According to the optical theorem $f(0) = \sigma_{tot}$. It follows from (4) that the real part vanishes (or grows slower than the imaginary part) for $t = 0$. For other momentum transfers $(t \neq 0)$

$$\operatorname{Im} A(s, t) / \operatorname{Re} A(s, t) = O(1) \qquad (s \to \infty). \tag{5}$$

We can apply these results to the case of πN scattering. The general covariant form of the πN scattering amplitudes is

$$F = F^0 \delta_{\alpha\gamma} + F^1 \tfrac{1}{2} [\tau_\alpha, \tau_\gamma],$$

$$F^i = A^i + \tfrac{1}{2} \gamma (p_1 + p_3) B^i, \qquad i = 0, 1. \tag{6}$$

The four amplitudes have the following properties:

$$A^k(s, u, t) = (-1)^k A^k(u, s, t),$$

$$B^k(s, u, t) = -(-1)^k B^k(u, s, t). \tag{7}$$

The imaginary parts of the amplitudes have the form (1), i.e.,

$$A_s^k(s, t) = f_k(t) s^{L_k(t)},$$

$$B_s^k(s, t) = g_k(t) s^{M_k(t)}, \qquad k = 0, 1. \tag{8}$$

Applying the above method and using (7) and (8) we obtain

$$\operatorname{Re} A^k(s, t) = f_k(t) \frac{1 + (-1)^k \cos \pi L_k(t)}{\sin \pi L_k(t)} s^{L_k(t)},$$

$$\operatorname{Re} B^k(s, t) = g_k(t) \frac{1 - (-1)^k \cos \pi M_k(t)}{\sin \pi M_k(t)} s^{M_k(t)}. \tag{9}$$

It is easy to show that the amplitudes A^k and B^k contribute to the asymptotic behavior if $L_k(0) = 1$ and $M_k(0) = 0$. It follows from (9) that if $L_1(0) = 1$ then the equation $f_1(0) = 0$ must be fulfilled since $\operatorname{Re} A^1(s,t)$ has to be finite at $t = 0$. But then according to (8), $A_s^1(s, 0) = 0$ and in a similar fashion $B_s^1(s, 0) = 0$.

If the asymptotic form of the amplitudes is given by the asymptotic series

$$\sum_{i=1} h_i(t) s^{l_i(t)}, \tag{10}$$

where $h_1(t) s^{l_1(t)}$ is the principal term of the expansion, and $l_1(0) = 1$ but $l_2(0) < 1$, then according to (8) and (9) $A_s^1(s, 0) \neq 0$ and we obtain the following asymptotic estimate:

$$A_s^1(s, 0) = c s^{l_2(0)} \tag{11}$$

and from this

$$\sigma_{tot}^{\pi^+ p} - \sigma_{tot}^{\pi^- p} = c' s^{l_2(0) - 1},$$

i.e., $l_2(0)$ can be measured.

The behavior of $l_2(t)$, the second Regge trajectory, is important for $\pi\pi$ scattering.[4]

The author deems it his pleasure to thank G. Domokos for valuable discussions.

Note added in proof (May 11, 1962). After completing the present paper we learned of the work of Gell-Mann, Frautschi, and Zachariasen (preprint 1962) where these authors arrive independently at similar results concerning the behavior of the real parts of the scattering amplitudes.

*Our symbols s, u and t agree with those of Mandelstam[3].

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Translated by M. Danos
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ON THE REMARKS OF S. I. ANDREEV AND M. P. VANYUKOV CONCERNING THE PAPER "EXPANSION OF THE CHANNEL IN INTENSE MINIATURE SPARKS"

B. A. DEMIDOV, Yu. F. SKACHKOV, and S. D. FANCHENKO

Submitted to JETP editor January 22, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 1427-1429 (May, 1962)

Andreev and Vanyukov propose[1] that in experiments reported by us in JETP[2] we have not, in fact, observed the hydrodynamic expansion of a channel of a spark, but rather the passage of a streamer through the observation slit. These authors base their proposal on results reported by Saxe[3] (Saxe's results are discussed in greater detail in [4]). Andreev and Vanyukov propose that to verify this suggestion one should make instantaneous photographs of the interelectrode gap, similar to those of Saxe, who used an exposure of approximately 10^{-9} sec.

Actually, our presently available experimental data contains adequate verification of our interpretation of the results and demonstrates the inapplicability of interpreting the results on the basis of the conclusions given by Saxe. This result is indicated by earlier experiments[5] carried out by us before publication of the paper in question in JETP.[2]

1. In the experiment described earlier, [5] in which static and time-resolved spark pictures were compared, the spark circuit was a Lebedev dipole with a total length of 10 mm and a discharge gap of 0.1 mm. The discharge was excited in nitrogen at a pressure of 30 atm. A magnified microscope image of the interelectrode gap was projected simultaneously on the input photocathodes of two electron-optical image converters (EIC) by means of a semitransparent mirror.

The input stage of EIC-1 provided a circular sweep of the spark image at a frequency of 300 Mc with a resolution time of approximately 1×10^{-11} sec. The image in EIC-2, however, was not swept. The EIC-2 served only to intensify the brightness of the spark image since it could not be photographed without amplification.

In Fig. 1 we compare the static and time-swept photographs of the spark; this comparison indicates that the time during which the interelectrode gap is bridged is no greater than 10^{-11} sec. Figure 2b shows what would be expected in 1b if one were to observe "streaming" of the streamer channel from electrode A to electrode B during the time of observation, as proposed by Andreev and Vanyukov. A careful comparison carried out for a large number of static and swept photographs reveals no pattern such as that shown in Fig. 2b. Consequently the exact shape of the

FIG. 1. Comparison of the swept and static pictures of the spark: a) static picture of a spark between electrodes AB (cf. Fig. 2); b) circular sweep of the spark image. One scale division corresponds to 2×10^{-10} sec.

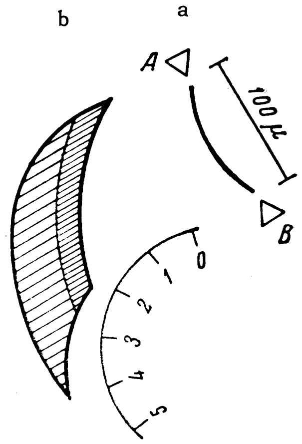

FIG. 2. Diagram showing how the photograph in Fig. 1 would look if the interelectrode gap were bridged in 5×10^{-5} sec.

brightly luminous channel of the spark, which is maintained during the course of the entire subsequent discharge process and fixed on the static photograph (cf. Fig. 1a), is established in a time smaller than the resolving time of the experiment.

It is evident that the described experiment is equivalent to that proposed by Andreev and Vanyukov, i.e., an instantaneous photograph of the initial stage of the discharge; our effective exposure was not 10^{-9} sec (as in the case of Saxe), but approximately 10^{-11} sec.

2. In a series of preliminary experiments swept slit images of the spark, similar to those in [2], were taken with the same interelectrode gap (approximately 10^{-2} cm in nitrogen, hydrogen and other gases at pressures up to 20—30 atm) but with different values of the electric parameters of the discharge circuit. Circuit No. 1 had the least capacitance and represented the Lebedev dipole described above. Circuit No. 2 consisted of two discs 5 mm in diameter with apertures at the center, through which the spark passed. Circuit No. 3 had a capacity of three pF and a higher inductance because of the artificial separation of the conductors connecting the capacity with the spark gap. Circuits No. 4 and No. 5 are described in [2] (capacities of 30 and 6300 pF respectively).

In each case the streak photograph gave a complete history of the production, development, and extinction of the emission from the discharge. The series of experiments that were carried out showed a similar pattern for all circuits, regardless of the difference of time scale. At the beginning, during the first quarter cycle of the oscillation of the discharge circuit, there occurs a rapid expansion of the luminous region, at a rate determined primarily by the parameter U/L. In four circuits having comparable inductance L the initial rate of expansion was found to be several times 10^6 cm/sec. The initial rate of expansion was much smaller (approximately 10^5 cm/sec) in the circuit with the higher inductance (circuit No. 3). In the circuits with comparable inductance the diameter of the luminous region at the end of the initial rapid expansion was proportional to the period of the self-oscillations of the circuit, approximately $1\,\mu$ for circuit No. 1, $5\,\mu$ for circuit No. 2, $100\,\mu$ for circuit No. 4 and 1 mm for circuit No. 5.

After the termination of the first quarter cycle of the oscillations the rate of expansion falls off sharply (one or two orders of magnitude) so that the diameter of the luminous region remains essentially constant during the following process. The general brightness of emission increases in the course of the initial rapid expansion and then remains approximately constant for one or more half cycles of the oscillations of the discharge circuit and then finally falls off sharply (because of the low quality factor of the circuits that were used).

The direct experiment (Sec. 1) and a comparison of experiments described in the present section show that for a given discharge gap the duration of the streamer stage of the discharge is shorter than the resolution time of the experiment described in [2]; the initial diameter of the expanding spark channel is not more than several microns so that it can be neglected in the determination of the expansion velocity.

The work reported by Saxe is based on observations in a spark gap 8 mm long in air, in which case the duration of the streamer stage is approximately 10^{-9} sec and the streamer channel diameter is approximately $100\,\mu$; for this reason the comparison made by Andreev and Vanyukov between our work and Saxe's is not appropriate. First, the discharge gap was 30—80 times shorter in our experiments. Under the assumption that the streamer velocity is the same as for Saxe, in our case the duration of the streamer stage would have to be 4×10^{-11} to 1×10^{-10} sec, that is to say, very much smaller than the resolution time of the experiment in [2]. It is likely that the difference in the streamer diameters arises for the same reason.

Second, in the experiments reported by Saxe, as in all other similar experiments, at the moment the gap is bridged by the streamer channel one observes a sharp increase in the luminous emission of the spark. Such a flash of emission after the initial expansion of the luminous region is not observed in any of the photographs obtained in our entire series of experiments. This result also indicates that in [2] we have been observing the channel stage of the spark exclusively.

Thus the comparison of the results obtained by Saxe serves to verify our analysis of the results that had been based on preliminary experiments.

3. In [2] we have estimated the derivative $(dI/dt)_0$ by direct measurement of the quantity U/L, pointing out how the estimate was made and indicating its limitations. Obviously, if account were taken of the effect of the time variation of the impedance of the spark, for example, indicating the applicability to our case of the results in [6], then the estimate would be precise. However, a number of considerations, in particular comparison with the results of other authors (cf. the table in [2]), show that the reported estimate is of the proper order of magnitude and that it gives a correct qualitative picture of the process in the spark; the possibility of any further improvement in accuracy is relatively small.

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Translated by H. Lashinsky
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