found earlier,[7,5] and is close to the 3.6×10^{-8} sr^{-1} predicted theoretically for the case of vector current conservation.[5] If we use the angular distribution calculated by Bludman and Young, then, from the data obtained by us, it follows that the total relative probability of the structure-dependent π^+-meson radiative decay, integrated over θ, for which the universal theory[5] gives the value 6×10^{-8} is limited by the inequality

$$\omega_{SD}(\pi^+ \to \gamma + e^+ + \nu)/\omega(\pi^+ \to \mu^+ + \nu) < 1.5 \cdot 10^{-7}.$$

After concluding the experiment described above, we have introduced a number of changes into the electronic circuitry, as a result of which it has been possible to improve considerably the resolution of the coincidence circuits, and to increase the selectivity of the detector of the stopping π^+ mesons. This enabled us to lower the background of chance coincidences by a factor of several times. In a series of experiments carried out with the new apparatus, it was found that the β decay of the π meson exists, and the first result for the value of its probability was confirmed. As a result of the measurement, during which the array was traversed by 0.6×10^{10} π^+ mesons, it was found that

$$\lambda = (1.1^{+1.0}_{-0.5}) \cdot 10^{-8}, \qquad G = (1.14 \pm 0.37)\, G_\beta$$

which confirms the correctness of the vector conservation hypothesis.

In conclusion, we wish to express our thanks to S. S. Gershteĭn, B. Pontecorvo, and O. V. Savchenko for helpful discussion.

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BEHAVIOR OF THE REAL PART OF THE SCATTERING AMPLITUDE AT VERY HIGH ENERGIES

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Submitted to JETP editor March 31, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 1425-1426 (May, 1962)

IN a previous paper[1] it was shown that the Mandelstam equation allows two types of asymptotic behavior of the imaginary part of the amplitude. One type of asymptotic behavior (which is unique in that only this can lead to constant total cross section) agrees precisely with the behavior of the Regge type:[2]*

$$A_s(s,t) = f(t)\, s^{L(t)}. \qquad (1)$$

If the total cross section approaches a constant as $s \to \infty$, then $L(0) = 1$, in agreement with the optical theorem. Below we will always assume that the total cross section approaches a constant: thus $A_s(s, t)$ has the form (1).

In the elastic scattering of isoscalar particles the asymptotic form of the imaginary part of the scattering amplitude in the second channel is easily obtained:

$$A_u(u,\ t) = f(t)\, u^{L(t)} \qquad (2)$$

for $|u| \gg 1$.

With the help of (1) and (2) one can obtain the explicit form of the real part of the amplitude:

$$\operatorname{Re} A(s,\ t) = \frac{f(t)}{\pi}\left\{s^2 \int_4^\infty \frac{s'^{L(t)-2}}{s'-s}\, ds' + (-s)^2 \int_4^\infty \frac{u'^{L(t)-2}}{u'+s}\, du'\right\}. \qquad (3)$$

In the relation (3) we have utilized the fact that if t is finite and $s \to \infty$ then $s \approx -u$. It follows from crossing symmetry that the second subtraction term (proportional to s) vanishes. The possibility of substituting the asymptotic form of the imaginary part into the dispersion relations can be easily verified: the contribution of the "nonasymptotic" region

$$s^2 \int_4^\lambda \frac{A_s(s',\ t)}{(s'-s)\, s'^2}\, ds' + u^2 \int_4^\lambda \frac{A_u(u',\ t)\, du'}{(u'-u)\, u'^2}$$

is of order O(1) (here λ is the "limit" of the region of the asymptotic behavior).

After integration one obtains from (3)

$$\operatorname{Re} A(s,\ t) = f(t)\, s^{L(t)}\, \frac{1+\cos \pi L(t)}{\sin \pi L(t)} + O(1). \qquad (4)$$

According to the optical theorem $f(0) = \sigma_{tot}$. It follows from (4) that the real part vanishes (or grows slower than the imaginary part) for $t = 0$. For other momentum transfers $(t \neq 0)$

$$\operatorname{Im} A(s, t) / \operatorname{Re} A(s, t) = O(1) \qquad (s \to \infty). \tag{5}$$

We can apply these results to the case of πN scattering. The general covariant form of the πN scattering amplitudes is

$$F = F^0 \delta_{\alpha\gamma} + F^1 \tfrac{1}{2} [\tau_\alpha, \tau_\gamma],$$

$$F^i = A^i + \tfrac{1}{2} \gamma (p_1 + p_3) B^i, \qquad i = 0, 1. \tag{6}$$

The four amplitudes have the following properties:

$$A^k(s, u, t) = (-1)^k A^k(u, s, t),$$
$$B^k(s, u, t) = -(-1)^k B^k(u, s, t). \tag{7}$$

The imaginary parts of the amplitudes have the form (1), i.e.,

$$A_s^k(s, t) = f_k(t) s^{L_k(t)},$$
$$B_s^k(s, t) = g_k(t) s^{M_k(t)}, \qquad k = 0, 1. \tag{8}$$

Applying the above method and using (7) and (8) we obtain

$$\operatorname{Re} A^k(s, t) = f_k(t) \frac{1 + (-1)^k \cos \pi L_k(t)}{\sin \pi L_k(t)} s^{L_k(t)},$$

$$\operatorname{Re} B^k(s, t) = g_k(t) \frac{1 - (-1)^k \cos \pi M_k(t)}{\sin \pi M_k(t)} s^{M_k(t)}. \tag{9}$$

It is easy to show that the amplitudes A^k and B^k contribute to the asymptotic behavior if $L_k(0) = 1$ and $M_k(0) = 0$. It follows from (9) that if $L_1(0) = 1$ then the equation $f_1(0) = 0$ must be fulfilled since $\operatorname{Re} A^1(s,t)$ has to be finite at $t = 0$. But then according to (8), $A_s^1(s, 0) = 0$ and in a similar fashion $B_s^1(s, 0) = 0$.

If the asymptotic form of the amplitudes is given by the asymptotic series

$$\sum_{i=1} h_i(t) s^{l_i(t)}, \tag{10}$$

where $h_1(t) s^{l_1(t)}$ is the principal term of the expansion, and $l_1(0) = 1$ but $l_2(0) < 1$, then according to (8) and (9) $A_s^1(s, 0) \neq 0$ and we obtain the following asymptotic estimate:

$$A_s^1(s, 0) = c s^{l_2(0)} \tag{11}$$

and from this

$$\sigma_{tot}^{\pi^+ p} - \sigma_{tot}^{\pi^- p} = c' s^{l_2(0) - 1},$$

i.e., $l_2(0)$ can be measured.

The behavior of $l_2(t)$, the second Regge trajectory, is important for $\pi\pi$ scattering.[4]

The author deems it his pleasure to thank G. Domokos for valuable discussions.

Note added in proof (May 11, 1962). After completing the present paper we learned of the work of Gell-Mann, Frautschi, and Zachariasen (preprint 1962) where these authors arrive independently at similar results concerning the behavior of the real parts of the scattering amplitudes.

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*Our symbols s, u and t agree with those of Mandelstam[3].

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ON THE REMARKS OF S. I. ANDREEV AND M. P. VANYUKOV CONCERNING THE PAPER "EXPANSION OF THE CHANNEL IN INTENSE MINIATURE SPARKS"

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Submitted to JETP editor January 22, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 1427-1429 (May, 1962)

ANDREEV and Vanyukov propose[1] that in experiments reported by us in JETP[2] we have not, in fact, observed the hydrodynamic expansion of a channel of a spark, but rather the passage of a streamer through the observation slit. These authors base their proposal on results reported by Saxe[3] (Saxe's results are discussed in greater detail in [4]). Andreev and Vanyukov propose that to verify this suggestion one should make instantaneous photographs of the interelectrode gap, similar to those of Saxe, who used an exposure of approximately 10^{-9} sec.

Actually, our presently available experimental data contains adequate verification of our interpretation of the results and demonstrates the inapplicability of interpreting the results on the basis of the conclusions given by Saxe. This result is indicated by earlier experiments[5] carried out by us before publication of the paper in question in JETP.[2]