EFFECT OF ππ INTERACTION ON s *AND* p *WAVES OF* πN *SCATTERING*

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On the basis of the Mandelstam representation, account is taken of the effect of ππ interaction in s- and p-waves of πN scattering. Transition to partial waves is effected by combining the dispersion relations for forward and backward scattering. The equations for s^-- and p^--waves are compared with experimental data on the πN-scattering phases. Satisfactory agreement is obtained for incident π-meson energies up to ~500 MeV in the laboratory coordinate system. It is shown that ππ interaction affects significantly the energy dependence of the s^-- and $p^-_{1/2}$-phase even at low energies, and up to ~400 MeV (in the laboratory system) it has but a slight effect on the behavior of the $p^-_{3/2}$-phase.

INTRODUCTION

IN the present paper we derive, with the aid of the double Mandelstam representation, equations for the partial πN-scattering waves at low energies with account of the ππ interaction. This paper is closely related with one by Efremov, Shirkov, and one of the authors [1], the notation of which is used throughout.

In the present paper we consider states with isotopic index (−), i.e., the functions α and $B^{(-)}$ [see (3.5) in [1]], the equations for which contain only the p wave of the ππ scattering process. The transition to partial waves is by combining the dispersion relations for $c = \pm 1$.[1)] The advantages of this method of obtaining the equations were discussed in detail previously [3].

In Sec. 1 we consider the form of the functions and $B^{(-)}$ in the region of reaction III ($\pi\pi \to N\bar{N}$), which follows from the unitarity condition. In Sec. 2 we take into account by the Muskhelishvili method the influence of the cut due to the reaction III, and discuss the ambiguities that arise in an account of the ππ interaction. The choice of the specific pion form factor is made in Sec. 3. In Secs. 4 and 5 equations are derived for the $s^{(-)}$ and $p^{(-)}$ partial amplitudes with account of the crossing symmetry. The principal terms of the equations satisfy, when expanded in powers of 1/M, the properties of crossing symmetry for a fixed nucleon. In Sec. 6 a comparison is made with the experimental data. The importance of taking the ππ interaction into account is shown.

[1)]This method was applied by Chu Hung-yuan[2] to πN scattering.

1. INVESTIGATION OF THE CUT DUE TO REACTION III

Various approaches to the study of πN scattering are connected with different methods of taking the influence of the cut due to reaction III into account. Consequently the correctness of taking this cut into account is important and must be demonstrated.

We consider the limitations imposed on the type of the functions α and $B^{(-)}$ by the condition of unitarity of the process III. In the two-meson approximation the condition for the unitarity of processes III has the simplest form for the partial amplitudes $f^l_\pm$:

$$\operatorname{Im} f^l_\pm = f^l_\pm \exp(-i\delta^l_I) \sin\delta^l_I. \qquad (1.1)$$

Here δ^l_I are the ππ-scattering phases and I is the total isotopic spin of the ππ system.

We shall use analytic continuation of the unitarity condition (1.1) into the nonphysical region of $4 < t < 16$ process III.[2)] Equations (1.1) are valid up to the first threshold of inelastic ππ scattering processes, where all the phases δ^l_I are real, and the higher waves are small. Therefore we assume that

$$\delta^l_I = 0, \qquad l \geqslant 2. \qquad (1.2)$$

Then $f^l_\pm$ with $l \geq 2$ are real functions, while J_{++} and J_{+-} have the form [3)]

$$J_{++} = \rho_1 \exp(i\delta^1_1) + \rho_2, \qquad J_{+-} = \rho_3 \exp(i\delta^1_1) + \rho_4, \quad (1.3)$$

where ρ_i are unknown functions.

[2)]We put $\mu = 1$ throughout.

[3)]For expressions for J_{++} and J_{+-} and their expansions in partial waves f_l cf. [1].

To determine the form of ρ_2 and ρ_4 we expand the functions J_{++} and J_{+-} without the pole terms. The region of convergence of the expansions is now given by the double integrals of the spectral representations (curves C_{13} and C_{23} in the Mandelstam notation). We note that the $f'_{\pm}$ also contain harmonics of the pole terms. Therefore, in order to use (1.1) without changing the structure of (1.3), we subtract from the functions J_{++} and J_{+-} the pole terms without the first harmonics. Confining ourselves then to the first terms of the expansions, we obtain the functions ρ_2 and ρ_4 in explicit form. We now are able to write down the expressions of interest to us for α and $B^{(-)}$:

$$\alpha = \tilde{\rho} e^{i\delta} + \frac{g^2}{4p_3^2} M(\Delta + \Delta_1),$$

$$B^{(-)} = \rho e^{i\delta} + g^2(\Delta - \Delta_0), \qquad (1.4)$$

where ρ and $\tilde{\rho}$ are unknown real functions, and

$$\Delta_0 = \frac{1}{2}\int_{-1}^{+1} \Delta d\cos\theta_3, \qquad \Delta_1 = \frac{3}{2}\int_{-1}^{+1} \cos^2\theta_3 \cdot \Delta\, d\cos\theta_3.$$

In terms of the variables ν and c, formulas (1.4) are valid when $\nu \geq C_R(c)$ (see [4]). We shall use them for $c = -1$.

2. $\pi\pi$ INTERACTION AND ITS CALCULATION

The usual method of changing over to the equations for partial waves by integrating over the interval $-1 \leq c \leq +1$ involves the use of expansions in Legendre polynomials in regions where these expansions are known to diverge [3]. The dispersion relations for $c = \pm 1$ make it possible to circumvent these difficulties [4,5].

The use of dispersion relations for backward scattering is the most convenient from the point of view of accounting for the influence of the $\pi\pi$ interaction. In the present paper the changeover to partial amplitudes will be via a combination of the dispersion relations for $c = \pm 1$. The number of relations is then sufficient to obtain the equations for the $s^{(-)}$ and $p^{(-)}$ partial amplitudes. The d-waves can be accounted for by including the dispersion relations for the derivatives with respect to c of the scalar πN-scattering amplitude functions at $c = -1$. However, the d-waves so determined will contain large errors [5].

Dispersion relations for the functions $\Phi = (\alpha, B^{(-)})$ with $c = -1$ have the form

$$\Phi(\nu) = \frac{1}{\pi}\int_0^{\infty} \frac{\operatorname{Im}\Phi(\nu')}{\nu' - \nu} d\nu' + \frac{1}{\pi}\int_{-\infty}^{-1} \frac{\operatorname{Im}\Phi(\nu')}{\nu' - \nu} d\nu' \qquad (2.1)$$

plus pole terms for $B^{(-)}$.

The anti-Hermitian parts of $\Phi(\nu)$ are specified on the cut $-\infty, -1$ by formulas (1.4). Strictly speaking, they are valid up to $\nu = -4$ (corresponding to $t = 16$), but it is usually assumed that the region of their validity is somewhat broader [6]. Formulas (1.4) represent the amplitude of the $\pi\pi \to N\bar{N}$ process in the nonphysical region $4 < t < 16$. The first terms are the p-wave amplitudes, while the second are sums of all the partial amplitudes with higher angular momenta. Neglect of terms with large l in the expansions of J_{++} and J_{+-} is equivalent to assuming that the higher angular momenta can be well approximated by the pole terms [7]. Then, in accord with (1.2), $\operatorname{Im} f_{\pm}^{l} = 0$ if $l \geq 2$. The latter follows from the properties of the functions Δ_0 and Δ_1, which are the "physical" branches of the analytic functions $\Delta_0(z)$ and $\Delta_1(z)$. The accuracy of the approximate expressions (1.4) calls for an additional analysis. In our case, however, it seems to be sufficient (see [8]).

The integral of reaction III along the cut contains the unknown functions ρ and $\tilde{\rho}$, which relate the πN scattering equations with the equations for $\pi\pi \to N\bar{N}$. Usually additional assumptions are made regarding these functions. Bowcock et al [9] replaced them by the values of ρ and $\tilde{\rho}$ at the point corresponding to the interaction resonance ($T = J = 1$). An analogous device is used by Hamilton et al. [10]

If we neglect the higher waves in (1.4), we have on the left cut for the functions $\Phi = (\alpha, B^{(-)})$ a linear boundary condition $\Phi^{(+)} = \exp(-i2\delta_1^1)\Phi^{(-)}$. On the right cut the functions Φ are expressed in terms of the known πN-scattering phases, and we arrive at a linear boundary problem for the functions Φ. [11] The solution methods are known [12]. This, however, raises the question of the choice of solution, which is not unique even in the class of functions that are bounded on the cut. Furthermore, the role of the higher waves of process III cannot be estimated in this manner.

Let us attempt to eliminate from the dispersion relations the unknown functions ρ and $\tilde{\rho}$, but in such a way as to take into account the higher partial waves of process III. To do so we use the following procedure. We denote by $F(\nu)$ the class of functions which are unique in the ν plane with cut $-\infty, -1$. On the cut

$$F^{(+)}(\nu) = e^{-2i\delta(\nu)} F^{(-)}(\nu). \qquad (2.2)$$

We impose no other limitations on $F(\nu)$. We consider in place of $\Phi(\nu)$ the function $\Phi(\nu)F^{-1}(\nu)$. We then have on the left cut

$$\operatorname{Im}\Phi(\nu)F^{-1}(\nu)=\left(\frac{1}{F^{(+)}(\nu)}-\frac{1}{F^{(-)}(\nu)}\right)\varphi(\nu),$$

where $\varphi(\nu)$ is the known contribution of the higher waves of process III. Thus, the dispersion relations for $\Phi(\nu)F^{-1}$ solve the problem posed above. Their specific form, however, depends on the behavior of $F(\nu)$ at infinity and on the distribution of the zeroes of $F(\nu)$. The ambiguity of the solution lies in the choice of the degree of the "subtraction" polynomial and in the determination of the residues at the zeroes of $F(\nu)$. The choice of a "physical" function $F(\nu)$ guarantees correctness of the calculation of the ππ interaction.

3. ELECTROMAGNETIC PION FORM FACTOR

The electromagnetic pion form factor $F_\pi(\nu)$ satisfies in the two-meson approximation the boundary condition (2.2). It is therefore natural to choose as the function $F(\nu)$ the form factor $F_\pi(\nu)$. The ambiguity of the solution of the integral equation (2.1) (which is linear on the cut $-\infty, -1$) is thereby reduced to the ambiguity in the determination of $F_\pi(\nu)$.

The general solution of (2.2) has the form

$$F_\pi(\nu)=P(\nu)e^{u(\nu)},$$

$$u(\nu)=-\frac{\nu}{\pi}\int_0^\infty\frac{\delta_1^1(k^2)\,dk^2}{(k^2+1)(k^2+1+\nu)}, \qquad (3.1)$$

where $P(\nu)$ is an arbitrary polynomial, δ_1^1 the phase shift corresponding to the state $J = T = 1$, and k^2 the modulus of the c.m.s. pion momentum in the ππ interaction. If we stipulate that the form factor $F_\pi(\nu)$ be bounded on the cut, then the degree of the polynomial $P(\nu)$ is determined by the behavior of the phase shift δ_1^1 at infinity. From the normalization condition $F_\pi(0) = 1$ it follows that $P(0) = 1$. The question of the ambiguity of $F_\pi(\nu)$ was discussed, in particular, in [13]. We use the generally accepted assumption that $P(\nu) \equiv 1$ (see [14]).

The specific form of the phase shift $\delta_1^1(k^2)$ is unknown. In accordance with the existing hypothesis concerning the resonance in the ππ-scattering wave with $J = T = 1$ we put

$$ak^3\operatorname{ctg}\delta_1^1=k_r^2-k^2. \qquad (3.2)^*$$

Carrying out the integration in (3.1) with the aid of the residue theorem, we obtain an exact expression for $F_\pi(\nu)$.[4)] For values of $\epsilon^2 = (ak_r)^2 \ll 1$ the form of $F_\pi(\nu)$ simplifies to

$$F_\pi(\nu)=\frac{k_r+1/(k_r+\varepsilon)}{k_r+\omega^2/(k_r+\varepsilon\omega)}, \qquad \omega^2>0;$$

$$|F_\pi(\nu)|=\left[\frac{(k_r^2+1)^2+\varepsilon^2(k_r^4-(k_r^2+1)/2)}{(k_r^2+\omega^2)^2+\varepsilon^2(k_r^4-(k_r^2+1)/2)}\right]^{1/2}$$

$$\times\left[\frac{k_r^2-\varepsilon^2\omega^2}{k_r^2-\varepsilon^2}\right]^{1/2}\left[\frac{k_r^2+1-a}{k_r^2+1+a}\right]^{1/2}, \quad \omega^2<0. \qquad (3.3)$$

If we carry out in $F_\pi(\omega > 0)$ additional expansion of the expression $1/(k_r + \epsilon\omega)$ (which obviously is valid when $\epsilon\omega/k_r \ll 1$), then the expression for $F_\pi(\nu)$ coincides with the expression given by Bowcock et al.[9]

The integral equation

$$F_\pi(\nu)=1+\frac{\nu}{\pi}\int_{-1}^{-\infty}\frac{e^{i\delta}\sin\delta\cdot F_\pi(\nu')}{\nu'(\nu'-\nu)}d\nu',$$

$$k^2+1=-\nu \qquad (3.4)$$

shows that the small vicinity $-k_r^2 = \nu + 1$ is the most important for $F_\pi(\nu)$ in the form (3.3). The integral over the far region determines inessential details of the behavior of $F_\pi(\nu)$ at small $\nu \sim \nu_r$. These properties are connected with the fact at the point ν_r we have $\operatorname{Re} F_\pi(\nu) \sim 1/\epsilon$, i.e., it has a clearly pronounced maximum.

It is possible, however, to choose the polynomial $P(\nu)$ so that it has a zero in the vicinity of ν_r. A form factor of this type is not bounded at infinity and the entire region of integration in (3.4) is important to it. The latter cannot be listed among the advantages of a form factor with a zero. The main objection against it is the relativistic model considered by Gell-Mann and Zachariasen [16]. We shall henceforth use expression (3.3) for the form factor.

4. EQUATIONS FOR $s^{(-)}$ AND $p^{(-)}$ PARTIAL WAVES

The dispersion relations for forward scattering will be written in the usual form

$$\Phi(s,+1)=\frac{1}{\pi}\int_{(M+1)^2}^\infty\operatorname{Im}\Phi(s',+1)$$

$$\times\left[\frac{1}{s'-s}+\frac{1}{s'-s}\right]ds' \qquad (4.1)$$

plus a pole term for $B^{(-)}$. Relation (4.1) is written without subtraction. From the Pomeranchuk theorem [17] it follows that the amplitude of elastic πN scattering with isotopic index (−) is bounded at infinity. An analogous assumption was used by Haber-Shain [18].

Dispersion relations for backward scattering

*ctg = cot.

4)An exact expression for $F_\pi(\nu)$ is given in [15].

for the functions $\Phi(\nu, -1)/F_\pi(\nu)$ will be written with one subtraction, since $F_\pi(\nu) \sim 1/\sqrt{\nu}$ as $\nu \to \infty$. Carrying out the subtraction at the point $\nu = 0$ we obtain

$$\Phi(\nu, -1) = \Phi(0, -1) F_\pi(\nu) + \frac{\nu}{\pi}\int_0^\infty \frac{\operatorname{Im}\Phi(\nu', -1)}{\nu'(\nu' - \nu)} \frac{F_\pi(\nu)}{F_\pi(\nu')} d\nu' + \frac{\nu F_\pi(\nu)}{2\pi i}\int_{-\infty}^{-1} G(\nu')\left[\frac{1}{F_\pi^{(+)}(\nu')} - \frac{1}{F_\pi^{(-)}(\nu')}\right]\frac{d\nu'}{\nu'(\nu' - \nu)} \tag{4.2}$$

plus a pole term for $B^{(-)}$. Here $F_\pi^{\pm}(\nu) = F_\pi(\nu \pm i0)$, and

$$G(\nu) = \begin{cases} g^2(\Delta - \Delta_1) & \text{for } B^{(-)} \\ g^2 M [M^2 + \nu]^{-1}(\Delta_1 + \Delta_2) & \text{for } \alpha \end{cases}.$$

The pole term for $B^{(-)}$ has the form

$$-g^2 \frac{2 - 1/M^2}{4M^2} \frac{1}{F_\pi(\nu_0)} \frac{1}{\nu - \nu_0}, \qquad \nu_0 = -1 + \frac{1}{4M^2}.$$

The transition from the scalar amplitudes $\Phi(\nu, \pm 1) \equiv (\alpha, B^{(-)})$ to the partial amplitudes $f^{(-)}_{s,p}$ can be readily carried out with the aid of the functions $f^{(-)}_{1,2}(\nu, -1)$, which are simply related with the functions α and $B^{(-)}$. In the calculation of $\operatorname{Im}\Phi(\nu, \pm 1)$ we confine ourselves to the largest of the πN-scattering phase shift, α_{33}. The subtraction constants for $\Phi(0, -1)$ are expressed in terms of the quantities

$$a^- = \tfrac{1}{3}(a_1 - a_3), \qquad a_1^- = \tfrac{1}{3}(a_{11} - a_{31}),$$
$$a_3^- = \tfrac{1}{3}(a_{13} - a_{33}).$$

This does not mean that they are all parameters, since subtraction is carried out only in (4.2) and not in (4.1). The explicit expressions for $\Phi(0, -1)$ have the form

$$\frac{\alpha(0, -1)}{4\pi} = \frac{1}{4M}\left[\frac{2M + 1}{2M} a^- - 2M(a_1^- - a_3^-)\right],$$
$$\frac{B^{(-)}(0, -1)}{4\pi} = \frac{a^-}{2M} + 2M(a_1^- - a_3^-). \tag{4.3}$$

Using (4.1)—(4.3) and changing over in (4.1) to the variable ν, we obtain expressions for Re $f^{(-)}_s$, Re $f^{(-)}_{p_{1/2}}$, and Re $f^{(-)}_{p_{3/2}}$, in terms of the large phase shift α_{33} and the subtraction constants. The effect of the $\pi\pi$ interaction is accounted for by the pion form factor $F_\pi(\nu)$.

5. CROSSING SYMMETRY AND EXPANSION IN 1/M

Starting from the crossing symmetry properties of the scalar functions $A^{(\pm)}(s, \bar{s}, t)$ and $B^{(\pm)}(s, \bar{s}, t)$ we can obtain in the 1/M approximation, by simple calculation, the following relations for the partial amplitudes:

$$f_s^{(\pm)}(\omega) \mp f_s^{(\pm)}(-\omega) = 0,$$
$$f_{p_{1/2}}^{(\pm)}(\omega) - f_{p_{3/2}}^{(\pm)}(\omega) \pm [f_{p_{1/2}}^{(\pm)}(-\omega) - f_{p_{3/2}}^{(\pm)}(-\omega)] = 0,$$
$$f_{p_{1/2}}^{(\pm)}(\omega) + 2f_{p_{3/2}}^{(\pm)}(\omega) \mp [f_{p_{1/2}}^{(\pm)}(-\omega) + 2f_{p_{3/2}}^{(\pm)}(-\omega)] = 0. \tag{5.1}$$

An account of the terms of type 1/M leads to coupling between the partial amplitudes; for example, p waves will enter into the first equation of (5.1)[19].

It is obvious that expansion in powers of 1/M in Eqs. (4.1) and (4.2) should yield for the partial waves a system of equations satisfying the symmetry properties (5.1). Carrying out these expansions we have, for example for $f^{(-)}_s$

$$\operatorname{Re} f_s^{(-)} = f^2\omega + \frac{\nu f^2}{\omega}\left[\frac{F_\pi(\nu)}{F_\pi(-1)} - 1\right] + \frac{F_\pi(\nu) - 1}{2}[\omega a^- + \nu(a_1^- - a_3^-)] + \frac{1}{2}[\omega a^- + \nu(a_1^- - a_3^-)] + \frac{1}{2\pi}\int_0^\infty d\nu' \operatorname{Im} f_{p_{3/2}}^{(-)}\left\{2\frac{\omega}{\nu'\omega'} + \frac{\nu}{\nu'^2}\right\} - \frac{\nu}{2\pi}\int_0^\infty \frac{d\nu'}{(\nu' - \nu)\nu'} \operatorname{Im} f_{p_{3/2}}^{(-)}\left(\frac{F_\pi(\nu)}{F_\pi(\nu')} - 1\right)\left(2\frac{\omega}{\omega'} + \frac{\nu}{\nu'}\right). \tag{5.2}$$

We can obtain analogously relations for $f^{(-)}_{p_{1/2}}$ and $f^{(-)}_{p_{3/2}}$.

An estimate of the influence of the higher waves of the $\pi\pi \to N\bar{N}$ process can be readily obtained from (4.2):

$$\frac{1}{2\pi i}\int_{-\infty}^{-1} \frac{G(\nu')}{\nu'(\nu' - \nu)}\left[\frac{1}{F_\pi^{(+)}(\nu')} - \frac{1}{F_\pi^{(-)}(\nu')}\right]d\nu' \approx \varepsilon^2 \frac{\text{const}}{\nu_r - \nu}. \tag{5.3}$$

Using the results of Bowcock et al[9] we find that $\epsilon^2 \approx 0.04$. Inasmuch as $\epsilon/k_r \lesssim 0.1$, we can put $\epsilon = 0$ in the first relation of (3.3) and in (5.3). We thus exclude from consideration from now on the higher waves of the $\pi\pi \to N\bar{N}$ process, and obtain for the form factor the expression

$$F_\pi(\nu) = (k_r^2 + 1)/(k_r^2 + \nu + 1). \tag{5.4}$$

In the approximation considered we can simplify relation (5.2) and the analogous relations for

Re $f^{(-)}_{p_{1/2}}$ and Re $f^{(-)}_{p_{3/2}}$. Namely, going to the limit as $\nu \to 0$, we obtain

$$a^- = 2f^2 - \frac{2}{3\pi}\int_0^\infty \frac{1}{\nu'\omega'} \operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\, d\nu',$$

$$a_1^- - a_3^- = \frac{1}{3\pi}\int_0^\infty \frac{1}{\nu'^2} \operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\, d\nu'. \tag{5.5}$$

Relations (5.4)—(5.5) now enable us to write dispersion relations of the type (5.2) in a form that clearly satisfies the crossing-symmetry conditions (5.1)

$$\operatorname{Re} f_s^{(-)} = a^-\omega F_\pi(\nu),$$

$$\operatorname{Re}(f^{(-)}_{p_{1/2}} - f^{(-)}_{p_{3/2}}) = \nu(a_1^- - a_3^-) \times \frac{F_\pi(\nu)}{2} - \frac{\nu}{2\pi}\int_0^\infty \frac{\nu'+\nu}{\nu'-\nu}\frac{\operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\, d\nu'}{\nu'^2} - \frac{\nu^2}{2\pi}\int_0^\infty \frac{\operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')}{\nu'^2(\nu'-\nu)}\left[\frac{F_\pi(\nu)}{F_\pi(\nu')} - 1\right] d\nu',$$

$$\operatorname{Re}(f^{(-)}_{p_{1/2}} + 2f^{(-)}_{p_{3/2}}) = -2\frac{\nu}{\omega}f^2 + a^-\omega[1 - F_\pi(\nu)] + \frac{2\nu\omega}{\pi}\int_0^\infty \frac{\operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')}{\nu'(\nu'-\nu)\omega}\, d\nu'. \tag{5.6}$$

6. COMPARISON WITH EXPERIMENT

In the derivation of (5.6) we have carried out subtraction in the dispersion relations for backward scattering at the point $\nu = 0$. This eliminates the contribution of the $\pi\pi$ terms to the scattering length a^-, for which we obtain exactly the formula of Chew, Goldberger, Low, and Nambu [20]. The numerical calculations of a^- with account of the 1/M corrections and using the phase shift α_{33}, taken from the paper of Layson [21], yield $a^- = 0.092$, which agrees with Orear's data [22]. The use of other known expressions for α_{33} alters the value of a^- little. Thus, the scattering length a^- in Eqs. (5.6) can be regarded as a parameter.

Inasmuch as the experimental data for a_1^- and a_3^- have been determined with an extreme degree of inaccuracy (for example, it is known that $a_1^- = 0.008 \pm 0.037$ and $a_3^- = -0.095 \pm 0.080$; see [23], which contains further references to the experimental material), it is meaningful to carry out additional subtraction in the second and third equations of (5.6) (at the point $\nu = 0$), and also regard the quantities a_1^- and a_3^- as parameters.

As a result we obtain a final system, which will then be compared with experiment

$$\operatorname{Re} f_s^{(-)} = a^-\omega F_\pi(\nu),$$

$$3\operatorname{Re} f^{(-)}_{p_{1/2}} = \nu\left\{a_1^-[\omega + 1 + F_\pi(\nu)] + a_3^-\left[2\frac{\nu}{\omega+1} + 1 - F_\pi(\nu)\right] + 2\frac{\nu}{\omega}f^2 + \frac{a^-\omega}{\nu}[1 - F_\pi(\nu) + \nu F_\pi'(\nu)|_{\nu=0}] - \frac{2\nu}{\pi}\int_0^\infty \operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\frac{d\nu'}{\nu'^2\omega'(\omega'+\omega)} - \frac{\nu}{\pi}\int_0^\infty \operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\frac{1}{\nu'^2(\nu'-\nu)}\left[\frac{F_\pi(\nu)}{F_\pi(\nu')} - 1\right] d\nu',$$

$$3\operatorname{Re} f^{(-)}_{p_{3/2}} = \nu\left\{a_3^-\left[2\omega + \frac{1+F_\pi(\nu)}{2}\right] + a_1^-\left(\frac{\nu}{\omega+1} - \frac{1-F_\pi(\nu)}{2}\right) + 2\frac{\nu}{\omega}f^2 + \frac{a^-\omega}{\nu}[1 - F_\pi(\nu) + \nu F_\pi'(\nu)|_{\nu=0}] + \frac{\nu}{\pi}\int_0^\infty \operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\frac{1}{\nu'^2(\nu'-\nu)}\left[1 + 2\frac{\omega}{\omega'}\right] d\nu' + \frac{\nu}{2\pi}\int_0^\infty \operatorname{Im} f^{(-)}_{p_{3/2}}(\nu')\frac{1}{\nu'^2(\nu'-\nu)}\left[\frac{F_\pi(\nu)}{F_\pi(\nu')} - 1\right] d\nu'. \tag{6.1}$$

The prime [of $F'_\pi(\nu)$] denotes differentiation with respect to ν.

The position of the $\pi\pi$ resonance is determined from a comparison of Re $f_s^{(-)}$ with the experimental data. In this case, a satisfactory description of the energy dependence of the s wave is obtained with $a^- = 0.08$ and $t_r = 22$ (Fig. 1).

The formulas for the p waves are not very sensitive to the values of a^{-1} and t_r and strongly depend on a_1^-, a_3^-, and f^2. The functions by which a_1^- and a_3^- are multiplied are increasing functions of

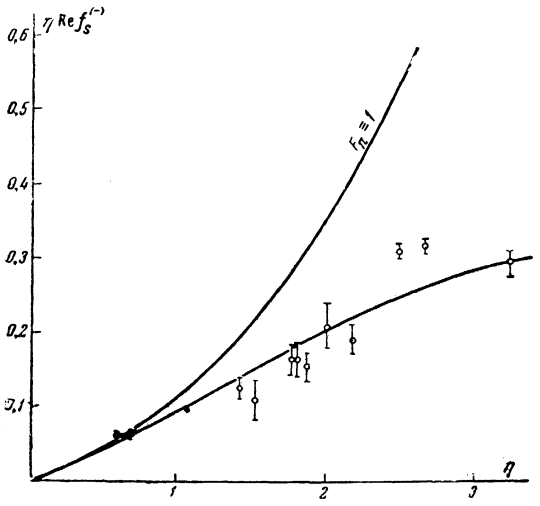

FIG. 1

ω. Therefore small changes in the subtraction constant greatly influence the behavior of Re $f^{(-)}_{p_{1/2}}$ and Re $f^{(-)}_{p_{3/2}}$ at $\eta \sim 2$—3. The dependence of Re $f^{(-)}_{p_{1/2}}$ on the energy (see Fig. 2) necessitates that the quantity a_1^- be negative. We note that if we use for Re $f^{(-)}_{p_{1/2}}$ an expression without additional subtraction, then the deduction concerning the sign of a_1^- does not change.

The numerical calculations show (Fig. 3) that for all reasonable values of a_1^- and a_3^- the $\eta \operatorname{Re} f^{(-)}_{p_{3/2}}$ curves fit the experiment well up to $\eta \sim 1.1$. In the resonance region Re $f^{(-)}_{p_{3/2}}$ passes through zero (see Fig. 3), but the experimental points lie above the curve. In our opinion this is a result of neglecting the small phase shifts in the integrand.

The effect of the $\pi\pi$ interaction on the $f^{(-)}_{p_{3/2}}$ wave is small. Inasmuch as the Chew-Low formula for α_{33} describes the experiment well only up to the resonance, we can expect the contribution of the $\pi\pi$ interaction, if it is important, to be noticeable above $\eta \sim 2$. The dashed curve representing the $\pi\pi$ effect satisfies this requirement (see Fig. 3).

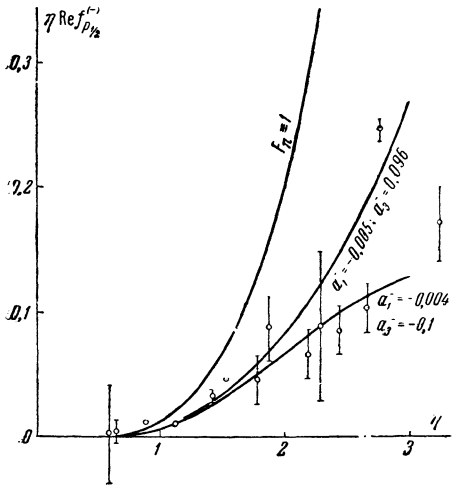

FIG. 2

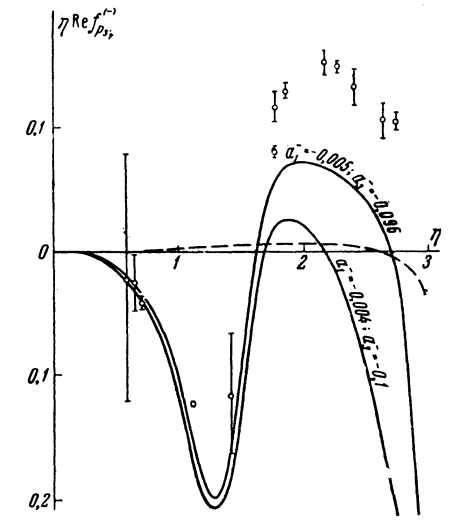

FIG. 3

CONCLUSIONS

1. The systems (5.6) and (6.1) for the partial waves take into account the influence of the $\pi\pi$ interaction on the πN scattering. Without the $\pi\pi$ interaction ($F_\pi(\nu) \equiv 1$) Eqs. (5.6) go over into the Chew-Low equations, while (6.1) go into the Chew-Low equations with corresponding subtraction. The final formulas (5.6) and (6.1) satisfy crossing-symmetry properties in the form (5.1). All the parameters have a clear-cut physical meaning.

2. The ambiguity in the account of the $\pi\pi$ interaction, which reduces to ambiguity in the determination of the electromagnetic pion form factor, is analyzed. A comparison with experiment is made for the specific form (5.4) of $F_\pi(\nu)$.

3. Satisfactory description of the $s^{(-)}$ wave is obtained for $a^- = 0.08$ and $t_r = 22$. At values $a_1^- \sim -0.004$, $a_3^- \sim -0.1$, and $f^2 = 0.087$, the $f^{(-)}_{p_{1/2}}$ wave reproduces well the energy dependence up to $\eta \sim 3$, while for $f^{(-)}_{p_{3/2}}$ there is a qualitatively correct dependence on η. The latter is connected with the crudeness of the assumptions made in the calculation of the integral in the sense of principal value. As expected, the $\pi\pi$ interaction effect in $f^{(-)}_{p_{3/2}}$ is small.

The sign of a_1^- ($a_1^- < 0$) is independent of whether subtraction is carried out in the equations for the p waves or not. It is a consequence of taking the crossing symmetry into account. If we neglect this symmetry and assume, as usual, that $\alpha_{ij} \approx a_{ij}\eta^3$, then $a_1^- > 0$.

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