of the $K_1^0$ and $K_2^0$ mesons, which is known [3] to be $\Delta m_K \sim 1/\tau(K_1^0)$ [where $\tau(K_1^0) \approx 10^{-10}$ sec is the lifetime of the $K_1^0$ meson], i.e., of the order of $G^2m^5$. Consequently the existence of the decay processes $K^0 \rightarrow \pi^- + e^+ + \nu$ and $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}$ leads to the conclusion that the cutoff $\Lambda$, up to which the theory of weak interactions of leptons is applicable, is comparatively small.

For a more concrete estimate we calculate the matrix element $\mathfrak{M}$ assuming the interaction Hamiltonian for the decay $K^0 \rightarrow \pi^- + e^+ + \nu$ to be of the form*

$$H = \frac{1}{\sqrt{2}} G\beta q_\mu \left(\bar{\psi}_\nu \gamma_\mu (1 + \gamma_5) \psi_e\right) \varphi_{K^0} \overset{+}{\varphi}_{\pi^-} + \text{H.c.} \qquad (1)$$

where $q_\mu$ is the momentum of the $K^0$ meson and $\beta$ is a real constant, $\beta^2 \approx 0.1$. Assuming for simplicity a form factor which depends on the pion momentum only we obtain for the matrix element $\mathfrak{M}$ (including also the contributions due to $K^0 \rightarrow \pi + \mu + \nu \rightarrow \bar{K}^0$)

$$\mathfrak{M} = \frac{1}{2}\frac{1}{(2\pi)^3} G^2\beta^2\Lambda^2 m_K \qquad (2)$$

(where $m_K$ is the mass of the K meson). With the normalization chosen the matrix element $\mathfrak{M}$ equals the difference in the masses of the $K_1^0$ and $K_2^0$ mesons due to the diagram in question: $\Delta m_K = \mathfrak{M}$. Introducing for $\Delta m_K$ the experimental value we obtain the following estimate

$$\Lambda \sim 0{,}5 m^2/M, \qquad (3)$$

i.e., $\Lambda$ turns out to be of the order of a nucleon mass.

We are thus led to the following conclusions: if the $K^0$ meson decays both according to the mode $K^0 \rightarrow \pi^- + e^+ + \nu$ and the mode $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}$, then it follows from the magnitude of the experi-