

*THE SINGLE-PARTICLE GREEN'S FUNCTION OF A FERMI-SYSTEM WITH ATTRACTIVE AND REPULSIVE INTERACTIONS*

M. Ya. AMUS'YA

Leningrad Physico-technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor February 26, 1962; resubmitted April 5, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 287-300 (July, 1962)

We have evaluated the single-particle Green's function of a spatially infinite fermion system with two-body interactions consisting of an attraction  $U$  of radius  $b$  and a strong repulsion  $V$  of radius  $a$ , with  $b \gg a$  and  $U \ll V$ . We expand the self-energy part  $\Sigma$  in terms of two parameters  $a\rho^{1/3}$  and  $1/b\rho^{1/3}$  in the intermediate range of densities  $\rho$  ( $b^3\rho \gg 1$  and  $a^3\rho \ll 1$ ). We take into account all diagrams which give a contribution to  $\Sigma$  which is not less than the contribution from the gas-approximation term which is quadratic in  $a$ . We obtain the single-particle excitation spectrum, an expression for the effective mass at the Fermi surface, the chemical potential, and the sound velocity. We have considered the influence of a self-compressed state of the system on the chemical potential and on the sound propagation velocity. We consider nuclear matter as an example of such a two-parameter system; and determine in this example the average energy per nucleon, the quasi-particle effective mass, and the symmetry energy.

## 1. INTRODUCTION

IN order that one can apply perturbation theory to a study of many-particle Fermi-systems, it is necessary that there exist a small parameter which is determined by the magnitude of  $\mathcal{U}$ , the interaction radius  $\alpha$ , and the state of the system—its density  $\rho$  (or the Fermi momentum  $p_0$ ). Up to the present two limiting cases were mainly considered: the gas approximation and the high-density approximation. These are valid, respectively, when  $p_0\alpha \ll 1$  and  $\mathcal{U}\alpha^2$  is arbitrary and when  $(p_0\alpha)^{-1} \ll 1$  and  $\mathcal{U}\alpha^2 < 1$ . However, in most many-particle systems occurring in nature, both fermion and boson systems, the interaction radius  $\alpha$  is large i.e.,  $(\rho^{1/3}\alpha)^{-1} \ll 1$ . Whereas at large distances relatively weak attractive forces operate, at small distances huge repulsive forces operate, causing the compressibility of the liquid to be small. It turns out that  $\mathcal{U}\alpha^2 \gg 1$ , and although  $(p_0\alpha)^{-1} \ll 1$ , it is impossible to apply perturbation theory with the parameter  $(p_0\alpha)^{-1}$ .

There are systems (for instance, nuclear matter) where it is possible to split the two-particle interaction potential  $\mathcal{U}$  into two parts,  $U$  ( $U < 0$ ) and  $V$  ( $V > 0$ ) with action radii  $b$  and  $a$  such that  $a\rho \ll 1$  and  $Va^2 \gg 1$ ,  $(bp_0)^{-1} \ll 1$  and  $Ub^2 < 1$ . The possibility then arises for a microscopic study of a number of systems by means of a perturbation theory involving two parameters. A pre-

vious paper by the present author<sup>[1]</sup> was devoted to considering such a system. In that paper we evaluated the average energy per particle,  $E_{av}$ , and we obtained an equation of state, i.e., the dependence of the pressure on the density. Since the repulsive energy increases with increasing density faster than the attractive energy, the pressure at a certain density is determined, as in the case of a gas of non-interacting particles, by the kinetic energy.

We considered especially the case where the pressure was equal to zero and  $\partial^2 E_{av} / \partial \rho^2 > 0$ : self-compressed systems. A study of the dependence of  $E_{av}$  on  $\rho$  shows that if  $p_0a \ll 1$  and  $(bp_0)^{-1} \ll 1$  and if the forces are Wigner forces, it is impossible to satisfy all the requirements for self-compression:  $E_{av} < 0$ ,  $\partial E_{av} / \partial \rho = 0$ , and  $\partial^2 E_{av} / \partial \rho^2 > 0$ . The occurrence of Majorana forces increases—as we show in detail in the present paper—the repulsive energy, and also enables us to obtain a negative  $E_{av}$ . Self-compression turns out to be possible in the systems considered only when there are present a strong repulsion and long-range exchange forces and the equilibrium density depends then strongly on the range of the repulsive forces.

In the present paper we continue the study of such a system and evaluate the single-particle

<sup>1)</sup>Henceforth quoted as I.

Green's function. It is well known<sup>[2]</sup> that the poles of the analytical continuation of the Green's function determine the single-particle excitation spectrum and the damping of the quasi-particles, and that the point where the imaginary part of the self-energy part  $\Sigma$  vanishes determines the chemical potential of the system. A knowledge of the chemical potential enables us to determine the low-frequency sound velocity. Of special interest is an elucidation of the influence of exchange forces on  $\Sigma$ , in particular upon the magnitude of the quasi-particle effective mass at the Fermi surface, the magnitude of the damping, and so on.

It is clear that it has only sense to speak about single-particle excitations under such circumstances that the imaginary part of  $\Sigma$ , the damping, is sufficiently small compared with the real part of  $\Sigma$ , and thus we are mainly interested in the Green's function at the Fermi surface, as will become clear from what follows.

We do not consider poles in  $\Sigma$ , which Migdal<sup>[3]</sup> has shown to correspond to the possibility of the occurrence of bound pairs such as Cooper pairs since the problem of the superfluidity of the systems considered will be studied elsewhere.

**2. CHOICE OF DIAGRAMS**

As in our previous paper, we shall use a diagram technique the details of which can be found in [2,4,5]; the features peculiar to the two-parameter problem were noted in I. Since  $Va^2 \gg 1$  and  $p_0a \ll 1$ , we shall sum, as far as the short-range potential is concerned, in the gas approximation, with the shaded square denoting  $V_{\text{eff}}$ , the sum of the ladder diagrams in  $V$  (for details see Fig. 2 of I). An ordinary wavy line indicates the long-range potential  $U$ , and a double wavy line  $U_{\text{eff}}$ , the sum of all diagrams, where the maximum number (equal to the order) of particles is connected in each order of perturbation theory in  $U$  (see Fig. 3 in I).

When evaluating and choosing the diagrams we shall aim to perform the calculations up to and including terms of order  $\sim (p_0, a)^2$ . The single-particle Green's function of non-interacting particles was determined in (I.2):

$$G_0^{-1}(\mathbf{p}) = p_\epsilon - p^2/2 + i\delta\theta(\mathbf{p});$$

$$\theta(\mathbf{p}) = (1 - 2n_p) = \begin{cases} 1, & |\mathbf{p}| > p_0 \\ -1, & |\mathbf{p}| \leq p_0 \end{cases} \quad (1)$$

If we take into account the fact that the scattering amplitudes of hard spheres do not depend strongly on the momenta and the fact, noted in I, that the integration over intermediate states oc-

curs over momenta which are not more than a few times  $p_0$ , we can replace  $V_{\text{eff}}$  by  $4\pi a$  in all mixed diagrams.

We need first take into account all irreducible diagrams of Fig. 1. Were it necessary to take into account terms  $\sim a^3$ , the main difficulty would lie in the appreciable increase in the number of mixed diagrams. Diagrams such as the one of Fig. 2 can lead to an appreciable contribution only at the Fermi surface, because of the long range of the potential  $U$ . However, for  $p \approx p_0$  the contribution from Fig. 2 is small. This enables us to drop the diagram of Fig. 2 and similar diagrams where one of the wavy lines  $U$  is replaced by the square  $V_{\text{eff}}$ , which is equal to  $4\pi a$  (for instance, Fig. 3). The contribution from diagrams such as the one of Fig. 4 is at least  $p_0b$  times smaller than that from diagram  $s$  of Fig. 1 which is taken into account (see I, where estimates are given).

In Fig. 1 we have thus given all basic types of irreducible diagrams, since the remaining diagrams give contributions which are either  $p_0b$  or  $1/p_0a$  times smaller and need not be considered. Taking them into account does not affect qualitatively the results obtained in the present paper.

To take all diagrams into account one must consider apart from those of Fig. 1 several self-

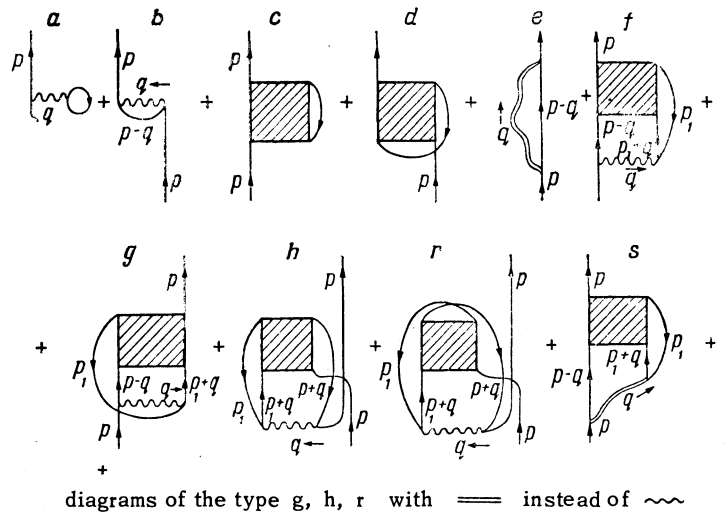


FIG. 1



FIG. 2

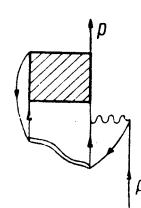


FIG. 3

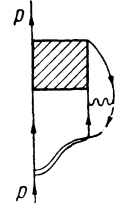


FIG. 4

energy parts, i.e., one must evaluate the diagrams of Fig. 1 using not the free-particle Green's function (1), but (I.3)

$$G^{-1}(\mathbf{p}) = p_\epsilon - p^2/2 - \Sigma_I(\mathbf{p}) + i\delta\theta(\mathbf{p}). \quad (2)$$

It is, however, clear that  $\Sigma_I(\mathbf{p})$  must not contain terms  $\sim a$  since they are taken into account separately.

### 3. SINGLE-PARTICLE EXCITATION SPECTRUM: CALCULATION OF THE SELF-ENERGY PART $\Sigma$

The single-particle excitation spectrum  $\epsilon$  at the Fermi surface is determined using the relation

$$\epsilon = \lambda + p_0(p - p_0)/\mu_{\text{eff}}, \quad (3)$$

where  $\lambda$  is the chemical potential,  $\mu_{\text{eff}}$  the effective quasi-particle mass at the Fermi surface, and

$$\lambda = \frac{p_0^2}{2} + \Sigma(p_0), \quad \frac{1}{\mu_{\text{eff}}} = 1 + \frac{1}{p_0} \left. \frac{\partial \Sigma}{\partial p} \right|_{p=p_0}.$$

We need thus know the self-energy part  $\Sigma$  at the Fermi surface with an accuracy of  $\sim p - p_0$  in order to determine the spectrum.

We first of all evaluate the contribution to  $\Sigma$  of all irreducible diagrams of Fig. 1 taking into account that the two-particle interaction is a non-exchange, central one.

1. One verifies easily that  $\Sigma_a(\mathbf{p}) = U(0)\rho$ ,  $\rho = p_0^3/3\pi^2$ .

2. The contribution from diagram 1b is equal to

$$\Sigma_b(p) = - \int U(q) n_{\mathbf{p}-\mathbf{q}} \frac{dq}{(2\pi)^3}. \quad (4)$$

The expansion of  $\Sigma_b(p)$  near  $p \approx p_0$  up to terms  $\sim p$  gives

$$\Sigma_b(p) = \Sigma_b(p_0) - \frac{p_0 - p}{4\pi^2} \int_0^{2p_0} U(q) \left(1 - \frac{q^2}{2p_0^2}\right) q dq, \quad (5)$$

$$\Sigma_b(p_0) = - \frac{1}{4\pi^2} \int_0^{2p_0} U(q) \left(1 - \frac{q}{2p_0}\right) q^2 dq. \quad (6)$$

3. The situation is much more complicated when we want to determine the contribution of the diagram of Fig. 1e. We evaluate the real part,  $\text{Re } \Sigma_e$  using the fact that the single-particle excitations are determined in two different ways: in terms of the variational derivative of the total energy density<sup>[6]</sup> with respect of the occupation numbers, and in terms of the Green's function poles, and these two definitions are clearly the same<sup>2)</sup>, i.e.,

<sup>2)</sup>Karpman<sup>[7]</sup> has rigorously shown the equivalence of these definitions for the vicinity of the Fermi surface.

$$\text{Re } \Sigma_e(\mathbf{p}) = \delta(\epsilon_{\text{corr}, \rho})/\delta n_\rho. \quad (7)$$

$\epsilon_{\text{corr}}$  is the correlation energy per particle which was evaluated in <sup>[8]</sup> and in I. Furthermore,  $\mathbf{p}$  is the momentum of a real particle or hole, i.e., there exists an unambiguous connection between  $\mathbf{p}$  and  $p_\epsilon$ , the fourth momentum component.

We have thus

$$\begin{aligned} \epsilon_{\text{corr}} &= - \frac{1}{2\pi\rho} \int q \frac{dq}{(2\pi)^3} U^2(q) \\ &\times \int_{-\infty}^{+\infty} ds \sum_{n=2}^{\infty} \frac{(-1)^n}{n} [Q_q(s)]^n U^{n-2}(q), \quad 0 < q < p_0. \end{aligned} \quad (8)$$

$$\begin{aligned} Q_q(s) &= 2 \int \frac{dl}{(2\pi)^3} n_l (1 - n_{\mathbf{q}+\mathbf{l}}) \\ &\times \int_{-\infty}^{+\infty} \exp\left\{itsq - |t| \left[\frac{1}{2}q^2 + \mathbf{q}\mathbf{l}\right]\right\} dt. \end{aligned} \quad (9)$$

Taking the variation of  $Q_q(s)$  with respect to  $n_{\mathbf{p}}$ , we find

$$\begin{aligned} \frac{\delta Q_q(s)}{\delta n_{\mathbf{p}}} &= (1 - n_{\mathbf{p}+\mathbf{q}}) \int_{-\infty}^{+\infty} \exp\left\{itsq - |t| \left[\frac{1}{2}q^2 + \mathbf{q}\mathbf{p}\right]\right\} dt \\ &- n_{\mathbf{p}-\mathbf{q}} \int_{-\infty}^{+\infty} \exp\left\{itsq - |t| \left[-\frac{1}{2}q^2 + \mathbf{q}\mathbf{p}\right]\right\} dt. \end{aligned} \quad (10)$$

Integrating in (10) over  $t$  and the direction of  $\mathbf{q}$  and taking as the upper limit  $p_0$  we find then near the Fermi surface the expression

$$\begin{aligned} \text{Re } \Sigma_e &= \text{Re } \Sigma_e(p_0) \\ &+ \frac{1}{4\pi^2} (p_0 - p) \int_0^{p_0} q dq \sum_{n=2}^{\infty} (-1)^n [Q_q(0)]^{n-1} U^n(q) \\ &- \frac{1}{4\pi^2} p_0 (p_0 - p) \int_0^{p_0} q^3 dq \int_{-\infty}^{+\infty} ds \\ &\times \sum_{n=2}^{\infty} (-1)^n [Q_q(s)]^{n-1} U^n(q) \frac{1}{(p_0^2 + s^2)^2}, \end{aligned} \quad (11)$$

$$\begin{aligned} \text{Re } \Sigma_e(p_0) &= - \frac{1}{8\pi^3} \int_0^{p_0} q^3 dq \int_{-\infty}^{+\infty} ds \\ &\times \sum_{n=2}^{\infty} (-1)^n [Q_q(s)]^{n-1} U^n(q) \frac{1}{p_0^2 + s^2}. \end{aligned} \quad (12)$$

Summing over  $n$  we find<sup>3)</sup>

<sup>3)</sup>We note that  $\Sigma_e(p_0) = \partial(\epsilon_{\text{corr}}\rho)/\partial\rho$  which is reduced to the calculation of  $\partial[Q_q(s)]/\partial\rho$ . The upper limit of integration over  $q$  in  $\epsilon_{\text{corr}}$  is usually equal to  $p_0$ , but when taking the variation with respect to  $\rho$  one need not take this dependence into account, since the limitation of the integrals over  $q$  by the limit  $p_0$  is not one of principle, but is connected with the fact that the upper limit and, in general, the large  $q$  region does not contribute to the integral because  $U(q)$  decreases steeply.

$$\begin{aligned} \operatorname{Re} \Sigma_e(p) &= \operatorname{Re} \Sigma_e(p_0) + \frac{1}{4\pi^2} (p_0 - p) \int_0^{p_0} q dq \frac{U^2(q) Q_q(0)}{1 + U(q) Q_q(0)} \\ &\quad - \frac{1}{4\pi^3} p_0 (p_0 - p) \int_0^{p_0} q^3 dq \int_{-\infty}^{+\infty} \frac{ds}{(p_0^2 + s^2)^2} \frac{U^2(q) Q_q(s)}{1 + U(q) Q_q(s)}, \end{aligned} \quad (13)$$

$$\operatorname{Re} \Sigma_e(p_0) = -\frac{1}{8\pi^3} \int_0^{p_0} q^3 dq \int_{-\infty}^{+\infty} \frac{ds}{p_0^2 + s^2} \frac{U^2(q) Q_q(s)}{1 + U(q) Q_q(s)}, \quad (14)$$

where in zeroth approximation in  $q$

$$Q_q(s) = \frac{p_0}{\pi^2} \left( 1 - \frac{s}{p_0} \operatorname{arctg} \frac{p_0}{s} \right), \quad Q_q(0) = \frac{p_0}{\pi^2} \left( 1 - \frac{q^2}{12p_0^2} \right). \quad (15)^*$$

4. Galitskiĭ<sup>[4]</sup> has evaluated the Green's function in the gas approximation. However, to find the contribution from the diagrams c and d we must take into account the self-energy part  $\Sigma_I$ . We use here the same approximation as in I, i.e., the approximation of  $\Sigma_I$  by the quadratic expression  $\Sigma_I = \sigma + p^2/2\mu_1 - p^2/2$ . One can then show<sup>4)</sup> that the effective repulsive potential will be not  $4\pi a$ , but  $4\pi a/\mu_1$ . In first approximation we have thus a renormalization of the effective repulsive interaction.

5. When evaluating  $\Sigma_{f,h,r,g}$  we must treat with care the fact that since  $U$  is a long-range potential the small- $q$  region gives the main contribution. The point is that upon integration over the intermediate states there occur sufficiently high powers of  $q$  ( $q^2, q^3$ ) to swamp the large magnitude of  $U(q)$  at small  $q$ . We must thus evaluate  $\Sigma_{f,h,r,g}$  accurately without restricting ourselves to small  $q$ . In the approximation considered where the short range part is replaced by  $4\pi a/\mu_1$  the matrix elements of  $f$  and  $g$  differ only in the numerical coefficient which arises from the summation over the spin states and in the sign, but the matrix elements of  $f$  and  $h$  are the same. We have then

$$\begin{aligned} \operatorname{Re}(\Sigma_f + \Sigma_g + \Sigma_h + \Sigma_r) &= 8\pi \frac{a}{\mu_1} P \int \frac{n_{p_1} (1 - n_{p-q}) (1 - n_{p_1+q}) + n_{p-q} n_{p_1+q} (1 - n_{p_1})}{q(p-p_1) - q^2 + p_\epsilon - p^2/2} \\ &\quad \times U(q) \frac{dp_1 dq}{(2\pi)^6}, \end{aligned} \quad (16)$$

where  $P$  is the principal value symbol.

Moreover, we must take into account reducible diagrams; to do this we must in the denominator add  $\Sigma_I$ :

$$\begin{aligned} p_\epsilon - (p-q)^2/2 - \Sigma_I(p-q) + p_1^2/2 + \Sigma_I(p_1) \\ - (p_1+q)^2/2 - \Sigma_I(p_1+q). \end{aligned} \quad (17)$$

If we consider the real part we get with the accuracy used here

$$p_\epsilon - p^2/2 - \Sigma_I(p) = 0. \quad (18)$$

We note that the integrand [apart from  $U(q)q^2$ ] is a smooth function of  $q$  near  $q=0$  even for real particles, and the region where it changes appreciably turns out to be larger than  $p_0$ . If  $U(q)q^2$  does not have a steep maximum near  $q=0$  and varies sufficiently little between 0 and  $p_0$  (for instance, the Yukawa potential) one can take  $\Sigma_I$  into account by introducing an effective mass  $\mu_1$ , and  $\operatorname{Re} \Sigma$  is determined from Eq. (16) with a instead of  $a/\mu_1$ . If, however, the potential  $U(q)q^2$  has a steep maximum for  $q \ll p_0$ , then we are interested not in the average effective mass  $\mu_1$  in the interval 0 to  $p_0$ , but in  $\mu_2$ , the effective mass at the Fermi surface.

Performing the calculation for the real part  $\operatorname{Re} \Sigma$  and real particles we get after rather protracted calculations an extremely complicated expression. Since the diagrams f...r of Fig. 1 considered here do not contribute to  $\mu_1$  we need find their magnitude only near the Fermi surface, namely, for  $(p-p_0)/p_0 \ll 1$  and it is necessary only to retain terms with  $(p_0-p)/p_0$  of at most the first degree. We then get<sup>5)</sup>

$$\operatorname{Re} \Sigma(p_0) = \frac{a}{2\pi^3} (\ln 2 - 1) \int_0^{2p_0} q^3 U(q) dq - \frac{2ap_0^3}{3\pi^3} \int_{2p_0}^{\infty} U(q) dq \quad (19)$$

or, if  $U(q)q^2$  has a steep maximum for small  $q$  we must replace in the first term  $a$  by  $a\mu_2/\mu_1$ . In deriving (19) we used the fact that we can expand  $(x-\alpha) \ln|x-\alpha|$  under the integral sign in

the expression  $\int_0^\alpha (x-\alpha) \ln|x-\alpha| dx$ , assuming

that  $x/\alpha \ll 1$ . In the same approximation

$$\begin{aligned} \operatorname{Re} \Sigma(p) &= \operatorname{Re} \Sigma(p_0) + \frac{p_0-p}{\pi^3} a p_0 \int_0^{2p_0} q dq U(q) \left( 1 - \frac{q^2}{3p_0^2} \right) \\ &\quad + \frac{(p_0-p) a p_0^2}{3\pi^3} \int_{2p_0}^{\infty} dq U(q). \end{aligned} \quad (20)$$

For the  $U(q)$  mentioned above we must replace  $a$  in the second term by  $a\mu_2/\mu_1$ .

\* $\operatorname{arctg} = \tan^{-1}$ .

<sup>4)</sup>A detailed consideration of the influence of  $\Sigma_I$  on the scattering amplitude and the effective potential is given in another paper.

<sup>5)</sup>At first glance it looks as if we can neglect the second integral; this is, however, not the case. In fact, the contributions from the first and the second integral are of the same order of magnitude.

We noted above that the contributions of the diagrams f and h of Fig. 1 are the same. However, if we understand them in the sense of Hugenholtz<sup>[9]</sup> ( $\uparrow$  denotes a particle and  $\downarrow$  a hole) the contributions are different and their difference is especially important for long-range potentials. For instance, if  $p > p_0$  the contribution from the term with  $n_{\mathbf{p}-\mathbf{q}}$  in (16) is small, while for  $p < p_0$  the contribution from the term with  $1 - n_{\mathbf{p}-\mathbf{q}}$  is small. On the Fermi surface the contributions from these terms are approximately the same. In Hugenholtz' notation the term with  $n_{\mathbf{p}-\mathbf{q}}$  corresponds to the diagrams h and r and the one with  $1 - n_{\mathbf{p}-\mathbf{q}}$  to f and g.

In the Brueckner theory the contribution from the diagrams h and r was called the "rearrangement energy." It is clear that the determination of the single-particle excitations in terms of the diagrams f and g only<sup>[10,11]</sup> is completely incorrect and this explains the anomalously large value of the rearrangement energy obtained by a number of authors. It is possible that an incorrect definition of the single-particle energy does not matter in the calculation of the ground-state energy but it is completely inadmissible when determining the single-particle excitation spectrum, the damping of the excitations, and so on.

6. We now calculate the remaining diagrams such as s. We start from the fact that  $U_{\text{eff}}(q)$  decreases so fast with increasing  $q$  that the main contribution comes from the region where  $q \ll p_0$ . This enables us to use the method of calculation applied in subsection 3 of the present section. We note that  $\text{Re } \Sigma_s$  can be found in the same way as in subsection 3, since all diagrams of the kind f and r with a double wavy line and also several other ones (Fig. 5) are obtained from (7) if we replace in  $\epsilon_{\text{corr}}$  given by (8) one of the  $U(q)$  by  $4\pi a$ . Moreover, since diagrams with one ordinary wavy line are taken into account in Fig. 1f...r, the summation in (8) must start from  $n = 3$ . Repeating all calculations which led from (8) to (13) and (14) we find

$$\begin{aligned} \text{Re } \Sigma_s &= \text{Re } \Sigma_s(p_0) \\ &- \frac{a}{2\pi} (p_0 - p) \int_0^{p_0} q dq \frac{2U^3(q) Q_q^3(0) + 3U^2(q) Q^2(0)}{[1 + U(q) Q_q(0)]^2} \\ &+ \frac{a}{2\pi^2} p_0 (p_0 - p) \\ &\times \int_0^{p_0} q^3 dq \int_{-\infty}^{+\infty} \frac{ds}{(p_0^2 + s^2)^2} \frac{2U^3(q) Q_q^3(s) + 3U^2(q) Q^2(s)}{[1 + U(q) Q_q(s)]^2}, \quad (21) \end{aligned}$$

$$\text{Re } \Sigma_s(p_0) = \frac{a}{4\pi^2} \int_0^{p_0} q^3 dq \int_{-\infty}^{+\infty} \frac{ds}{p_0^2 + s^2} \frac{2U^3(q) Q_q^3(s) + 3U^2(q) Q^2(s)}{[1 + U(q) Q_q(s)]^2}. \quad (22)$$

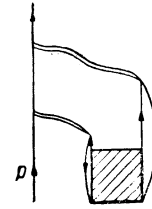


FIG. 5

To take into account the contribution from the reducible diagrams in the approximation used in this paper we see easily that we must in (13), (14), (21), and (22) replace  $Q$  by  $Q' = \mu_2 Q$  and in (21) and (22) replace  $a$  by  $a/\mu_1$ .

#### 4. THE SINGLE-PARTICLE EXCITATION SPECTRUM: THE EFFECTIVE MASS ON THE FERMI SURFACE, THE CHEMICAL POTENTIAL

In the preceding section we obtained an expression for the Green's function self-energy part  $\Sigma$  which depended on the effective masses  $\mu_1$  and  $\mu_2$ ; these we shall now determine. We have noted already that terms proportional to  $a$  need not be taken into account in the Green's functions when evaluating  $\Sigma(p)$  with the accuracy considered here. The best accuracy in determining  $\mu_2$  is obtained as follows: we evaluate the diagrams b and e of Fig. 1 assuming some value of  $\mu_2'$  and we find  $\Sigma_I(p, \mu_2')$ , which we use to determine  $\mu_2''$ , and so on, until self-consistency is reached. Having thus determined  $\Sigma_I$  we can approximate  $\Sigma_I$  by the best quadratic expression over a wide range of  $p$  from 0 to  $p_0$  and thus determine  $\mu_1$ . A self-consistent determination of  $\mu_1$  and  $\mu_2$  from  $\Sigma(p)$  and not from  $\Sigma_I$  leads to exceeding the accuracy of the calculation, even if we forget the very large complications.

Let us consider a mixture of attractive forces, a Wigner force  $W$  and a Majorana force  $M$  ( $W + M = 1$ ), where we shall assume for the sake of simplicity that their radial dependence is the same. As an example we give the results of the calculations for a Yukawa-type long-range potential

$$U = \frac{U_0}{vr} e^{-vr} \quad \text{or} \quad U(q) = \frac{4\pi U_0}{v} \frac{1}{q^2 + v^2}.$$

We find the expression for  $\mu_2$  in the vicinity of the Fermi surface from

$$\frac{1}{\mu_2} = 1 + \frac{1}{p_0} \left. \frac{\partial \Sigma_I}{\partial p} \right|_{p=p_0}, \quad \Sigma_I = \Sigma_a + \Sigma_b + \Sigma_e.$$

If  $U$  is such that it is sufficient to restrict ourselves to  $\Sigma_a$  and  $\Sigma_b$ , then the whole of the long-

range part of the potential is taken into account by the self-consistent field in which the system with two-body repulsions is situated. We have from (5) and (13)

$$\begin{aligned} \frac{1}{\mu_2} = & 1 + \sum_{i=0,1} \frac{(2i+1)}{4\pi^2\rho_0} \left\{ \int_0^{2\rho_0} q U_i(q) \left(1 - \frac{q^2}{2\rho_0^2}\right) dq \right. \\ & - \int_0^{\rho_0} q dq \frac{U_i^2(q) Q'_q(0)}{1 + U_i Q'_q(0)} \\ & \left. + \frac{\rho_0}{\pi} \int q^3 dq \int \frac{ds}{(\rho_0^2 + s^2)^2} \frac{U_i^2(q) Q'_q(s)}{1 + U_i(q) Q'_q(s)} \right\}, \\ U_0 = & (W - M/2) U, \quad U_1 = -MU/2. \end{aligned} \quad (23)$$

We have already noted that in the evaluation of  $\Sigma_{f...g}$  the integration over the intermediate states leads to the occurrence of fairly high powers of  $q$  ( $q^2, q^3$ ). The contribution of the second order to the correlation self-energy part must thus be taken into account exactly by means of (20) and not in the Gell-Mann-Brueckner approximation in which Eq. (15) was obtained.

As an example we give the results of the calculation which are valid both when  $UQ \ll 1$  and when  $UQ \gg 1$ , restricting ourselves to terms  $\sim U^2$  in the last integral<sup>6)</sup>

$$\begin{aligned} \frac{1}{\mu^2} = & 1 + \sum_{i=0,1} (2i+1) \left[ U_{0i}'' (\ln 2\eta - 1) \right. \\ & \left. - \frac{U_{0i}''}{2} \ln(1 + 4U_{0i}''\mu_2\eta^2) + \frac{2}{3} U_{0i}''^2 \ln 2\eta \right]; \\ \eta = & \rho_0/\nu, \quad U_{00}'' = (W - M/2) U^0/\nu\pi\rho_0, \\ U_{01} = & -MU^0/2\nu\pi\rho_0. \end{aligned} \quad (24)$$

When  $W/2 < M$  the value of  $1/\mu_2$  remains  $> 1$ . Similarly,  $1/\mu_1 > 1$  if  $W/2 < M$ . This means that when there is an appreciable admixture of exchange forces the effective repulsion is strengthened, while for purely non-exchange forces the presence of a long-range force leads to a weakening of the repulsion. The contribution from diagrams c and d of Fig. 1 to the effective mass will thus be<sup>[4]</sup>

$$\mu_{c,d}^{-1} = -8(7 \ln 2 - 1) \rho_0^2 a^2 / 15\pi^2 \mu_1. \quad (25)$$

We find from (5), (13), (20), (21), and (24) for the effective mass on the Fermi surface the expression

<sup>6)</sup>It may turn out that as  $\nu \rightarrow 0$ , i.e., when we take the limit to a constant potential,  $1/\mu_2 \rightarrow \infty$  which is absurd. However, this is not the case. Indeed, when  $\nu = 0$  we shall have  $U_{\nu=0}(q) = U''^0 \delta(q)$  and in order that  $U_{\nu}(q)_{\nu \rightarrow 0} = U_{\nu=0}(q)$  it is necessary that  $U^0 = U''^0 \nu^{1+\alpha}$  with  $\alpha > 0$ . The contribution of  $\Sigma_b$  to  $1/\mu_2$  tends thus to 0 as  $\nu \rightarrow 0$ , as is evident from physical considerations.

$$\begin{aligned} \frac{1}{\mu_{eff}} = & \frac{1}{\mu_2} - \frac{a}{\pi^3} \int_0^{2\rho_0} q dq U(q) \left(1 - \frac{q^2}{3\rho_0^2}\right) - \frac{a\rho_0}{3\pi^3} \int_{2\rho_0}^{\infty} dq U(q) \\ & - \frac{a}{2\mu_1\pi^2} \sum_{i=0,1} (-1)^i (2i+1) \\ & \times \int_0^{\rho_0} q^3 dq \int_{-\infty}^{+\infty} ds \left[ \frac{1}{(\rho_0^2 + s^2)^2} - \frac{\pi}{\rho_0 q^2} \delta(s) \right] \\ & \times \frac{2U_i^3(q) Q_q^3(s) + 3U_i^2(q) Q_q^2(s)}{[1 + U_i(q) Q'_q(s)]^2} - \frac{8}{15\pi^2} (7 \ln 2 - 1) \frac{\rho_0^2 a^2}{\mu_1}. \end{aligned} \quad (26)$$

A study of  $1/\mu_{eff}$  enables us to estimate whether there are exchange forces present in the system and how large they are. In any case, if  $1/\mu_{eff}$  is appreciably larger than unity there are certainly exchange forces present. As before, we give as an example the results of our calculations for the case  $UQ \gg 1$ , limiting ourselves in the last integral to terms  $\sim U^2$ :

$$\begin{aligned} \frac{1}{\mu_{eff}} = & \frac{1}{\mu_2} - \frac{4\zeta U^0}{\pi\mu_1} \left( \ln 2\eta - \frac{1}{2} \right) + \frac{\zeta}{\mu_1\pi} \sum_{i=0,1} (-1)^i (2i+1) \\ & \times \left\{ 2U_{0i}'' \ln |1 + 4U_{0i}''\eta^2\mu_2| - U_{0i}'' - \frac{12}{\pi} U_{0i}''^2 \ln \eta \right\} \\ & - \frac{8}{15\pi^2} (7 \ln 2 - 1) \frac{\zeta^2}{\mu_1}, \end{aligned} \quad (27)$$

where  $\zeta = p_0 a$ ,  $U^{0''} = U^0/\nu\pi\rho_0$ .

We now determine the chemical potential, which is well known to be equal to the quasi-particle energy for  $p = p_0$ <sup>[2]</sup> and equal to the energy of splitting off one particle, and which can thus be measured experimentally. From<sup>[4]</sup> and (6), (14), (19), and (22) we find

$$\begin{aligned} \lambda = & U_0(0) \rho + \frac{\rho_0^2}{2} \\ & - \frac{1}{(2\pi)^2} \int_0^{2\rho_0} (U_0(q) - 3U_1(q)) q^2 dq \left(1 - \frac{q}{2\rho_0}\right) \\ & + \frac{a}{2\pi^3} (\ln 2 - 1) \int_0^{2\rho_0} q^3 U(q) dq \\ & - \frac{2a\rho_0^3}{3\pi^3} \int_{2\rho_0}^{\infty} U(q) dq - \frac{1}{8\pi^3} \sum_{i=0,1} (2i+1) \int_0^{\rho_0} q^3 dq \\ & \times \int_{-\infty}^{+\infty} \frac{ds}{\rho_0^2 + s^2} \frac{U_i^2(q) Q'_q(s)}{1 + U_i(q) Q'_q(s)} \\ & \times \left[ 1 - (-1)^i \frac{2a\pi}{\mu_1} Q'_q(s) \frac{3 + 2U_i(q) Q'_q(s)}{1 + U_i(q) Q'_q(s)} \right] \\ & + \frac{1}{2\mu_1} \rho_0^2 \left[ \frac{4}{3\pi} \rho_0 a + \frac{4}{15\pi^2} (11 - 2 \ln 2) \rho_0^2 a^2 \right], \end{aligned} \quad (28)$$

which in the simplest case, considered here, gives

$$\begin{aligned} \lambda = & \frac{4}{3\pi} \left( W - \frac{M}{2} \right) \eta^3 U^0 + \frac{1}{2} p_0^2 + 2 \frac{(2M-W)}{\pi} U^0 \eta \left( 1 - \frac{\pi}{4\eta} \right) \\ & + \frac{\mu_2 (W^2 - WM + M^2) U^0 \eta^2}{\pi^2 p_0^2} \left\{ 2 (\ln 2 - 1) \left( \ln 2\eta - \frac{1}{2} \right) \right. \\ & \left. - \frac{1}{9} + \frac{3 \ln 2 \zeta \mu_2 (2 \ln \eta - 1)}{2\pi \mu_1} \right\} + \frac{4}{\pi^2} \zeta \eta U^0 \left( \ln 2 - \frac{4}{3} \right) \\ & + \frac{1}{2\mu_1} p_0^2 \left[ \frac{4}{3\pi} \zeta + \frac{4}{15\pi^2} (11 - 2 \ln 2) \zeta^2 \right], \end{aligned} \quad (29)$$

where, as in (24), the second order contribution to the correlation self-energy part is taken into account accurately by means of (19). The elementary excitation spectrum at the Fermi surface is thus determined by Eq. (3) with  $\lambda$  and  $\mu_{\text{eff}}$  taken, respectively, from Eqs. (28), (29), (26), and (27).

One checks easily that one can obtain the expression for  $\lambda$  given here by using the formula  $\lambda = (\partial E_0 / \partial N)_{\Omega}$  where  $E_0$  is the total energy of the system of  $N$  particles in the volume  $\Omega$ . According to the definition of  $\lambda$  given by Galitskiĭ and Migdal<sup>[2]</sup>,  $\lambda = \epsilon_{p_0}$ , i.e.,

$$\epsilon_{p_0} = [\partial (E_{\text{av}} N) / \partial N]_{\Omega} = E_{\text{av}} + N (\partial E_{\text{av}} / \partial N)_{\Omega},$$

and we have thus for self-compressed systems in accordance with the Van Hove-Hugenholtz theorem<sup>[11]</sup>  $\epsilon_{p_0} = E_{\text{av}}$ .

It is of interest to note that as  $p \rightarrow \infty$  all  $\Sigma$  except  $\Sigma_a$ ,  $\Sigma_c$ , and  $\Sigma_d$  tend to zero. Hence

$$\lim \text{Re} \Sigma(p) = \Sigma_a + \text{Re} \Sigma_{c,d} \cong (W - M/2) U(0) \rho + \frac{2p_0^3 a}{3\pi} \quad (30)$$

According to Galitskiĭ<sup>[4]</sup>  $\Sigma_{c,d} \rightarrow \text{const}$  as  $p \rightarrow \infty$ . Indeed, the amplitude  $a$  decreases and  $\text{Re} \Sigma_{c,d}$  also decreases but at any rate less than  $\Sigma_e$ , and so on. A particle with a large momentum ( $p \gg p_0$ ) will thus move in some self-consistent field created by the long-range potential and will interact with the other particles as a hard sphere. We note that (30) is valid for  $U(q)$  which are such

that  $\int_0^{\infty} U(q) q^2 dq < \infty$ , i.e.,  $U(r)|_{r=0}$  is finite.

This requirement does not lead in the system considered here to a limitation of the generality since the presence of a strong repulsion at small distances ( $r < a$ ) enables us to change  $U(r)$  arbitrarily at small  $r$ , i.e., for large momenta. A study of the dispersion law for particles with large momenta enables us thus to study the forces acting in the system<sup>7)</sup>.

<sup>7)</sup>This deduction is not conclusive, since damping may be so large for large momenta that it clearly means less to talk about a single-particle spectrum. The actual possibility of determining the forces  $U$  and  $a$  experimentally depends on their magnitude.

The expression (28) which we obtained for  $\lambda$  enables us to determine the low-frequency sound velocity. It is well known that (see, for instance, [12] 8)

$$c^2 = \partial P / \partial \rho = \rho \partial \lambda / \partial \rho. \quad (31)$$

We then find from (28)

$$\begin{aligned} c^2 \cong & \left( W - \frac{M}{2} \right) U_0 \rho + \frac{p_0^2}{3} + \frac{(M-W/2)}{3(2\pi)^2} \int_0^{p_0} \frac{q^3}{\rho_0} U(q) dq \\ & + \sum_{i=0,1} \frac{(2i+1)(-1)^i}{3(2\pi)^2 \rho_0} \int_0^{p_0} q^3 dq \\ & \times \int_{-\infty}^{+\infty} \frac{ds}{p_0^2 + s^2} \left\{ \frac{U_i^2(q) Q'_q(s)}{[1 + U_i(q) Q'_q(s)]^2} \left[ (-1)^i U_i(q) Q'_q(s) \right. \right. \\ & \left. \left. + 2\pi Q'_q(s) \frac{a}{\mu_1} \frac{3 - U_i(q) Q'_q(s)}{1 + U_i(q) Q'_q(s)} \right] \right\} + \frac{4p_0^4}{3\pi^3} a (\ln 2 - 1) U(2p_0) \\ & - \frac{2ap_0^3}{3\pi^3} \int_{p_0}^{\infty} U(q) dq + \frac{p_0^2}{3\mu_1} \left[ \frac{2}{\pi} p_0 a + \frac{8}{15\pi^2} (11 - 2 \ln 2) p_0^2 a^2 \right]. \end{aligned} \quad (32)$$

The presence in (32) of terms of both signs leads to the fact that  $c^2 = 0$  or even less than zero for well-defined values of the parameters of the system  $U$ ,  $a$ ,  $W$ ,  $M$ , and  $p_0$ , i.e., sound cannot propagate. For instance  $c^2 > 0$  if  $M/2$  is appreciably larger than  $W$  and  $c^2 < 0$  if  $W > M/2$  and  $U$  is sufficiently large.

It is of interest to note that  $c^2$  is always positive for self-compressed systems. Indeed, in that case

$$c^2 = \frac{\partial}{\partial \rho} \rho^2 \frac{\partial E_{\text{av}}}{\partial \rho} = 2\rho \frac{\partial E_{\text{av}}}{\partial \rho} + \rho^2 \frac{\partial^2 E_{\text{av}}}{\partial \rho^2} > 0, \quad (33)$$

since  $\partial E_{\text{av}} / \partial \rho = 0$  and  $\partial^2 E_{\text{av}} / \partial \rho^2 > 0$ . From this it is clear that for self-compressed systems, i.e., for all liquids occurring in nature, even estimates of the sound velocity in terms the first derivative of  $\lambda$ , evaluated using only part of the interaction (for instance, using the hard-sphere formula) and hence not taking into account that for the equilibrium density  $\lambda = E_{\text{av}}$ , have no physical meaning.

## 5. APPLICATION TO MANY-NUCLEON SYSTEMS

The results obtained in the preceding sections can be applied to a study of nuclear matter, an equilibrium system consisting of the same number of neutrons and protons at a density which is the same as the density of the interior regions of heavy nuclei. Indeed, the poles of the single-par-

<sup>8)</sup>We note that when we determine  $c$  from (28) we need not differentiate  $\int_0^{p_0} q^3 dq$  with respect to the upper limit for the reasons given earlier (see footnote <sup>3)</sup>).

title Green's function enable us to determine the energy spectrum of the  $(N+1)$ -st particle coming into a system of  $N$  particles ( $N \gg 1$ ).

It is well known that the experimental data on the scattering of nucleons by nuclei in a wide range of energies (with  $E > 0$ ) of the incoming particle can be interpreted as the scattering by some average potential. The results obtained enable us not only to understand this fact but also, at least qualitatively, to find the potential. Indeed, one can introduce in the many-body problem quasi-single-particle states, the wave functions of which satisfy the equation

$$\left(i \frac{\partial}{\partial t} + \frac{\nabla^2}{2}\right) \psi(x) - \int d^4x' \Sigma(x, x') \psi(x') = 0, \quad (34)$$

which in the momentum representation yields for a spatially-homogeneous system near  $E_0 - p^2/2 - \Sigma(p, E_0) = 0$  the relation

$$\left[ E_0 - \frac{p^2}{2} - \text{Re} \Sigma(p, E_0) - i \frac{\text{Im} \Sigma(p, E_0)}{1 - \partial \text{Re} \Sigma(p, E) / \partial E |_{E=E_0}} \right] \times \psi(p, E_0) = 0. \quad (35)$$

We evaluate  $\text{Re} \Sigma$  for nuclear matter at  $E > 0$ . To do this we assume that Serber forces ( $W = M = 1/2$ ) operate between the nucleons and we approximate the part corresponding to the attraction by a square well potential with a well depth of 30 MeV and a radius of 2.3 F. We assume that the core radius (repulsive core) is 0.4 F.<sup>[13] 9)</sup> The equilibrium density (for which  $\partial E_{\text{av}} / \partial \rho = 0$ ,  $\partial^2 E_{\text{av}} / \partial \rho^2 > 0$ ) of nuclear matter is taken to be the same as that in the interior regions of heavy nuclei, corresponding to a Fermi momentum of  $1.47 \times 10^{13} \text{ cm}^{-1}$ .

Since nuclear matter is a two-component system, we must generalize the results obtained earlier to this case. We shall, however, not give again the expressions for  $\lambda$  and  $\mu_{\text{eff}}$  in a two-component system for arbitrary  $W$  and  $M$ , since they are very cumbersome, but carry out the whole calculation in using suitable generalization of Eqs. (26) and (28).

We consider excitations that are sufficiently close to the Fermi surface. The criterion for closeness is the smallness of the ratio  $(E - \lambda) / p_0^2 \ll 1$ , which enables us to determine the spectrum up to  $E \sim 10$  MeV. In the calculations given in the following we shall start from the fact that the two-parameter approximation is valid in nuclear matter where  $p_0 b \approx 3.5$  and  $p_0 a \approx 0.6$ . It

is completely obvious that without a detailed evaluation of the omitted diagrams it is impossible to assume that 3.5 is a large parameter or 0.6 a small one. We can, however, check by a direct calculation that the two-parameter approximation is applicable. From (3), (26), and (28) we get for nuclear matter<sup>10)</sup>

$$E = -13.3 + 12.4(p - p_0). \quad (36)$$

Since  $\lambda = E_{\text{av}}$  in self-compressed systems, we get  $E_{\text{av}} = -13.3$  MeV, in good agreement with the first term in the semi-empirical Weizsäcker nuclear mass formula,  $-15.5$  MeV.

Expressing  $p$  in terms of  $E$ , the energy of the incoming particle, we find for  $\text{Re} \Sigma$  near the Fermi surface

$$\text{Re} \Sigma = \lambda - p_0^2/2 + (1/\mu_{\text{eff}} - 1)(E - \lambda) \quad (37)$$

or  $\text{Re} \Sigma = (-54 + 0.32 E)$  MeV. This result agrees well with the data given in <sup>[14]</sup>. If  $E < 0$  the solutions of (35) describe the quasi-single-particle excitations in the system.

We now consider the case of variable density and shall assume that the dependence of the density of the system on the coordinate is such that  $\text{Re} \Sigma$  is equal at each point  $r$  to its value for a homogeneous system of density  $\rho = \rho(r)$ . In that approximation we can evaluate the optical potential of heavy nuclei.<sup>11)</sup> The aim of the calculation will be a proof of the qualitative agreement with the parameters of the potential wells used in phenomenological calculations. We can use the formulae for  $\text{Re} \Sigma$  and (26) and (28) until  $p_0 b$  becomes appreciably larger than unity, i.e., until  $\rho(r)/\rho(0) \approx 0.2$ . The results of the calculation are given in Fig. 6. The optical model potential is

$$U(r) = U_1(r) + U_2(r)E. \quad (38)$$

We see the rather steep decrease of the potential  $U_1$  at the surface. As far as  $U_2$  is concerned, it increases somewhat at  $\rho = 0.6\rho_0$ . It is, however, possible that this is connected with the inaccuracies in the determination of  $\mu_{\text{eff}}$  which according to the calculations decreases somewhat at the surface while it is clear that  $\mu_{\text{eff}} \rightarrow 1$  as  $\rho \rightarrow 0$ .

The results obtained, as well as the approach to the nuclear problem by starting from the single-particle Green's function, enable us to give a

<sup>10)</sup>We have included terms  $-a^3$  in the calculation of  $\lambda$ .

<sup>11)</sup>In reality we must consider the influence of the inhomogeneity of the density more accurately. We shall, however, not do this here since we are only interested in the qualitative aspects of the problem.

<sup>9)</sup>As one can verify, the results are not very sensitive to the actual form of the potential. Since we are interested in qualitative estimates, the limitation in the choice of the interaction is not essential.



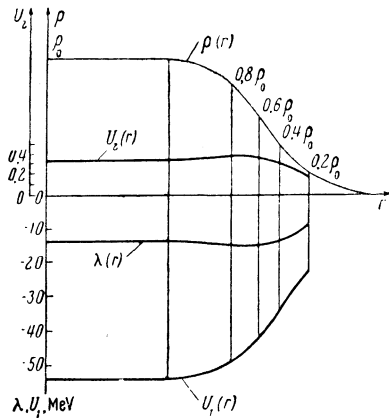


FIG. 6

qualitative foundation of the shell model. We saw that in a many-particle system with strong interactions there exists a branch of the excitation spectrum (quasi-single-particle branch) with wave functions described by Eq. (35) with a potential well defined by (38), i.e., by the shell model equation. It then becomes clear that the applicability of the shell model to a nucleus does not at all mean the independence of the motion of the nucleons in the nucleus, but the weakness of the interaction of the quasi-particles at the Fermi surface. Moreover, we see a direct connection between the shell-model potential describing the excitation spectrum for  $\lambda < E < 0$  and the optical potential ( $E > 0$ ). The close agreement between their parameters of the phenomenological theories seems surprising, yet it is clear from (35) and (38) that this is in fact the same potential, taken in the one case for  $E - \lambda \leq 3$  to 4 MeV (shell model) and in the other case for  $E - \lambda > 15$  MeV. We must make here one important observation: the excitation spectrum studied by the shell model corresponds in the many-body-theory language to the particle and hole spectrum at the Fermi surface. However, if we may neglect the residual particle-hole interaction, their excitation energy is equal to the sum of the excitation energies of a particle and a hole separately and is determined by (35) and (38).

By way of another example in which the earlier results can be used, we evaluate the symmetry energy. Up to now we have considered the case where the number of protons is equal to that of the neutrons. If the number of neutrons is not equal to that of the protons the average energy per nucleon increases. According to Weizsäcker's formula

$$\Delta E_{av} = 22 \left( (\rho_p - \rho_n) / (\rho_p + \rho_n) \right)^2 \text{ MeV},$$

where  $\rho_p$  and  $\rho_n$  are the proton and neutron den-

sities,  $\rho_p + \rho_n = \rho$ . The quantity  $\Delta E_{av}$  is called the symmetry energy. Let the initial density of the system be  $\rho$  ( $\rho_p = \rho_n = \rho/2$ ). The increase in the energy per unit volume of the system when we change the proton and neutron densities from  $\rho/2$  to  $\rho_p$  and  $\rho_n$  will then be equal to the sum of the energies of the additional quasi-particles. In other words,

$$\Delta E_{av} = \frac{1}{\pi^2 \rho} \left[ \int_{\rho_0}^{\rho_{0p}} \left( \lambda + \frac{\rho_0 (\rho - \rho_0)}{\mu_{eff}} \right) \rho^2 d\rho + \int_{\rho_0}^{\rho_{0n}} \left( \lambda + \frac{\rho_0 (\rho - \rho_0)}{\mu_{eff}} \right) \rho^2 d\rho \right], \quad (39)$$

where  $\rho_{0n}$  and  $\rho_{0p}$  are, respectively, the neutron and the proton Fermi momenta. We must assume in (39) that  $\rho_p + \rho_n = \rho$ . Let  $\rho_p - \rho_n \ll \rho$ ; we have then from (39)

$$\Delta E_{av} = \frac{1}{6\mu_{eff}} \rho_0^2 \left( \frac{\rho_p - \rho_n}{\rho_p + \rho_n} \right)^2 = 20 \left( \frac{\rho_p - \rho_n}{\rho} \right)^2, \quad (40)$$

i.e., we get good agreement with Weizsäcker's formula. In the derivation we neglected interaction between the quasi-particles. We can, however, take such a small change in density that this neglect is fully justified.

We can approach Eqs. (37) and (40) from a slightly different point of view. According to the derivation, both  $\Delta E_{av}$  and the term  $\sim E$  in (37) are determined by  $\mu_{eff}$  on the Fermi surface, and this enables us to determine this quantity experimentally. It is clear that both (37) and (40) lead to values of  $\mu_{eff}$  close to unity.

Just as the symmetry energy, we can also determine the compressibility of the nuclear matter  $K = 9\rho^2 \partial^2 E_{av} / \partial \rho^2$ . If the average energy was  $E_{av}^0$  for some density  $\rho_0$ , we have for a density  $\rho$  [ $(\rho - \rho_0) / \rho \ll 1$ ]

$$E_{av} = \frac{1}{\rho} \left[ E_{av}^0 \rho_0 + \frac{2}{\pi^2} \int_{\rho_0}^{\rho_0'} \left( \lambda + \frac{\rho_0 (\rho - \rho_0)}{\mu_{eff}} \right) \rho^2 d\rho \right], \quad (41)$$

where  $\rho_0$  and  $\rho_0'$  are the Fermi momenta corresponding to  $\rho_0$  and  $\rho$ . We find from (41)

$$\partial E_{av} / \partial \rho |_{\rho=\rho_0} = -E_{av} / \rho_0 + \lambda / \rho_0, \quad (42)$$

$$K = 18 (E_{av} - \lambda) + 3\rho_0^2 / \mu_{eff}. \quad (43)$$

For the self-compressed system which interests us, nuclear matter,  $\lambda = E_{av}$ , i.e.,  $\partial E_{av} / \partial \rho = 0$ , and  $\partial^2 E_{av} / \partial \rho^2 > 0$  and

$$K \cong 350 \text{ MeV}. \quad (44)$$

Equation (44) expresses  $K$  in terms of  $\mu_{eff}$  and thus in terms of (40). This enables us in practice to find  $K$  experimentally.

## 6. CONCLUSIONS

In the present paper we have extended I and considered the single-particle Green's function. We have shown that a study of  $\text{Re } \Sigma(p)$  at high energies, of the excitation spectrum, of the sound velocity, and of the chemical potential enables us to determine several parameters characterizing the two-body interaction. We have ascertained that the possibility of propagating low-frequency sound depends essentially on whether the system is self-compressed and also on the character of the forces, namely the ratio of the Wigner to the Majorana forces.

The introduction of the effective mass  $\mu_2$  at the Fermi surface and an effective mass  $\mu_1$  averaged over a momentum range from 0 to  $p_0$  enabled us to take into account reducible self-energy diagrams. We showed that  $\mu_1$  and  $\mu_2$  depend essentially on the character of the forces, which in turn leads to a dependence on the character of the effective repulsive forces.

The application of the results obtained to systems consisting of particles possessing isospin requires very elementary generalizations.

Further problems of a two-parameter system, in particular, the two-particle Green's function, will be considered in another paper.

The system considered is a convenient model which can find an application in the study of nuclear matter and liquid  $\text{He}^3$ . Indeed, it is clear that the existence of a self-compressed state at finite densities requires the presence of both repulsive and attractive forces. One needs therefore only verify that indeed the experimentally determined parameters  $p_0a$  and  $(p_0b)^{-1}$  are small. At the present we have obtained by a direct calculation results which enable us to assert that for the experimentally known nucleon-nucleon forces, nuclear matter is a two-parameter system with small parameters  $p_0a$  and  $(p_0b)^{-1}$ . One can hope that this may enable us to develop a consistent microscopic theory of nuclear matter free from the objection usually raised against the Brueckner model. The value obtained in the last section for the average energy per nucleon in a

nucleus ( $-13.3$  MeV), for the real part of the optical potential, and for the symmetry energy convince us of the reasonableness of the results obtained using the present model for nuclear matter and heavy nuclei. The calculation is, however, still very rough. One needs both use more realistic nucleon-nucleon forces (for instance, Gammel-Christian-Thaler forces, and not Serber forces) and study the influence of the inhomogeneity of the density. Because of this a detailed study of real systems will be done elsewhere.

In conclusion the author finds it a pleasant duty to express his deep gratitude to Prof. A. A. Sliv and also to G. M. Shklyarevskii for many discussions and valuable advice.

<sup>1</sup>M. Ya. Amus'ya, JETP **41**, 429 (1961), Soviet Phys. JETP **14**, 309 (1962).

<sup>2</sup>V. M. Galitskiĭ and A. B. Migdal, JETP **34**, 139 (1958), Soviet Phys. JETP **7**, 96 (1958).

<sup>3</sup>A. B. Migdal, JETP **40**, 684 (1961), Soviet Phys. JETP **13**, 478 (1961).

<sup>4</sup>V. M. Galitskiĭ, JETP **34**, 151 (1958), Soviet Phys. JETP **7**, 104 (1958).

<sup>5</sup>A. Klein and R. Prange, Phys. Rev. **112**, 994 (1958).

<sup>6</sup>L. D. Landau, JETP **30**, 1058 (1956), Soviet Phys. JETP **3**, 920 (1956).

<sup>7</sup>V. I. Karpman, JETP **39**, 185 (1960), Soviet Phys. JETP **12**, 133 (1961).

<sup>8</sup>D. A. Kirzhnits, JETP **37**, 585 (1959), Soviet Phys. JETP **10**, 414 (1960).

<sup>9</sup>N. M. Hugenholtz, Physica **23**, 533 (1957).

<sup>10</sup>K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).

<sup>11</sup>N. M. Hugenholtz and L. Van Hove, Physica **24**, 363 (1958).

<sup>12</sup>A. A. Abrikosov and I. M. Khalatnikov, UFN **66**, 177 (1958), Soviet Phys. Uspekhi **1**, 68 (1959).

<sup>13</sup>G. M. Vagradov and D. A. Kirzhnits, JETP **38**, 1499 (1960), Soviet Phys. JETP **11**, 1082 (1960).

<sup>14</sup>I. S. Shapiro, UFN **75**, 61 (1961), Soviet Phys. Uspekhi **4**, 674 (1962).

Translated by D. ter Haar