DETERMINATION OF THE AMPLITUDES FOR π-MESON PRODUCTION IN NEUTRINO-NUCLEON COLLISIONS BY MEANS OF DISPERSION RELATIONS

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A study is made of the properties of the amplitudes for π-meson production in neutrino-nucleon collisions; the one-dimensional dispersion relations are written for these amplitudes, and solutions of the integral equations are obtained on the assumption that the (33) P wave predominates and that the contribution of other waves to the imaginary parts of the amplitudes can be neglected. Comparison of the results with experiment will provide a test of the V-A theory of weak interactions.

AMONG experimental studies of the weak interaction, experiments with neutrinos are of great interest, since neutrinos do not take any direct part in the electromagnetic and strong interactions. The possibility of experiments with neutrinos has been discussed in many papers (cf. e.g., [1]). The cross sections for π-meson production in the collision of a neutrino or antineutrino with a nucleon,

$$\nu(\bar{\nu}) + N \to e^-(e^+) + \pi + N, \tag{1}$$

have also been estimated by Azimov[2] and by Berman.[3] In the present paper the amplitudes for the processes (1) at low energies are determined by means of dispersion relations.

As is well known, the V-A theory of the weak interaction[4] has successfuly explained the experimental data on β decay, μ-meson decay, and π-meson decay. According to this theory the Lagrangian of the weak interaction is of the form

$$L = (G/\sqrt{2})\, J_\mu J_\mu^+, \tag{2}$$

where J_μ is the sum of the lepton currents $i\bar{\nu}\gamma_\mu(1+\gamma_5)e + i\bar{\nu}\gamma_\mu(1+\gamma_5)\mu$ and the currents of the strongly interacting particles: the strangeness-conserving current $j_\mu = j_\mu^V + j_\mu^A$ and the strangeness-nonconserving current $S_\mu = S_\mu^V + S_\mu^A$. The vector part of the current j_μ is an isotopic component of the conserved isovector $\mathbf{V}_\mu$, whose third component is proportional to the isovector electromagnetic current of the strongly interacting particles:

$$\mathbf{V}_\mu = \frac{i}{\sqrt{2}}\bar{N}\boldsymbol{\tau}\gamma_\mu N + i\sqrt{2}\,\Pi^*\mathbf{T}\left(\frac{\overleftarrow{\partial}}{\partial x_\mu} - \frac{\overrightarrow{\partial}}{\partial x_\mu}\right)\Pi,$$

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \qquad \Pi = \begin{pmatrix} \pi_+ \\ \pi_0 \\ \pi_- \end{pmatrix}; \tag{3}$$

$$j_\mu^V = V_\mu^1 + iV_\mu^2, \qquad j_\mu^{V+} = V_\mu^1 - iV_\mu^2. \tag{4}$$

With neglect of effects of strange particles the axial part of the current j_μ is also an isotopic component of an isovector

$$\mathbf{A}_\mu = \frac{i}{\sqrt{2}}\bar{N}\boldsymbol{\tau}\gamma_\mu\gamma_5 N, \tag{5}$$

$$j_\mu^A = A_\mu^1 + iA_\mu^2, \qquad j_\mu^{A+} = A_\mu^1 - iA_\mu^2 \tag{6}$$

and therefore the current j_μ also has this property:

$$j_\mu = I_\mu^1 + iI_\mu^2, \qquad j_\mu^+ = I_\mu^1 - iI_\mu^2, \tag{7}$$

where I_μ^α is an isovector.

According to the relations (2) and (7) the matrix elements of the processes (1) are of the form

$$M_\nu = \frac{iG}{\sqrt{2}}\bar{u}_e\gamma_\mu(1+\gamma_5)\,u_\nu\langle\pi N\,|\,I_\mu^1 + iI_\mu^2\,|\,N\rangle,$$

$$M_{\bar{\nu}} = \frac{iG}{\sqrt{2}}\bar{v}_\nu\gamma_\mu(1+\gamma_5)\,v_e\langle\pi N\,|\,I_\mu^1 - iI_\mu^2\,|\,N\rangle. \tag{8}$$

The calculation of the cross sections of these processes requires a study of the matrix elements $\langle\pi N\,|\,I_\mu^\alpha\,|\,N\rangle$ of the current I_μ^α between the state of one nucleon and the states of the system πN. The vector parts of these matrix elements also occur in the matrix elements for photoproduction and electroproduction of π mesons from nucleons. The low-energy photoproduction has been studied in many papers (cf. e.g., [5-8]). In [9] and [10] the electroproduction of π mesons from nucleons has been treated as production of a π meson by a virtual photon—virtual photoproduction. The singularities in the momentum transfer of the electron are partly taken care of by the introduction of electromagnetic form-factors of the nucleon and π meson.

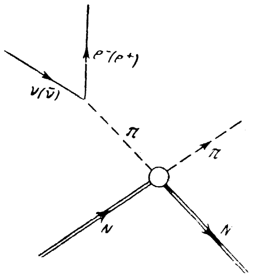

FIG. 1

In the present paper the neutrinic production of π mesons from nucleons is treated in an analogous way. The singularities in the momentum transfer of the leptons are also dealt with by the introduction of neutrinic form-factors of nucleon and π meson. We note that the axial-vector part of the matrix elements has a pole corresponding to the diagram of Fig. 1, but the contributions of this pole, and also those of the pseudoscalar effective neutrinic form-factor of the nucleon, are very small because of the smallness of the electron mass. For zero electron mass these contributions are zero. Hereafter we shall suppose that the electron mass is zero and shall not include these contributions.

Let us introduce the following notations: k_1, k_2, and q are the respective four-momenta of the neutrino (or antineutrino), the electron (or positron), and the π meson; p_1 and p_2 are the four-momenta of the nucleon in the initial and final states; $P = (p_1 + p_2)/2$, $k = k_1 - k_2$; M and m are the masses of nucleon and π meson; and θ is the angle between the momenta $\mathbf{k}$ and $\mathbf{q}$ in the center-of-mass system of the πN system.

The scalar amplitudes depend on three invariant independent variables:

$$\nu = -Pk/M, \quad \nu_B = kq/2M, \quad k^2. \tag{9}$$

We assume that these amplitudes satisfy a dispersion relation without subtraction in the variable ν for fixed values of the other variables. As in the case of photoproduction, in a region of energies and angles satisfying the relation

$$\cos\theta \approx (k^0 q^0 - k^0_{\text{th}} m)/|\mathbf{k}||\mathbf{q}| \tag{10}$$

(k^0_{th} is the threshold value of k^0 for given k^2) the dispersion integrals do not involve the unobservable region. For small energies and corresponding angles satisfying Eq. (10) the essential contribution to the imaginary parts of the amplitudes is that from the resonance (33) P wave, and we can neglect the contributions of other waves. The agreement between theory and experiment for photoproduction in this range of energies and angles allows us to hope that the results obtained in this paper are also valid in this region. In other ranges of angles there can be some deviation because of the influence of the nonphysical region. [8]

To shorten the writing we shall denote $i\bar{u}_e\gamma_\mu(1+\gamma_5)u_\nu$ or $i\bar{v}_\nu\gamma_\mu(1+\gamma_5)v_e$ by ϵ_μ. The mass of the electron is taken to be zero. Furthermore we then have

$$\varepsilon k = 0. \tag{11}$$

Let us set

$$\varepsilon_\mu \langle \pi_\beta N \mid I^\alpha_\mu \mid N\rangle = \varphi^*_\beta \bar{U}_2 T_{\beta\alpha} U_1, \tag{12}$$

where φ_β, U_1, and U_2 are the wave functions for the meson and for the nucleon in its initial and final states. As has been pointed out, I^α_μ is an isovector. Therefore the isotopic structure of $T_{\beta\alpha}$ is analogous to that of the amplitudes for the elastic scattering of a π meson by a nucleon:

$$T_{\beta\alpha} = \tfrac{1}{2}\{\tau_\beta, \tau_\alpha\} T_{(+)} + \tfrac{1}{2}[\tau_\beta, \tau_\alpha] T_{(-)}. \tag{13}$$

From invariance arguments, the Dirac equation for the nucleon, and the relation (11) it follows that $T_{(\pm)}$ is of the form

$$T_{(\pm)} = \sum_{i=1}^{6} (V^i_{(\pm)}\gamma_5 + A^i_{(\pm)})\, T_i,$$

where

$$\begin{array}{llll} T_1 = \varepsilon\gamma & (-)\,(-) & T_4 = 2i\varepsilon p & (+)\,(-) \\ T_2 = i\,(\varepsilon\gamma)\,(k\gamma) & (+)(-); & T_5 = (\varepsilon q)\,(k\gamma) & (-)\,(-). \\ T_3 = i\varepsilon q & (-)\,(+) & T_6 = 2\,(\varepsilon P)\,(k\gamma) & (+)\,(+) \end{array} \tag{14}$$

Regarded as functions of ν, ν_B, and k^2, the independent scalar amplitudes $V^i_{(\pm)}$ and $A^i_{(\pm)}$ have definite crossing-symmetry properties: the signs + and − in Eq. (14) indicate the parity of these amplitudes under the substitution $\nu \to -\nu$, $\nu_B \to \nu_B$, and $k^2 \to k^2$; the first column is for $V^i_{(+)}$, the second for $A_{(+)}$, and the amplitudes $V^i_{(-)}$ and $A^i_{(-)}$ have the opposite parities.

We assume that the scalar amplitudes $V^i_{(\pm)}$ and $A^i_{(\pm)}$ satisfy dispersion relations without subtraction in the variable ν for fixed ν_B and k^2. Omitting effects of $\pi\pi$ interaction, we write the dispersion relations in the form

$$\operatorname{Re} H_i(\nu, \nu_B, k^2) = C_i(\nu_B, k^2) + R_i\left[\frac{1}{\nu_B - \nu} \pm \frac{1}{\nu_B + \nu}\right] + \frac{1}{\pi} P\int_{\nu_0}^{\infty} d\nu' \operatorname{Im} H_i(\nu', \nu_B, k^2)\left[\frac{1}{\nu' - \nu} \pm \frac{1}{\nu' + \nu}\right],$$

$$\nu_0 = \nu_B + (2Mm + m^2)/2M. \tag{15}$$

The signs + or − in these dispersion relations

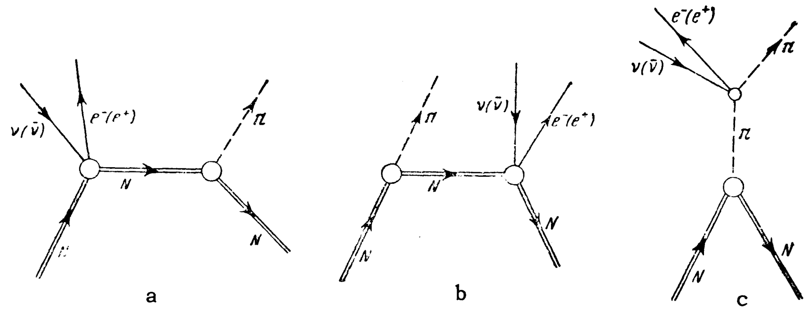

FIG. 2

are determined by the crossing-symmetry properties (14) of the individual amplitudes. The constants $C_i(\nu_B, k^2)$ and the residues R_i are determined by the Born term corresponding to the diagrams of Fig. 2.

According to Eqs. (3) and (4) the contribution of the diagram 2, c is determined by the electromagnetic form-factor of the π meson. Because of the weak-magnetism effect [11] the contribution of the vector parts of diagrams 2, a, b is completely determined by the electromagnetic form-factors and the anomalous magnetic moments of the nucleons. As for the contribution of the axial parts, it is determined by the axial β-decay form-factor, which has been calculated by Goldberger and Treiman. [12]

When we take into account only the contribution of the resonance (33) P wave to the imaginary parts of the amplitudes, the linear integral equations obtained from the dispersion relations and the unitarity condition can be solved by means of the method given in [7]. Here it is convenient to go over to the new variable

$$x = \nu - \nu_B.$$

The approximate solutions of the integral equations are of the form

$$H_i(x, \nu_B, k^2) = C_i(\nu_B, k^2) + R^i\left(\frac{1}{\nu_B - \nu} \pm \frac{1}{\nu_B + \nu}\right) + \frac{e^{i\delta(x)}}{\pi}\int\limits_{1+m/2M}^{\infty} dy \sin\delta(y)\, b_i(y, \nu_B, k^2) \times \exp\{\rho(x, \nu_B) - \rho(y, \nu_B)\}\left[\frac{1}{y - x - i0} \pm \frac{1}{y + x + 2\nu_B}\right],$$

with (16)

$$\rho(x, \nu_B) = \frac{1}{\pi}\,\mathrm{P}\int\limits_{1+m/2M}^{\infty} dx'\,\delta(x')\left[\frac{1}{x' - x} + \frac{1}{x' + x + 2\nu_B}\right]$$

for $\delta(\infty) = 0$,

$$\rho(x, \nu_B) = \frac{(x + \nu_B)^2}{\pi}\,\mathrm{P}\int\limits_{1+m/2M}^{\infty} dx'\delta(x')\,\frac{1}{[x' + \nu_B]^2} \times\left[\frac{1}{x' - x} + \frac{1}{x' + x + 2\nu_B}\right] \quad \text{for } \delta(\infty) = \pi. \qquad (17)$$

Here $\delta(x)$ is the phase of the (33) P wave of πN scattering.

The quantities b_i are determined by the projection of the Born term on the (33) state. Concrete expressions for R_i, C_i, and b_i have been given by the writer in [13].

We have obtained the invariant amplitudes of the processes (1) by means of dispersion relations, starting from the hypotheses of the V-A theory [4,11] of the weak interaction. Comparison with experiment will provide a test of these hypotheses.

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Translated by W. H. Furry

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ERRATA

Vol	No	Author	page	Correction
15	6	Turov	1098	The article contains an erroneous statement that weak ferromagnetism cannot exist in any cubic crystal (with collinear or weakly noncollinear antiferromagnetic structure. This was found to be true only for crystal classes T and T_h, and for others weak ferromagnetism will appear in antiferromagnets with magnetic structure type $3^+ 4^-$, and only due to invariants of third and higher orders in the antiferromagnetism vector L. Consequently a line (14) should be added to the table on p. 1100: $14 \mid 207\text{-}230 \mid \text{Cubic} \mid 3^+, 4^- \mid M_xL_x(L_y^2 - L_z^2) + M_yL_y(L_z^2 - L_x^2) + M_zL_z(L_x^2 - L_y^2) \mid \text{VI}$ The Cartesian axes are directed here parallel to the fourfold symmetry axes. The tensors $g^{(1)}$ and $g^{(2)}$ for this (sixth) group of weakly ferromagnetic structures will be identically equal and isotropic: $g_{\alpha\beta}^{(1)} = g_{\alpha\beta}^{(2)} = g\delta_{\alpha\beta}$
16	1	Valuev	172	At the end of the article there are incorrect expressions pertaining to $K_{\mu 3}$ decay. The correct formula can be easily obtained from the main formula of the article by putting $g_S = g_T = 0$. The tangent of the angle between the $\mid m \mid^2$ curve and the $\cos\theta$ axis will be $\approx \beta_e$ if $g_{V2}/g_{V1} = -0.5$ and ≈ 0 if $g_{V2}/g_{V1} = 4.5$ and $\beta_e \sim 1$, so that in fact the difference in the angle correlations between these cases is even somewhat stronger than indicated in the article.
16	1	Zhdanov et al	246	The horizontal parts of curves 2 and 3 in Fig. 2 should be drawn with solid lines (they correspond to the asymptotic calculated values of the ionization losses, i.e., to the region in which the theory describes the relation between g/g_0 and the particle energy exactly).
16	1	Deutsch	478 & 481	When account is taken of thermoelectric processes it is necessary to add in the first curly bracket of (24) the term $A = 3v_0^2 H_y c(\alpha_{xz} - \alpha_{zx})/2$ and in Eq. (31) the term $A/9$.
16	1	Nguyen	920 Eqs. (4), (6), (7), & (8)	The combinations $V^1 \pm V^2$, $A^1 \pm A^2$, and $I^1 \pm I^2$ should be divided by $\sqrt{2}$.
16	1	Gershtein et al	1097 Eq. (1)	Reads $G/\sqrt{2}$, should read $G/2$
16	5	Gurevich		An error has crept into Eq. (30). The right half of this formula is actually equal to $\varepsilon_E \frac{\delta_{kz}}{2\pi E}\left[F_0(\varepsilon) + 2\varepsilon \frac{d}{d\varepsilon} F_0(\varepsilon)\right]$.