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A METHOD OF MEASURING THE MOMENTUM OF ELECTRONS IN A METAL

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A series of size effects, which are determined by the relationships between the size of electron orbits and the size of a metallic sample, can be observed when a sample having a long free path is placed in a constant magnetic field. Phenomena of similar type were discovered a relatively long time ago in dc measurements of conductivity. They can be observed, however, in much more distinct form in measurements of high-frequency impedance, owing to the presence of a supplementary parameter with the dimensions of length—the skin depth δ. A similar effect was first found by Khaĭkin,[1] who discovered the disappearance of cyclotron resonance upon decrease of the field, starting with that field for which the diameter of the electron orbit is comparable with the thickness of the sample.

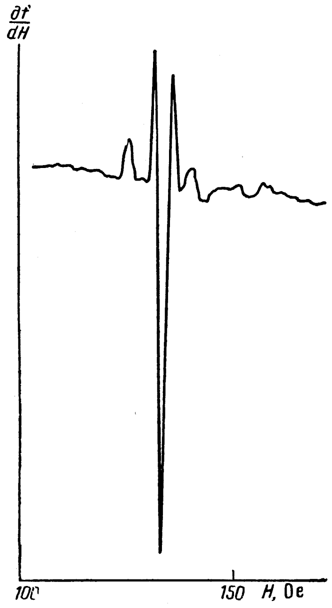
For observation of the size effects, however, one can also use frequencies much lower than in Khaĭkin's experiment, on the order of 10^6 cps, at which there is no cyclotron resonance, but the condition $d \gg \delta$ is well satisfied (d is the diameter of the electron orbit).

Let us consider a flat slab with a constant magnetic field parallel to its surface. The electrons move along helices with axes parallel to the surface of the metal; the major part of the electron trajectories passes deep in the metal, where there is no high frequency field; on returning to the skin layer, the electrons find there a high frequency field in the same phase as it was during the preceding passage through the skin layer. The reason for this is that the field does not have time to change significantly during the time of rotation of the electron in orbit ($\sim 10^{-9}$ sec). The dependence of the impedance on the field does not have, however, a resonance character, since the condition of constant phase of the electric field for all passages of the electron through the skin layer is fulfilled for all fields. When the field is increased the radius of the electron orbit decreases, and the number of returns of the electron through the skin layer during the free path time increases. However, the electron returns to the skin layer only if the diameter of its orbit is less than the thickness of the sample. In the contrary case, it is scattered on the surface of the crystal. Thus for that value of the field at which the orbit diameter of the electron on the extremal section of the Fermi surface becomes equal to the thickness of the slab, a certain singularity should be observed in the field dependence of the impedance. The character of this singularity depends on the variation of the dispersion in the vicinity of the extremal section. For example, using a method analogous to that used by Heine,[2] it is easy to show that for a quadratic dispersion law the curve has a kink, and if the part of the Fermi surface under consideration is a tube of constant cross section, then the singularity is a discontinuity.

We observed this phenomenon experimentally. A flat sample was inserted into the coil of the tank circuit of an oscillator. The cross section of the coil was an elongated rectangle. A constant magnetic field was applied parallel to the surface of the sample. The oscillation frequency varied with the magnitude of field, owing to the variation of the reactive component of the impedance of the sample. The dependence of the frequency on the magnitude of the field was measured by a modulation method; the modulation frequency was 20 cps.

The samples were highly purified monocrystalline tin (about $10^{-4}\%$ impurities), grown from a melt in dismountable crystal molds. At helium temperatures the mean free path of the electrons in the samples reached apparently $(1-3) \times 10^{-1}$ cm: the skin depth at 1—5 Mcs was about 10^{-4} cm.

One of the curves obtained is shown in the figure. The sample had a thickness of 0.54 mm. The [100] axis was perpendicular to the surface of the slab; a high-frequency and a constant magnetic field were directed along [001] axis. The temper-



ature of this experiment was 3.75°K and the frequency was f = 2.8 Mcs.

As expected, the position of the line depends on the thickness of the sample and depends neither on the frequency nor on the inclination of the constant field relative to the surface, within a limit of several degrees. The intensity of the effect increases with decreasing temperature, approximately doubling in the interval from 4.2 to 2.9°K. If the line is recorded more slowly and the modulation amplitude decreased, a fine structure can be observed in the line, but its reproducibility from sample to sample has not yet been investigated. Generally speaking the shape of the line may depend on many factors. For example, it should reflect the fact that when the diameter of the orbit is somewhat smaller than the thickness of the sample the electron passes through the skin layer on both sides of the slab during each revolution, thereby making a supplementary contribution to the conductivity.

In principle, the same phenomenon can be observed at high "cyclotron" frequencies, and it was actually observed by Khaĭkin[3] at $\sim 10^{10}$ cps ("non-resonant orbit cutoff"). In this case, however, the singularity in the dependence of the impedance on the field is connected with the scattering of electrons returning to the skin layer at a phase different from the phase of the field. This aggravates the observation conditions.

The phenomenon described yields, generally speaking, the same information about the Fermi surface as the cyclotron resonance cutoff gives —the magnitude of the Fermi momentum in the extremal cross section in a direction normal to both the surface and the magnetic field vector. By rotating the field in the plane of the sample we can plot the cross section of the Fermi surface in this plane. In our case, however, the phenomenon takes place against a background of a smoother variation of the impedance. That is unquestionably a favorable circumstance and offers hope of obtaining some additional information about the behavior of electrons in a metal. On the other hand, we do not obtain here the effective mass for this cross-section simultaneously with the value of the momentum of the electron. This makes the interpretation of the results and comparison with cyclotron-resonance data more difficult.

The numerical values we obtained agree almost completely with the data of Khaĭkin. In particular, we obtained a value 5.7×10^{-20} g-cm/sec for the diameter of the orbit in momentum space in the [100] direction. The part of the surface referred to is a tube of almost constant cross section, which favors the observation of the effect. However, under the same conditions, we were able to observe other much weaker singularities in the plot of the dependence of $\partial f/\partial H$ on H, in fields of about 90 and 105 Oe, which are probably connected with other portions of the Fermi surface. Thus, the dimensional effect under anomalous skin-effect conditions presents one more convenient way of studying the Fermi surfaces of metals.

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