THEORY OF ACOUSTIC CYCLOTRON RESONANCE IN METALS

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A theoretical study is made of the features of the bulk cyclotron resonance (C.R.) excited in metals by high frequency sound ($\omega t_0 \gg 1$, where ω is the frequency, and t_0 is the relaxation time). The form, width and position of the resonance peaks of the acoustical absorption coefficient are studied as functions of the frequency, the magnetic field, the temperature, the topology and the shape of the Fermi surface, etc. The usual cyclotron and magneto-acoustic resonances are closely interrelated in acoustic cyclotron resonance (A.C.R.). In distinction from electromagnetic C.R., it is possible to have A.C.R. when **k** and **H** are not perpendicular to one another. Experimental observation of A.C.R. allows one to determine not only the extremal values of the effective masses and diameters on the Fermi surface, but also the effective masses and mean velocities on an arbitrary cross section, the direction and period of open trajectories, etc. The theory is in good agreement with recent A.C.R. observations by Roberts[7] on gallium.

1. INTRODUCTION

UNTIL recently most experimental and theoretical work concerned with the absorption of sound in metals in a magnetic field involved comparatively low acoustical frequencies, where $\omega t_0 \ll 1$ (ω is the angular frequency of the sound and t_0 is the characteristic relaxation time of the electrons). In this frequency range various magneto-acoustic effects occur which are associated with the spatial periodicity of the acoustic field in the metal.[1-5]

These effects include, first, Pippard-type[2] oscillations of the absorption coefficient α, first observed by Bömmel[1] and studied theoretically by V. Gurevich.[3] They appear when **k** is perpendicular to **H** (**k** is the wave vector of the sound and **H** is the magnetic field) and the period in the inverse field is directly related to the extremal dimensions of the electron orbit in the propagation direction of sound. Second, resonant oscillations of α should occur when **k** and **H** are not perpendicular, owing to the drift motion of electrons in the direction of **k**.[4] The period of these resonant oscillations is determined[4] by the extremal values of $\bar{v}_k/\Omega$, the mean electron displacement in the direction of **k** averaged over the orbit. Also related are the resonant oscillations of the absorption with **k** perpendicular to **H**, due to open periodic trajectories; these oscillations were predicted theoretically and observed experimentally by Galkin and Korolyuk.[5] Finally, the acoustic absorption coefficient as a function of H^{-1} undergoes periodic increases (jumps) of various types, which have been studied in [3-5].

In all these effects there is no change of the acoustic field in a time of one mean free path ($\omega \ll v \approx 1/t_0$). The alternating field can, therefore, be considered as static at the electron trajectory, and only the spatial periodicity need be considered. At high frequencies ($\omega t_0 \gg 1$) the resonance conditions depend significantly on the frequency of the sound.

The present work is a theoretical study of acoustic attenuation at high frequencies when bulk cyclotron resonance (C.R.) is excited in the metal at the multiple frequencies $\omega = n\Omega$ (Ω is the cyclotron frequency). Mikoshiba[6] has considered the possibility of acoustical cyclotron resonance (A.C.R.). Recently Roberts[7] observed A.C.R. in single crystals of gallium at $\omega/2\pi = 115$ Mc. The observation of A.C.R. at such a comparatively low frequency was possibly because of the large mean free path ($t_0 \sim 10^{-8}$ sec, $\omega t_0 \sim 5$). Roberts used a very ingenious method to separate A.C.R. from Pippard-type oscillations: if the polarization of the sound is changed (with ω = const), the wavelength changes, and thus the Pippard-type maxima are displaced, while the A.C.R. maxima are not displaced. Thus, in a single experiment, one is able to determine not only the extremal diameters, but also the effective masses. Apparently, Reneker also observed A.C.R. in Bi.[8] There is no detailed theoretical analysis of these effects in the literature at present.

The characteristic features of A.C.R. are associated with the fact that the acoustic wave vector **k** has a definite value in the metal, in distinction from C.R. in an electromagnetic field, where, owing to the skin effect, waves with various values of k are propagated into the metal. It is obvious that the vector **k** has a well-defined value if the acoustic absorption coefficient α is small compared with the inverse wavelength. In the absence of a magnetic field[9] $\alpha \sim (n\mu\omega)/(\rho v s^2)$ (where n, μ, and v are the density, Fermi energy and velocity of the electrons, $s = \omega/k$ is the velocity of sound, and ρ is the density of the metal). For an electron concentration of the order of one electron per atom, the condition $\alpha \ll k$ will be satisfied for metals with mass numbers $A \gtrsim 10$ ($\alpha/k \sim mv/Ms \sim A^{-1}$, where M is the mass of the ion), i.e., practically always.

The physical mechanism of A.C.R. is easily understood. If the vectors **k** and **H** are mutually perpendicular, the mean displacement of an electron along **k** is zero in a precession period, and an electron interacts most effectively with the alternating field when it falls in a plane of constant phase of the sound wave at a suitable moment ($\omega = n\Omega$). The advancing front of the sound wave acts as the "skin layer" (or gap in a cyclotron). After a period $2\pi/\Omega$ of precession in the magnetic field the electron is situated in equivalent constant-phase planes of the advancing sound wave, so that resonance occurs. The resonance is most clearly displayed when the frequencies Ω are equal for all electrons, i.e. for quadratic dispersion. When the dispersion law for the electrons is not quadratic, as is usual,[10] the electrons which resonate are those with extremal cyclotron frequencies $\Omega_{ext} = eH/cm_{ext}$ (i.e., extremal effective masses).

When the vectors **k** and **H** are not perpendicular to each other, the resonance is complicated by the Doppler effect, because the Doppler frequency shift $\mathbf{k} \cdot \bar{\mathbf{v}} = k_H \bar{v}_H$ is different for electrons with different projected velocities $\bar{v}_H(p_H)$. Those electrons for which the value of $(\omega - k_H \bar{v}_H)/\Omega$ is extremal participate in the resonance. Resonance is associated both with the temporal and the spatial periodicity of the acoustic field. The analysis performed below shows that the experimental study of A.C.R. allows one to obtain information on the extremal diameters and extremal effective masses of electrons at the Fermi surface, and also to determine the effective mass and mean velocity at any section of it.

The present work is a direct continuation of previous work[4] which is referred to below as I.

2. ABSORPTION OF SOUND IN ZERO MAGNETIC FIELD

The acoustic attenuation when H = 0 and $\omega t_0 \gg 1$ was determined by Akhiezer, Kaganov and Lyubarskiĭ,[9] who calculated the quantum mechanical probability of the direct absorption by an electron of a quantum of the external acoustical field. In order to avoid certain calculations in what follows, it is convenient for us to obtain the same results using the classical kinetic equation.

The complete system of equations consists of the kinetic equation for the electronic distribution function, Maxwell's equation for the eddy fields, and the condition for electrical neutrality in the metal:

$$\partial\chi/\partial t + (\mathbf{v}\nabla)\chi + (\delta\chi/\delta t)_{st} = \mathbf{v}(\nabla\delta\varepsilon + e\mathbf{E}), \quad (2.1)$$

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = -4\pi c^{-2}\partial\mathbf{j}/\partial t, \quad (2.2)^*$$

$$\rho' \equiv -2eh^{-3}\int d^3p\,\chi\partial f_0/\partial\varepsilon = ne\operatorname{div}\mathbf{u}, \quad (2.3)$$

$$\mathbf{j} \equiv -2eh^{-3}\int d^3p\,\mathbf{v}\chi\partial f_0/\partial\varepsilon. \quad (2.4)$$

Here $\chi\partial f_0/\partial\epsilon$ is the non-equilibrium contribution to the Fermi distribution function $f_0(\epsilon) = [e^{(\epsilon-\mu)/T}+1]^{-1}$; $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity, $\epsilon(\mathbf{p})$ is the energy, **p** is the momentum, μ is the chemical potential, e is the charge, and ρ' is the non-equilibrium part of the charge density of the electrons; the energy of interaction of the electrons with the acoustic wave is

$$\delta\varepsilon = \lambda_{ik}(\mathbf{p})\,u_{ik} + p_i v_k \partial u_i/\partial x_k + \mathbf{p}\dot{\mathbf{u}}.$$

In distinction from the low frequency case ($\omega t_0 \ll 1$, $kl \ll 1$) where the kinetic equation is linearized with respect to the instantaneous state of equilibrium in the alternating external field,[11] the equation at high frequencies is linearized with respect to $f_0(\epsilon)$. The coupling of the electrons with the sound is due to the force $-\nabla\delta\epsilon$ acting on an electron. The absorption coefficient has the usual form[3,4].

$$\alpha = \frac{1}{h^3 W}\int d^3p\,\chi^*\left(\frac{\delta\chi}{\delta t}\right)_{st}\left(-\frac{\partial f_0}{\partial\varepsilon}\right), \quad (2.5)$$

where W is the energy flux density in the acoustic wave.

Because the relaxation time does not appear in the answer, we replace the collision integral by a relaxation time t_0:

$$(\partial\chi/\partial t)_{st} = \nu\chi \quad (\nu = 1/t_0).$$

For a plane monochromatic wave

$$\chi = g(\nu - i\omega + i\mathbf{k}\mathbf{v})^{-1}, \qquad g = \mathbf{v}(\nabla\delta\varepsilon + e\mathbf{E}). \quad (2.6)$$

*rot = curl.

From (2.3) we find the longitudinal component of the field, $\mathbf{E}\cdot\mathbf{k}/k$, and from (2.2) and (2.4) the eddy part. We use the relation

$$(\nu - i\omega + i\mathbf{kv})^{-1} = \pi\delta(\omega - \mathbf{kv}) + iP(\omega - \mathbf{kv})^{-1}$$

(P signifies the principal part), because $|\omega - \mathbf{k}\cdot\mathbf{v}| \gg \nu$.

We find after straightforward calculation:

$$\alpha = \frac{\pi}{h^3 W}\int d^3\mathbf{p}\,\delta(\varepsilon - \mu)\,\delta(\omega - \mathbf{kv})\,|\delta\dot{\varepsilon} - \langle\delta\dot{\varepsilon}\rangle - e\mathbf{E}_\perp\mathbf{v}_\perp|^2, \quad (2.7)$$

where the brackets $\langle\ \rangle$ imply averaging over the Fermi surface:

$$\langle\varphi\rangle = \int d^3\mathbf{p}\,\delta(\varepsilon - \mu)\,\varphi \Big/ \int d^3\mathbf{p}\,\delta(\varepsilon - \mu).$$

The components of the eddy field are determined from the system of equations

$$\left(\frac{(kc)^2}{4\pi i\omega} - \hat{\sigma}_\perp\right)\mathbf{E}_\perp = \frac{2ie}{h^3}P\int d^3\mathbf{p}\,\frac{1}{\mathbf{kv}}\mathbf{v}_\perp\,\delta(\varepsilon - \mu)(\lambda_{ik}\dot{u}_{ik} - \langle\delta\dot{\varepsilon}\rangle). \quad (2.8)$$

It is apparent that the absorption is determined only by the symmetrical part of the tensor $\partial u_i/\partial x_k$. The components of the electrical conductivity tensor $\sigma_\perp$ have the form

$$\sigma_{\alpha\beta} = 2\pi e^2 h^{-3}\int d^3\mathbf{p}\,\delta(\varepsilon - \mu)\,\delta(\omega - \mathbf{kv})\,v_\alpha v_\beta. \quad (2.9)$$

It follows from (2.8) and (2.9) that the contribution to the absorption from the eddy fields depends markedly on the ratio between the acoustical wavelength $\lambda = 2\pi k^{-1}$ and the effective length of an electromagnetic wave in the metal $\delta = (c^2/4\pi\omega\sigma)^{1/2}$. This circumstance has been remarked upon by Gurevich and Pippard. If

$$\beta = (kc)^2/4\pi\omega\sigma \sim (v/s)(\delta_0/\lambda)^2 \qquad (\delta_0^2 = mc^2/4\pi ne^2) \quad (2.10)$$

is small compared with unity, the contribution of the electrical fields to α can be neglected. In the converse limiting case the eddy fields make a contribution to the absorption comparable in order of magnitude with the deformation absorption. For "good" metals, the inequality $\omega t_0 \gg 1$ is satisfied only for frequencies $\omega/2\pi \sim 10^9$ to 10^{10} cps and β already becomes of the order of unity for $\omega/2\pi \sim 10^8$ cps ($n \sim 10^{22}\,\mathrm{cm}^{-3}$, $v/s \sim 10$). It is clear that under these conditions the eddy fields can be neglected.

3. ACOUSTIC CYCLOTRON RESONANCE WHEN k AND H ARE MUTUALLY PERPENDICULAR

We consider the absorption of sound in an external magnetic field. In the system (2.1) — (2.4) only the form of the kinetic equation changes:

$$[-i(\omega - \mathbf{kv}) + \nu]\,\chi + \partial\chi/\partial\tau = \mathbf{v}(\nabla\delta\varepsilon + e\mathbf{E}) + \partial\delta\varepsilon/\partial\tau. \quad (3.1)$$

Here $\partial/\partial\tau = e\mathbf{H}c^{-1}[\mathbf{v}\times\partial/\partial\mathbf{p},]$ τ describes the time of motion of the electron in its orbit in the magnetic field. The appearance of the term $\partial\delta\epsilon/\partial\tau$ is associated with the fact that the Lorentz force $ec^{-1}\partial\epsilon/\partial\mathbf{p}\times\mathbf{H}$ is determined by the total velocity $\mathbf{v} + \partial\delta\epsilon/\partial\mathbf{p}$. This term must be retained in the resonance region $\omega \sim \Omega$ ($\Omega = eH/mc$ is the cyclotron frequency, $m = (2\pi)^{-1}\partial S/\partial\epsilon$ is the effective mass[12]). We denote the righthand side of (3.1) by $g(\epsilon, p_z, \tau)$. We will not calculate the electrical fields, because their contribution to the absorption is the same as, or less than that of terms in $\delta\epsilon$, which, in their turn, are known only in order of magnitude. The resonance effects considered below are determined only by the kinematics of electrons with an arbitrary dispersion law in a magnetic field. The magnitude of the attenuation depends only on the magnitude and explicit form of g. Because the electrons absorb sound when $\mathbf{k}\cdot\mathbf{v} = \omega$, we take $|g| \sim |\delta\dot{\epsilon}| \sim \mu\omega ku$.

Using the results of I, it is not difficult to write down the expression for the absorption coefficient:

$$\alpha - \alpha_0 = eH(h^3Wc)^{-1}\,\mathrm{Re}\int dp_z\{1 - \exp(2\pi i\beta - 2\pi\gamma)\}^{-1} \times \sum_{\substack{0<\tau_\alpha<T \\ 0<\tau_\alpha-\tau_\beta\leqslant T}} g_\alpha^* J_\alpha^* g_\beta J_\beta \exp\left[\int_{\tau_\alpha}^{\tau_\beta}(\nu - i\omega + i\mathbf{kv})\,d\tau\right], \quad (3.2)$$

where

$$\alpha_0 = \pi(h^3W)^{-1}\int d^3p\,|g^2|\,\delta(\varepsilon - \mu)\,\delta(\omega - \mathbf{kv}),$$

$$\beta = (\mathbf{k}\overline{\mathbf{v}} - \omega)/\Omega, \qquad \gamma = \overline{\nu}/\Omega, \qquad T(p_z) = 2\pi/\Omega, \quad (3.3)$$

$$J_\alpha = \int_{-\infty}^{\infty} d\tau\,\exp\left[\tfrac{1}{2}i(\mathbf{kv}'_\alpha)\tau^2 + \tfrac{1}{6}i(\mathbf{kv}''_\alpha)\tau^3\right]. \quad (3.4)$$

The index α signifies that the function is evaluated at the point $\tau = \tau_\alpha(p_z)$ (τ_α is the root of the equation $\mathbf{kv}(\tau, p_z) = \omega$ in the range $0 < \tau_\alpha < T$). The bar signifies averaging over the period:

$$\overline{\psi} = T^{-1}\int_0^T d\tau\,\psi(\tau).$$

The term in $\mathbf{k}\cdot\mathbf{v}''_\alpha$ can be neglected if $|\mathbf{k}\cdot\mathbf{v}''_\alpha|^2 \ll |\mathbf{k}\cdot\mathbf{v}'_\alpha|^3$. In formula (3.3) we have used the identity

$$eHc^{-1}d\tau\,dp_z\,d\varepsilon = d^3\mathbf{p}$$

and replace J_α by the expression

$$J_\alpha = (2\pi/|\mathbf{kv}'_\alpha|)^{1/2}\exp\left(\tfrac{1}{4}\pi i\,(\mathrm{sign}\,\mathbf{kv}'_\alpha)\right).$$

All the resonance effects are associated with the presence of the resonance multiplier B, which is

$$B(p_z) = [1 - \exp(-2\pi\gamma - 2\pi i\beta)]^{-1}. \quad (3.5)$$

1) A.C.R. should appear most clearly when the wave vector **k** is perpendicular to **H**. In this case for closed trajectories $\mathbf{k}\cdot\bar{\mathbf{v}} = 0$ and

$$B(p_z) = [1 - \exp\{-2\pi(i\omega + \bar{\nu})/\Omega(p_z)\}]^{-1}. \quad (3.6)$$

For a quadratic dispersion law $\epsilon(\mathbf{p}) = \mu_{ik}p_i p_k/2$, the cyclotron frequency $\Omega = eHc^{-1}(\mu_{xx}\mu_{yy} - \mu_{xy}^2)^{1/2}$ does not depend on p_z, and if the scattering is due to impurities, then $\nu(p_z)$ is also constant. In this case B does not depend on p_z, and can be taken outside the integral sign to obtain the result

$$\alpha = \alpha_0 \,\mathrm{Re\,cth}\,[\pi(i\omega + \bar{\nu})/\Omega]. \quad (3.7)^*$$

Resonance occurs at the multiple frequencies $\omega = n\Omega$. Near to resonance

$$\alpha = \alpha_0\bar{\nu}\Omega/\pi[(\omega - n\Omega)^2 + \bar{\nu}^2]. \quad (3.8)$$

For $\omega = n\Omega$

$$\alpha_n \approx \alpha_0\omega/\pi n\bar{\nu}. \quad (3.9)$$

A.C.R. is modulated by the usual magnetoacoustic resonance of the Pippard type[2,3] (or, as it is still called, geometric resonance) associated with the presence of oscillating exponents in the cross terms ($\tau_\alpha \neq \tau_\beta$) under the integral in (3.2). For an ellipsoid

$$\alpha = \alpha_0 \,\mathrm{Re\,cth}\,[\pi(i\omega + \bar{\nu})/\Omega] + \frac{4\pi eH}{h^3 Wck}\,\mathrm{Re}\,B\int dp_z\,|g_1 g_2|\,|v'_{k1}v'_{k2}|^{-1/2}\sin\left(\int_{\tau_1}^{\tau_2}(\mathbf{kv} - \omega)\,d\tau\right). \quad (3.10)$$

Gurevich showed[3] that the last term causes oscillations of α, the period of which is determined by the extremum with respect to p_z of the phase

$$\int_{\tau_1}^{\tau_2} d\tau\,(\mathbf{kv} - \omega) \approx (kc/eH)\,(p_y(\tau_2) - p_y(\tau_1))$$

(**k** is along the x axis, **H** along the z axis). The relative amplitude of these oscillations is of the order $(kD)^{-1/2}$, where $D = cd_{max}/eH$ (d_{max} is the maximal diameter of the Fermi surface).

2) It is well known that most metals have a complex non-quadratic dispersion law. In this case the effective mass depends significantly on p_z, and resonance is "smeared out." We shall consider the case when there are only two points τ_1, τ_2, where $\mathbf{k}\cdot\mathbf{v}(\tau) = \omega$ on the curve $\epsilon(\mathbf{p}) = \mu$, p_z = const. We write α as

$$\alpha = \alpha_1 + \alpha_2,$$

where

$$\alpha_1 = \frac{eH}{2h^3Wc}\,\mathrm{Re}\int dp_z\,\mathrm{cth}\,\pi(\gamma + i\beta)\sum_{\alpha=1}^{2}|g_\alpha J_\alpha|^2; \quad (3.11)$$

$$\alpha_2 = \frac{2eH}{h^3Wc}\,\mathrm{Re}\int dp_z B(p_z)\,\mathrm{Re}\left\{g_1^* g_2 J_1^* J_2 \exp\left[i\int_{\tau_1}^{\tau_2}(\mathbf{kv} - \omega)\,d\tau\right]\right\}. \quad (3.12)$$

The expression for α_1 can be transformed if Eq. (2.10) of I is used for J_α; in fact, we have

$$\alpha_1 = \frac{\pi}{h^3W}\,\mathrm{Re}\int d^3\mathbf{p}\,|g^2|\,\delta(\varepsilon - \mu)\,\delta(\mathbf{kv} - \omega)\,\mathrm{cth}\,\pi\frac{i\omega + \bar{\nu}}{\Omega} = \frac{\pi}{h^3Wk}\,\mathrm{Re}\int_{\mathbf{kv}=\omega} d\varphi\,|g^2|\,\frac{1}{Kv}\,\mathrm{cth}\,\pi\frac{i\omega + \bar{\nu}}{\Omega}, \quad (3.13)$$

where φ is the azimuthal angle in velocity space, with the polar axis along **k**; integration proceeds along the curve $\omega = \mathbf{k}\cdot\mathbf{v}$ on the Fermi surface, and K is the Gaussian curvature.

The values of φ close to the extremum $\Omega^{-1}(\varphi) \sim m(\varphi)$ make the principal contribution to α_1. If ω and H are such that $\omega \approx n\Omega_{ext}$, then

$$\alpha_{1\,res} = \frac{2\pi}{h^3Wk}\sum_{(ext)}\left(\frac{|g^2|}{Kv}\right)_{ext} M(\Delta, b, \xi),$$

$$\xi = \bar{\nu}/\omega, \quad \Delta = (\omega - n\Omega_{ext})/\omega, \quad b = (2m)^{-1}(\partial^2 m/\partial\varphi^2)_{ext}. \quad (3.14)$$

The symbol Σ_{ext} signifies summation of the expressions at all points φ_μ for which $\Omega(\varphi_\mu) = \Omega_{ext}$;

$$M(\Delta, b, \xi) = \frac{1}{\pi n}\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{1}{\xi + i(\Delta + b\varphi^2)}\,d\varphi = \frac{1}{n}\left|\frac{(\xi^2 + \Delta^2)^{1/2} - \Delta\,\mathrm{sign}\,b}{b(\xi^2 + \Delta^2)}\right|^{1/2}. \quad (3.15)$$

Formula (3.15) describes the form of the resonance line. The maximum of M, which is $3^{3/4}(2n|\xi b|^{1/2})^{-1}$, is attained when $\Delta = -3^{-\frac{1}{2}}\xi$ sign b. At resonance the absorption is $(\omega/\bar{\nu})^{1/2}$ times smaller than for a quadratic dispersion law:

$$\alpha_{1n} \sim \alpha_0 n^{-1}(\omega/\bar{\nu})^{1/2}. \quad (3.16)$$

The line has an asymmetrical shape; in fact, for $|\Delta| \gg \xi$ and $\Delta b < 0$ the value is $M \approx \sqrt{2}n^{-1}|\Delta b|^{-1/2}$, whereas for $|\Delta| \gg \xi$, $\Delta b > 0$, the value is $M \approx \xi[n|2b|^{1/2}|\Delta|^{3/2}]^{-1}$ which is significantly smaller (in the ratio $\sim\xi/|\Delta|$) than on the other side of the line. The asymmetry manifests itself on the resonance curve in the fact that the absorption is different at different minima.

An effective-mass extremum with respect to φ is attained at least at a central section and a point of contact. When studying cyclotron resonance at a point of contact some caution is required, be-

*cth = coth.

cause, when **k** is perpendicular to **H**, the quantity $\mathbf{k}\cdot\mathbf{v}$ tends to zero, and the use of formula (3.2) obtained by the method of stationary phase with respect to kr requires justification. Analysis shows that close to this point the phase $\int_{\tau_1}^{\tau_2}\mathbf{k}\cdot\mathbf{v}d\tau$ is of order $kr\,|\delta p_z/p_{max}|^{1/2}$. The relative width of the interval $|\delta p_z/p_{max}|$, which produces resonance, is, of course, of order ν/ω. Therefore, for $kr \gg (\omega t_0)^{1/2}$, the results given are also valid for a contact point of elliptic type. At a hyperbolic (saddle) contact point (such points can occur only on a non-convex Fermi surface), where the effective mass tends to infinity,[12] there is no C.R. In the inverse limiting case there is no resonance at the frequency corresponding to the elliptic contact point.

When evaluating α_2, which causes modulation of the C.R. by oscillations of the geometric resonance type, we must consider the competition between the two "sharp" functions under the integral; the resonance function and the rapidly oscillating exponential function. Close to the central section the "sharpness" of $B(p_z)$ is characterized by the quantity

$$|\delta p_z| \sim p_{max}(\nu/\omega)^{1/2},$$

and the "sharpness" of the exponential by the quantity

$$|\delta p_z| \sim p_{max}(kr)^{-1/2} \sim p_{max}(s/v)^{1/2}.$$

Because under actual conditions at frequencies $\omega/2\pi \sim 10^9 - 10^{10}$ cps and $\nu \sim 10^8 - 10^9$ cps the ratio $\omega s/\nu v$ is small, the exponential must be considered a "sharper" function than $B(p_z)$. Therefore, the contribution from $p_z = 0$ has the form

$$\alpha_2 = \frac{2(2\pi)^{3/2}eH}{h^3Wc}\left(\frac{|g_1^2|\,\mathrm{Re}\,B}{|\mathrm{kv}_1'|\,kD''|^{1/2}}\right)_{p_z=0}\sin\left(kD-\frac{\pi}{4}\right)$$
$$\sim \frac{\alpha_0}{(kD)^{1/2}}\mathrm{Re}\,B\sin\left(kD-\frac{\pi}{4}\right), \qquad (3.17)$$

where $D = cd/eH$, $d = 2p_{max}$ is the central diameter of the Fermi surface in the direction $\mathbf{k}\times\mathbf{H}$ and $D'' = (\partial^2 D/\partial p_z^2)_{p_z=0}$. The relative value of the second term (α_2) is small in comparison with α_1: close to resonance it is of the order $(s\omega/v\nu)^{1/2} \ll 1$ (by hypothesis), and far from it, of the order $(kD)^{-1/2} \sim (s/v)^{1/2}$.

When the resonance function $B(p_z)$ is sharper than the oscillating exponent, the absorption coefficient is

$$\alpha = \frac{2\pi eH}{h^3Wc}\left|\frac{g_1^2}{\mathrm{kv}_1'}\right|\left[1+\sin\left(kD-\frac{\pi\omega}{\Omega}\right)\right]M(\Delta, b, \xi), \quad (3.18)$$

where $b = (2m)^{-1}\partial^2 m/\partial p_z^2$. The values of all functions are taken at the central section $p_z = 0$; M is determined by formula (3.15). In this case the modulation is very pronounced, and geometric resonance can cause the absorption to fall almost to zero (see also [4]). However, to observe the effect extremely high acoustic frequencies and very pure specimens are required in order to satisfy the condition $\omega t_0 \gg v/s \sim 10^2 - 10^3$.[1)]

When C.R. occurs at the central section or at a contact point, the oscillating exponential in (3.12) is the sharpest function ($|\delta p_{z\,exp}/\delta p_{zB}| \sim (\omega t_0)^{1/2}/kD \ll 1$) and the absorption coefficient α_2 is given by a formula of the type (3.17).

4. CYCLOTRON RESONANCE WHEN THE VECTORS k AND H ARE NOT MUTUALLY PERPENDICULAR

a) We now study the resonance features of the acoustic absorption when the angle between **k** and **H** differs significantly from a right angle, and $\mathbf{k}\cdot\mathbf{v}$ is not zero. In this case combined cyclotron and magneto-acoustic resonance should be observed when $|\omega \pm \mathbf{k}\cdot\bar{\mathbf{v}}| = n\Omega$. If Ω is changing relatively slowly, then, on a closed convex Fermi surface, the phase of the resonance index $(\omega - \mathbf{k}\cdot\bar{\mathbf{v}})/\Omega$ has no extremum with respect to p_z in the range of values of p_z where $\mathbf{k}\cdot\mathbf{v}(\tau, p_z) = \omega$. The resonant contribution to the absorption is provided by electrons close to the limiting values $p_z = p_{z\,lim}$ ($p_{z\,lim}$ is the value of p_z for which the two points τ_1 and τ_2 merge into one another). Using the results of I we can at once write down an expression for the attenuation:

$$\alpha_{lim} = \frac{eH\,|g^2|}{6h^3Wc\,|\beta'|}\left(\frac{6}{|\mathrm{kv}''|}\right)^{2/3}\Gamma^2\left(\frac{1}{3}\right)\left\{1+\frac{2\,\mathrm{sign}\,\beta'}{\pi}\mathrm{arc\,tg}\,\Delta/\gamma\right\}, \qquad (4.1)^*$$

where

$$\beta = (\mathbf{k}\bar{\mathbf{v}} \pm \omega)/\Omega, \qquad \beta' = d\beta/dp_z, \qquad \mathrm{kv}'' = kd^2\mathrm{v}/d\tau^2,$$

$\Delta = |\beta| - n$ is the "detuning" of the resonance. The multiplier in front of the curly brackets is of order $\alpha_0(kr)^{-2/3}$. Formula (4.1) describes periodically occurring increases of α. For $\beta'\Delta > 0$, $|\Delta| \gg \gamma$ the expression in the curly brackets is equal to two, and for $\beta'\Delta < 0$ it is small ($\sim\gamma/|\Delta|$). The variation of α_{lim} on H is shown schematically in the figure. The curve has a saw-toothed

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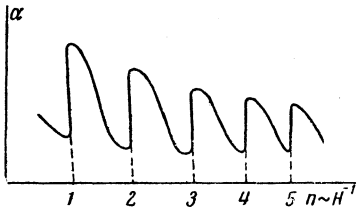
1)An analogous situation can occur also in the electromagnetic field. Averaging over the wave vector due to the skin effect leads in this case, as shown by Azbel',[13] to resonance spreading of the field into the bulk of the metal and a number of other interesting effects.

*arctg = $\tan^{-1}$.



The schematic form of the variation with the absorption at the limiting point $\alpha_{\lim}$ with the magnetic field when $d\beta/dp_z > 0$.

form. The magnitude of an individual jump is

$$\Delta\alpha = \frac{eH\,|g^2|}{3h^3Wc\,|\beta'|}\left|\frac{6}{kv''}\right|^{2/3}\Gamma^2\left(\frac{1}{3}\right) \sim \alpha_0\,(kr)^{-2/3}. \quad (4.2)$$

The absorption increases with increasing magnetic field when $\beta' < 0$, and decreases when $\beta' > 0$. The derivative curve $d\alpha/dH$ should show resonance spikes of Lorentzian form:

$$d\alpha/d\ln H \sim \alpha_0\,(kr)^{1/3}\,\mathrm{sign}\,\beta'\cdot\gamma\,/(\Delta^2 + \gamma^2). \quad (4.3)$$

In distinction from the low frequency case $\omega \ll \nu$, when the quantity ω was neglected in the expression for $\beta = |\mathbf{k}\cdot\mathbf{v} - \omega|/\Omega$, and both the limiting points $p_z = \pm p_{z\,\lim}$ gave equal absorption, at high frequencies the resonance spikes from both limiting points are split up and give two systems of resonance peaks. Their position and periods are different:

$$(\Delta H^{-1})_\pm = e/m_{\lim}c\,|\mathbf{kv} \pm \omega|_{\lim},\ \beta_\pm = n. \quad (4.4)$$

From the splitting of the resonances it is possible to determine directly the effective mass m and $\mathbf{k}\cdot\bar{\mathbf{v}}/k$ at the limiting point for a given orientation of the vectors **k** and **H**. In other words, by changing the mutual orientation of the vectors **k** and **H**, one can, in practice, find the effective mass and the mean velocity (over the period) at any section of the Fermi surface.

b) When the Fermi surface is not convex, the function $\beta(p_z) = (\mathbf{k}\cdot\mathbf{v} \pm \omega)/\Omega$ can attain an extremum with respect to p_z inside and at the limit of the range of integration in (3.2), which is defined by the values $-p_{z\,\lim} < p_z < p_{z\,\lim}$. For values of ω and H such that β_{ext} is close to a whole number, resonant oscillations arise:

$$\alpha = \frac{\pi eH}{h^3Wc}\left|\sum_\alpha |\mathbf{kv}'_\alpha|^{-1/2} g_\alpha \exp\left(\int_0^{\tau_\alpha} (\mathbf{kv} - \omega)\,d\tau\right)\right|^2_{ext} nM(\Delta, b, \gamma);$$

$$\Delta = \beta_{ext} - n, \quad \beta_{ext} = (\mathbf{k}\bar{\mathbf{v}} \pm \omega)/\Omega\,|_{ext},$$

$$b = \tfrac{1}{2}(\partial^2\beta/\partial p_z^2)_{ext}, \quad |\Delta|,\ \gamma \ll 1. \quad (4.5)$$

A simple calculation leads to the result

$$\alpha = \alpha'\left[\frac{(\gamma^2+\Delta^2)^{1/2} - \Delta\,\mathrm{sign}\,b}{\gamma^2+\Delta^2}\right]^{1/2}, \quad (4.6)$$

$$\alpha' \sim \alpha_0\,(kr)^{-1/2}. \quad (4.7)$$

The relative width and shape of the resonance curve is the same as in the case of A.C.R. for non-quadratic dispersion [see (3.14) and (3.15)]. The difference between resonant oscillations of this type and those which occur at low frequencies $\omega \ll \nu$,[4] lies in the splitting of the resonant frequencies at symmetrical points (similar to that treated above for limiting points).

We might remark that for some angle of inclination the limiting point coincides with an extremum of the function $\beta(p_z)$. Then formula (4.6) retains its form, but the value of α' is increased:

$$\alpha' = \frac{eH\,|g^2|}{3h^3Wc\,|b|^{1/2}}\left|\frac{6}{kv''}\right|^{2/3}\Gamma^2\left(\frac{1}{3}\right) \sim \frac{\alpha_0}{(kr)^{1/6}}. \quad (4.8)$$

Finally, the extremum of the function $\beta(p_z)$ can be attained in the range of values of p_z where there are no solutions of the equation $\mathbf{k}\cdot\bar{\mathbf{v}}(\tau, p_z) = \omega$. Straightforward analysis shows that in this case the distribution function of the electrons, and, consequently, the coefficient of absorption, have no resonance features (there is no resonance multiplier $B(p_z)$ in the expression for α).

c) We consider briefly the anisotropy of the absorption for small departures of the vectors **k** and **H** from the condition of mutual orthogonality. In the low frequency case $\omega \ll \nu$ the attenuation is markedly anisotropic for small departures of the vectors **k** and **H** from mutual orthogonality.[4] For A.C.R. this anisotropy is no longer significant. Let $\pi/2 - \varphi$ be the angle between **k** and **H** ($\varphi \ll 1$). For $\varphi \ll (kl)^{-1}$ the terms in $\mathbf{k}\cdot\mathbf{v}$ can in general be neglected. For the range of angles $(kl)^{-1} \ll \varphi \ll s/v$ the Doppler frequency shift $\mathbf{k}\cdot\mathbf{v}$ causes a small displacement of the resonance position and a small broadening of the line. For example, when there is an arbitrary dispersion law and there is resonance at the central section, the frequency shift is of order $(\varphi v/s)^2$, and the additional broadening is $\sim(\varphi v/s)^4$. On further increasing φ, ($\varphi \gtrsim s/v$), the Doppler shift $\mathbf{k}\cdot\mathbf{v}$ becomes of order ω, and we use the preceding analysis in its entirety.

We emphasize that the existence of resonance when **k** and **H** are not perpendicular is essentially related to the fact that the vector **k** has a definite magnitude. The indeterminacy of the vector **k** explains the absence of electromagnetic C.R. in an inclined magnetic field[9,14]: the averaging of the Doppler shift over **k** causes the resonance to disappear. Therefore, at high frequencies $\omega \gg \nu$ and in a strong magnetic field when the attenuation increases, A.C.R. when **k** is perpendicular to **H** should be sharper than when **k** and **H** are not perpendicular.

5. SINGULARITIES OF RESONANCE ON OPEN PERIODIC TRAJECTORIES

The presence of trajectories of such a type greatly influences the singularities of the magneto-acoustic effects and leads to oscillations of a new resonance type.[4,5] The peculiarity of these oscillations is that in open periodic trajectories the mean displacement of an electron along the propagation direction of the sound $\mathbf{k}\cdot\overline{\mathbf{v}}/\Omega = k_\eta \hbar c/2\pi aeH$ is not zero when $\mathbf{k}$ is perpendicular to $\mathbf{H}$, and, significantly, does not depend on p_z. (The axis of η is perpendicular to $\mathbf{H}$ and to the mean direction ξ of the open trajectory, a^{-1} is the reciprocal lattice period in the ξ direction.) Because the transition (when p_z is changed) from open periodic to closed trajectories takes place through a hyperbolic contact point (for example, on a corrugated-cylinder-type surface) it can be expected that the effective mass at the axial section $p_z = 0$ will be minimal. If the angle θ between the vector $\mathbf{k}$ (in the plane $\mathbf{k}\perp\mathbf{H}$) and the direction of the open trajectory ξ is not very large ($\theta < \theta_0$; see Fig. 3a in [4]), so that the line $\mathbf{k}\cdot\mathbf{v} = \omega$ passes through the axial section $p_z = 0$, then C.R. is described by the resonance function

$$B(p_z) = \left\{1 - \exp\left[-2\pi\left(\frac{\bar{\nu} - i\omega}{\Omega(p_z)} \pm i\frac{k_\eta \hbar c}{eHa}\right)\right]\right\}^{-1}. \quad (5.1)$$

Resonant absorption should occur when $|\omega/\Omega_{min} \pm k_\eta \hbar c/eHa| = n$ and is given by (3.14) — (3.16), in which the frequency ω should be replaced by the quantity $|\omega \pm k\hbar \sin\theta/am_{min}|$. As a function of H^{-1} there will be observed beats of two frequencies, from which can be determined both the mass at the axial section and the direction and period of the open periodic trajectory. Resonance, as in the case of closed trajectories, will be lightly modulated by oscillations from the central section.[4]

For some angle $\theta = \theta_0$ (Fig. 3b in [4]) two closed non-intersecting lines $\mathbf{k}\cdot\mathbf{v} = \omega$ come together, and with further increase in θ, become two open periodic curves (Fig. 3c in [4]). In the range of angles $\theta > \theta_0$ C.R. occurs at frequencies $\Omega(p_{z0})$, where p_{z0} separates the range of values of p_z for which $\mathbf{k}\cdot\mathbf{v} - \omega$ tends to zero from those regions for which $\mathbf{k}\cdot\mathbf{v} \neq \omega$. If p_{z0} falls on open periodic trajectories, the contribution to the absorption from p_{z0} is given by (4.1) — (4.3), and C.R. will be observed on the curve $d\alpha/dH$ (the absorption shows jumps). However, the order of magnitude estimate (4.2) for $\Delta\alpha$ is essentially altered; in fact

$$\Delta\alpha \sim \alpha_0 (kr)^{1/3}, \quad (5.2)$$

because in the expression $\beta = (\mathbf{k}\cdot\overline{\mathbf{v}} \pm \omega)/\Omega$ the first component does not depend on p_z and $|\beta'| \sim \omega|d\Omega^{-1}/dp_z|$. Therefore $d\alpha/dH$ also increases by a factor of approximately kr as compared with the case of closed trajectories:

$$d\alpha/d\ln H \sim \alpha_0 (kr)^{4/3}\,\mathrm{sign}\beta'\cdot\gamma/(\gamma^2 + \Delta^2). \quad (5.3)$$

Resonance of this type apparently allows us to determine the effective mass not only on an axial section, but also in practice on any open periodic trajectory (except in the immediate neighborhood of a saddle point, where $\Omega = 0$ and there is no resonance). Knowing m we can also determine $\bar{v}_y$ for these trajectories.

Resonance at p_{z0} disappears on increasing θ as p_{z0} approaches the saddle point ($\theta = \theta_k$). When $\theta > \theta_k$ the curve $\mathbf{k}\cdot\mathbf{v} = \omega$ changes completely into closed trajectories, and we use the analysis made for closed trajectories. For example, in the case of the "corrugated cylinder" there will only be C.R. at an elliptical contact point, when $\theta > \theta_k$ and $\mathbf{k}\perp\mathbf{H}$.

The anisotropy of the absorption at open periodic trajectories relative to the inclination of the vector $\mathbf{k}$ from the plane $\mathbf{k}\perp\mathbf{H}$ has the same character as for closed trajectories.

When there are open non-periodic or greatly extended closed trajectories (for example, $\mathbf{H}$ almost perpendicular to the axis of the cylinder), the contribution from them, which depends on the magnetic field, is insignificant.[4]

6. CONCLUDING REMARKS

The temperature variation of the absorption coefficient involves the temperature variation of the collision frequency ν. In zero magnetic field ν disappears from the expression (2.7) for α. In the resonance region electrons absorb sound only when they move with the wave: $\omega = \mathbf{k}\cdot\mathbf{v}$. When such electrons collide with phonons, it is most probable that they will leave the region of phase space $\mathbf{k}\cdot\mathbf{v} = \omega$, and the phonon part of the collision frequency is, therefore, $\nu_f \sim T^3$ (and not the usual $\nu_f \sim T^5$; see also [3,15]). If electron-phonon collisions dominate over impurity scattering (ν_d = const) and over electron-electron collisions ($\nu_e \sim T^2$), the temperature variation is of the form $\nu = \nu_f \sim T^3$. These conclusions are, of course, valid in the non-quantum range $\hbar\omega$, $\hbar\Omega \ll T$. When the quantum conditions $\hbar\Omega \gg T$ are satisfied, hypersound can excite bulk quantum C.R. of the type predicted by I. Lifshitz.[16]

It should be remarked that "Fermi-liquid" effects are only important when $\omega \gg \nu$ in the A.C.R. region (when $\omega \ll \nu$ they play no part).

The "Fermi-liquid" properties of the electrons cause an additional broadening of the resonance only in the very high frequency region, when $\omega t_0 \sim (kr)^2$ (for non-quadratic dispersion) and $\omega t_0 \sim kr$ (for quadratic dispersion); this is in complete analogy with the case of electromagnetic C.R.[17]

Comparison of the theory developed above with Roberts' experiments[7] shows that there is good agreement. Even at a comparatively low frequency ($\omega/2\pi$ = 115 Mc) the cyclotron maxima are clearly resolved and the modulation of the C.R. by oscillations of the Pippard type is clearly displayed.

Undoubtedly, further experimental observation and study of A.C.R. will allow a number of important properties of the dispersion law of electrons in a metal to be determined. With the aid of A.C.R. it is possible to determine simultaneously in the same experiments the extremal diameters, the effective masses, and the mean velocities of the electrons. These data are by themselves adequate to determine completely the Fermi surface and the electron velocities on it. However, in addition, the study of resonance when **k** and **H** are not perpendicular to one another makes it possible to find not only the extremal effective masses and mean velocities, but also their values at any section of the Fermi surface. A.C.R. can be used to discover and determine the characteristics of open periodic trajectories.

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36