THE ORIGIN OF HIGH-ENERGY COSMIC-RAY MUONS AND PHOTONS AND THE HYPERON HYPOTHESIS

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The differential spectrum of cosmic-ray 10^{11}—10^{14} eV muons has been calculated under the assumption that the particle carrying away the major fraction (0.7—0.8) of the primary particle energy is a hyperon. The spectrum obtained is in a good agreement with experimental data. However, the calculated negative excess contradicts experimental results at an energy of ~ 100 BeV. It is shown that this discrepancy can be eliminated assuming that the charge distribution of the hyperons is determined by the statistical weight in the isotopic space. The differential spectrum photons in the energy range 10^{11}—10^{14} eV at a high altitude has been calculated.

1. Peters [1] has recently evolved a hypothesis according to which the hyperons play a considerable role in high-energy interactions. The hypothesis is based on experimental results obtained with accelerators [2] concerning the increase of the number of hyperons per interaction with the energy. Since all baryons possess a nuclear charge and, consequently, hyperons are to a certain extent "produced from nucleons" in nucleon interactions, it is natural to assume that some properties of secondary nucleons will be common to all baryons. In particular, it is possible that the features of the energy spectrum of the baryons should be similar to those of the secondary nucleon spectrum. Many arguments have been invoked (the altitude dependence of extensive air showers, energy spectrum of secondary particles in high-energy stars, etc.) to support the hypothesis that, in interactions involving light nuclei, a nucleon carries away a considerable fraction (~ 0.6—0.7) of the primary nucleon energy. Thus, if the hypothesis of Peters were correct, one would expect that a hyperon carries away ~ 0.7—0.8 of the primary particle energy E_0.

Although the above reasons form a not too convincing basis for the hypothesis, there exists a fact which is difficult to explain without recourse to it. According to the experimental data, [3,4] the range of high-energy nuclear-active particles in water is by a factor of 1.5 greater than in air. This difference can be understood assuming that the particle carrying away the main fraction of the energy is a hyperon. (Such an explanation has already been proposed a long time ago. [3,4]) In the present article, we shall consider several other consequences of the hypothesis, which can serve as its experimental test.

2. Let us consider the energy spectrum of cosmic-ray muons. The spectrum was calculated on the basis of the hyperon hypothesis by Bhabba and Pal. [5] However, a number of simplifications were made in the calculation,[1)] and the obtained agreement of the results with the experiment is not convincing.

We shall assume in the calculation that the Λ, Σ^+, Σ^0, and Σ^- hyperons are produced with the same probability equal $1/4$.[2)] Moreover, we consider the following decay schemes (w —relative probability of the decay channel, E_π —mean pion energy in the hyperon rest system):

a) $\Lambda^0 \to p + \pi^-$, $w = 1$, $E_\pi = 0.17$ BeV;
b) $\Sigma^+ \to p + \pi^0$, $w = 1/2$, $E_\pi = 0.23$ BeV;
$\Sigma^+ \to n + \pi^+$ $w = 1/2$;
c) $\Sigma^0 \to \Lambda + \gamma$, $w = 1$;

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1)The simplifications are as follows: 1) secondary processes were neglected; 2) the "pionization" process (i.e., the production of pions in addition to those originating in the hyperon decay) was not taken into account; 3) hyperon interactions were not taken into account; 4) the primary nucleon spectrum was taken from [6], while the most recent data[7,8] reduce the number of primary particles by a factor of approximately two; 5) the inelasticity coefficient distribution was neglected.

2)Since the masses of Λ and Σ particles are roughly equal, their statistical weights are also approximately equal. The mass of the Ξ hyperon is considerably greater, and we therefore neglect the probability of its production.

d) $\Sigma^- \to n + \pi^-$, $w = 1$, $E_\pi = 0.23$ BeV.

The mean lifetime τ_π of a hyperon with respect to the decay into charged π mesons amounts then to 2×10^{-10} sec.

Let us calculate the probability W of the production of muons with energy E_μ in the decay of hyperons produced directly by primary particles (which we assumed to be nucleons). Assuming that the muon is emitted in the rest system of the pion, and that the pion in the rest system of the hyperon is emitted with a constant energy equal to the average one, we can write

$$W(E_\mu) = u_\Gamma u_\pi \int_0^{x_0} x^{u_\Gamma}\, dx \int_x^{x_0} y^{u_\pi - u_\Gamma - 1} dy \int_y^{x_0} e^{-z} z^{-(u_\pi+1)}\, dz,$$
$$u_{\Gamma,\pi} = k/c\,(E_{\Gamma,\pi}/M_{\Gamma,\pi})\,\tau_{\Gamma,\pi}, \tag{1}$$

where x_0 is the atmospheric depth expressed in nuclear mean-free-path units which we assumed to be identical for all nuclear-active particles (nucleons, hyperons, and pions), $k \sim 8000$, $\tau_{H,\pi}$ is the lifetime of hyperons and pions, respectively in their rest system, and $E_{H,\pi}$, $M_{H,\pi}$ are the energy and mass of the hyperon and pion, respectively. Since $x_0 \gg 1$, we have:

$$W = u_\Gamma u_\pi / (u_\Gamma + 1)(u_\pi + 1). \tag{2}$$

The number $n_1(E_\pi = 1.3\,\bar{E}_\mu)$ of pions with energy in the range from E_π to $E_\pi + dE_\pi$ produced in the decay of hyperons of the first interaction is

$$n_1(E_\pi)\, dE_\pi = C dE_\pi \int_{E_{0\,min} = 5E_\pi}^{\infty} P(E_0)\, \varphi(E_0, E_\pi)\, dE_0, \tag{3}$$

where $P(E_0)\,dE_0$ is the differential primary cosmic-ray spectrum which, for energies $E_0 \gg 10$ BeV, can be written in the form [7-9]

$$P(E_0)\, dE_0 = 1.8\, E_0^{-2.65}\, dE_0 \text{ particles/cm}^2 \text{ sr sec}$$

(E_0 is measured in BeV, and $dE_0 = 1$ BeV), $C = 7/8$ is the probability that a charged pion is produced in the hyperon decay,³⁾ and $\varphi(E_0, E_\pi)\,dE_\pi$ is the probability that a pion with energy between E_π and $E_\pi + dE_\pi$ is produced in the primary nucleon interaction. The quantity $\varphi(E_0, E_\pi)$ is calculated from the distribution function $\Phi(k)$ of the inelasticity coefficient which can be written in the form [10]

$$\Phi(k) = 0.8\delta(k - 0.2) + 0.3.$$

We have then

$$\varphi(E_0, E_\pi) = 0.8\delta(E_\pi - 0.2\, E_0) + 0.3/E_0. \tag{4}$$

Consequently,

$$n_1(E_\pi)\, dE_\pi = 10^{-1} E_\pi^{-2.65} dE_\pi. \tag{5}$$

It follows from Eqs. (2) and (5) that the number of muons $N_1(E_\mu)\,dE_\mu$ in the interval dE_μ is

$$N_1(E_\mu) = \frac{6\cdot 10^{-2} E_\mu^{-2.65} dE_\mu}{(1 + E_\mu/100)(1 + 4E_\mu/100\,000)}. \tag{6}$$

The contribution of the consecutive generations can easily be assessed if we take into account that: 1) the energy remaining after the i-th collision is $\alpha^i E_0$, where $\alpha = 1 - k \sim 0.6$—0.7, and 2) the collision probability of a particle of the i-th generation at a depth between x and x + dx is equal to $[e^{-x} x^{i-1}/(i-1)!]\,dx$. We have then,

$$W_i(E_\mu)\, dE_\mu = \frac{u_\pi u_\Gamma}{(i-1)!} \int_0^{x_0} x^{u_\Gamma + i - 1} dx \int_x^{x_0} y^{u_\pi - u_\Gamma - 1}\, dy \int_y^{x_0} e^{-z} z^{-(u_\pi+1)} dz$$
$$= u_\pi u_\Gamma / (u_\pi + i)(u_\Gamma + i) \ (\text{for } x_0 \gg 1); \tag{7}$$

$$N_i(E_\mu)\, dE_\mu = C(\alpha^\gamma)^i \cdot 1.3 \cdot 10^{-1} E_\mu^{-2.65} dE_\mu / (1 + iE_\mu/100) \times (1 + 4iE_\mu/100\,000). \tag{8}$$

For $\alpha = 0.7$ and $C = 7/8$ we have

$$N_i(E_\mu)\, dE_\mu = (0.5)^i \cdot 1.1 \cdot 10^{-1} E_\mu^{-2.65} dE_\mu / (1 + iE_\mu/100) \times (1 + 4iE_\mu/100\,000).$$

We shall calculate now the number of muons produced as a result of "pionization" in the first interaction. Using the empirical relation [10,11] $\bar{n} = aE_0^{1/4}$, where a = 3, for the multiplicity, and assuming that all secondary pions have the same energy, we find easily that

$$N'(E_\mu)\, dE_\mu = 3 \cdot 10^{-2} E_\mu^{-2.87} dE_\mu / (1 + E_\mu/100). \tag{9}$$

The integral muon spectrum corresponding to the sum $\sum_{i=1}^{\infty} N_i + N'$ is shown in Fig. 1. The experimental values collected in [5] are shown in the figure. It can be seen that the theoretical curve fits the experimental data. It is especially conspicuous that the slope of the spectrum increases at $E_\mu \sim 10^{13}$ eV. This feature of the spectrum agrees qualitatively with the results obtained recently at great depths underground. [12]

The following question arises: Is it possible to explain the muon spectrum without recourse to the hyperon hypothesis? In fact, the formulas given above indicate that the experimental muon energy spectrum can roughly be explained assuming that a pion carries away ~ 15—20% of the energy of each primary nucleon.⁴⁾

In our calculations we have made the assumption that the energy of all secondary pions (apart

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³⁾We neglect the energy difference between the Λ particle produced in the $\Sigma^0 \to \Lambda^0 + \gamma$ decay and the Σ^0 hyperon.

⁴⁾Cf. the earlier estimates of Zatsepin[13] and Grigorov.[14]

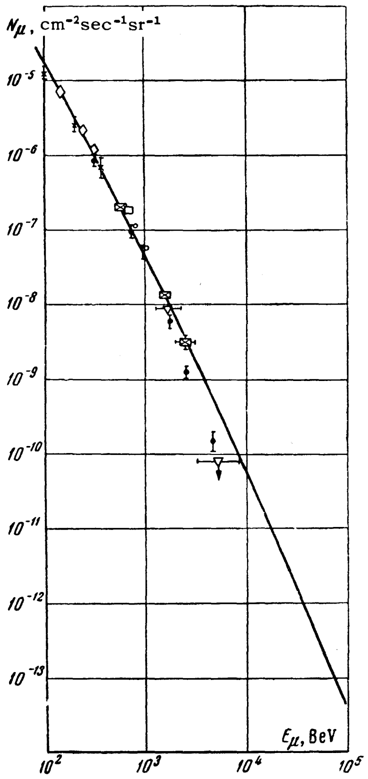

FIG. 1. Integral spectrum of μ mesons at sea level, calculated according to the hyperon hypothesis.

from those produced in hyperon decay) is the same. In reality, the secondaries may have very different energies. According to the hydrodynamical theory of multiple particle production, one of the secondary particles can carry away up to 50% of the primary particle energy. [15] Unfortunately, the hydrodynamical theory cannot give an exact value of this quantity. If we assume that one of the pions carries away a considerable fraction ($\sim$ 50%) of the total energy spent for pion production, and if we choose α = 0.5—0.7, we obtain a rough agreement with the observed intensity of cosmic-ray muons. In order to decide between the two hypotheses, it is especially important to analyze the muon spectrum in the range $E_\mu \geq 10^{13}$ eV and to look carefully for a possible increase in its steepness. This feature of the muon spectrum reflects the hyperon origin of the muons.[5)]

In conclusion, it should be noted that if we assume that the energy spent on "pionization" is mainly used for the production of new particles and is not converted into kinetic energy (i.e., the multiplicity is proportional to $\sim E_0^{1/2}$, as predicted by the Heisenberg theory) then the calculated muon intensity (neglecting the hyperons) will be considerably less than the observed one.

3. Let us consider the energy spectrum of muons of one sign. From equations (8) and (9) we have, for positive muons,

$$N^+(E_\mu)\,dE_\mu = 1.6\cdot 10^{-2}E_\mu^{-2.65}\,dE_\mu\sum_{i=1}^{\infty}\frac{\alpha^{i\gamma}}{(1+iE_\mu/100)(1+4iE_\mu/100\,000)} + \frac{1.5\cdot 10^{-2}E_\mu^{-2.87}dE_\mu}{1+E_\mu/100}, \qquad (10)$$

$$N^-(E_\mu)\,dE_\mu = 10^{-1}E_\mu^{-2.65}\,dE_\mu\sum_{i=1}^{\infty}\frac{\alpha^{i\gamma}}{(1+iE_\mu/100)(1+4iE_\mu/100\,000)} - \frac{1.5\cdot 10^{-2}E_\mu^{-2.87}dE_\mu}{1+E_\mu/100}. \qquad (11)$$

The differential spectra of both positive and negative muons are shown in Fig. 2.

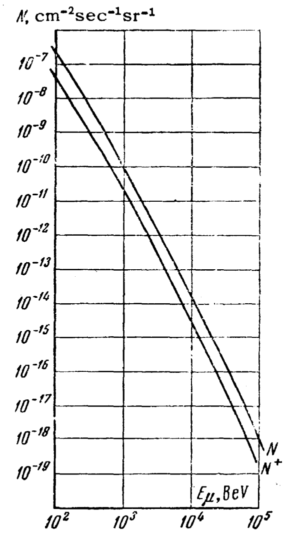

FIG. 2. Differential spectrum of negative and positive μ mesons at sea level calculated according to the hyperon hypothesis.

The negative excess is

$$\frac{N^- - N^+}{N^- + N^+} = \left[8\cdot 10^{-2}E_\mu^{-2,65}\sum_{i=1}^{\infty}\frac{\alpha^{i\gamma}}{1+4iE_\mu/100\,000}\right] \times\left[12\cdot 10^{-2}E_\mu^{-2.65}\sum_{i=1}^{\infty}\frac{\alpha^{i\gamma}}{1+4iE_\mu/100\,000} + 3\cdot 10^{-2}E_\mu^{-2,87}\right]^{-1}, \qquad (12)$$

which, for $E_\mu \ll$ 25,000 BeV gives, approximately,

$$(N^- - N^+)\,/\,(N^- + N^+) \sim 8E_\mu^{0,21}/\,(12E_\mu^{0,21} + 3) \qquad (13)$$

and for $E_\mu \sim$ 50—100 BeV

$$(N^- - N^+)\,/\,(N^- + N^+) \sim 0{,}6. \qquad (14)$$

This value is in violent disagreement with experimental data, [16,17] according to which in the energy

[5)]Another possible explanation could lie in an increase in the steepness of the primary cosmic ray spectrum. However, the analysis of extensive air shower data in the range 10^{14}–10^{15} eV does not support such a conclusion (see, e.g., [2-13]).

range under consideration[6)]

$$(N^- - N^+) / (N^- + N^+) \sim -0.1.$$

Obviously, it is difficult to explain the discrepancy within the framework of the hyperon hypothesis. The assumption that the hyperon effects do not play a considerable role below $E_0 \leq 10^{12}$ eV and that muons are mainly produced as a result of a multiple process is very arbitrary.[7)] In that case, all effects due to hyperons would be masked by multiple processes at higher energies, too.

The only way to remove the discrepancy between the theory and the experiment is to make use of the fact that the incident particles are positively charged. It is evident, however, that if this charge is uniformly distributed among all secondary particles (15 to 20 in the range under consideration) then, remaining within the framework of the hyperon hypothesis, we can compensate only for a small fraction ($\sim 10\%$) of the negative excess. It is therefore necessary to assume that the positive charge for its greater part is distributed among secondary baryons only.

For a quantitative estimate[8)] we shall use the single-meson scheme. We shall make the following assumptions: 1) the charge distribution of the baryons is given by the statistical weight in the isospin space, 2) the incident particle is a proton, 3) the target nuclei consist of protons and neutrons in equal numbers, and 4) the incident particle continues to move forward in the c.m.s. after the collision.[9)]

The estimates were carried out for two types of diagrams (see Fig. 3). There are 15 possible diagrams of type I, and six of type II.[10)]

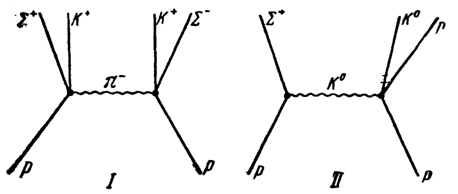

FIG. 3

For diagrams of type I, the ratio of Σ particles of different sign is

$$n(\Sigma^+) : n(\Sigma^0) : n(\Sigma^-) = 1.4 : 1 : 0.6.$$

This corresponds to a ratio $n(\mu^+) : n(\mu^-) = 1:4$ instead of $1:6$ for the case of a uniform charge distribution. If II dominates, then $n(\mu^+) : n(\mu^-) = 1:2$.

The ratio increases even more if we assume that the Λ particle is emitted in a bound state $Y_1^* = [\Lambda\pi]$ with isotopic spin $T = 1$ (for Σ particles the probability of the resonant state $[\Sigma\pi]$ is relatively small, $\sim 4\%$ of the probability of $[\Lambda\pi]$ and we can therefore neglect the contribution of the $[\Sigma\pi]$ resonance [18,19]).

The single-meson scheme predicts under similar approximations that the two factors (the nonuniformity of charge distribution of the Σ baryons and the production of the Y_1^* isobar) approximately compensate for the negative excess. Moreover, the effect of charge redistribution between the hyperons on the total muon spectrum is small and may be neglected.

Another attempt to account for the negative excess at energies of $\sim 10^2$ BeV can be based on the existence of (πN) resonance states in the interaction.[11)] To be specific, we limit ourselves to the discussion of the $T = S = 3/2$ isobar. Making the same assumptions as before, it is easy to calculate the isobar distribution with respect to the isospin projections. It is found that isobar projections $+3/2$ and $+1/2$ predominate, and that the ratio of π^+ to π^- mesons in the isobar decay is approximately $10:1$. In order to compensate for the negative excess, it is necessary that isobars are produced in approximately 50% of the events (for $E_0 \sim 10^{12}$ eV).

4. We shall calculate the energy spectrum $N(E_\gamma)$ of photons produced at high altitudes. If $x_0 \ll 30$ g/cm², then the probability of secondary electromagnetic processes is small and we can assume that all photons originate in the decay of mesons or hyperons produced in the first interaction. In this sense, the photons can be considered

6)The existence of such a discrepancy was mentioned by Peters.[1]

7)Thus, e.g., according to the hydrodynamical theory, the energy fraction carried away by a fast pion varies as $\sim E_0^{-1/4}$ and, consequently, one cannot expect a sharp change in the character of the elementary interaction at these energies.

8)Of course, all the estimates given in the following are too crude to serve as a proof of the charge compensation. They only testify to its plausibility.

9)This assumption, in line with the concept of peripheral collisions, gives the upper limit of the compensating effect of the primary charge. Another limiting assumption, that of both baryons moving after the collision forward, gives the lower limit of the ratio $n(\Sigma^+)/n(\Sigma^-)$, which is by a factor of about 1.5 smaller than the one obtained under the first assumption.

10)Under the assumption made, the K mesons "produced" in the vertices of the diagrams should also be energetically preferred. An estimate of the effect transgresses the bounds of this article.

11)See [10,20] concerning the production of resonance states in the interaction of cosmic-ray particles.

as "primary" if they originate in the following decays:

a) π^0 meson decay;

b) $\Sigma^0 \rightarrow \Lambda + \gamma$;

c) $\Sigma^+ \rightarrow p + \pi^0 \rightarrow p + 2\gamma$

(π^0 and Σ particles produced in the first interaction).

The total number of photons (in units of particles/sec-cm^2-sr) produced in process a) is

$$N_1(E_\gamma)\,dE_\gamma = (1 - e^{-x_0})\,0.008\,E_\gamma^{-2.87} dE_\gamma;$$

in process b)

$$N_2(E_\gamma)\,dE_\gamma = (1 - e^{-x_0})\,0.001\,E_\gamma^{-2.65} dE_\gamma;$$

and in process c)

$$N_3(E_\gamma)\,dE_\gamma = (1 - e^{-x_0})\,0.01\,E_\gamma^{-2.65}\frac{dE_\gamma}{1 + 6E_\gamma/100\,000}.$$

The total flux of photons at the depth of 20–30 g/cm^2 can be compared with the integral flux of photons of 470 BeV energy measured by Duthie et al.[21] It turns out that there is a beautiful agreement between the calculated value (2×10^{-7} cm^{-2}sr^{-1}) and the experimental one [$(2-3) \times 10^{-7}$].

The above estimates of the photon spectrum are made under the assumption that all hyperons are produced with equal probability. If we assume that $n(\Lambda) : n(\Sigma^+) : n(\Sigma^0) = 1 : 2 : 1$, which corresponds roughly to the scheme represented by the diagram of type II, we obtain (in units of particles/sec-cm^2-sr):

$$N_1(E_\gamma)\,dE_\gamma = (1 - e^{-x_0})\cdot 0.008 E_\gamma^{-2.87}\,dE_\gamma,$$
$$N_2(E_\gamma)\,dE_\gamma = (1 - e^{-x_0})\cdot 0.001 E_\gamma^{-2.65}\,dE_\gamma,$$
$$N_3(E_\gamma)\,dE_\gamma = (1 - e^{-x_0})\cdot 0.02 E_\gamma^{-2.65}\frac{dE_\gamma}{1 + 6E_\gamma/100\,000}.$$

In this case, the main contribution to the spectrum comes from photons produced in the $\Sigma^+ \rightarrow p + \pi^0$ decay. Because of this, the steepness of the differential photon spectrum in the high-energy range (10^{12}–10^{13} eV) increases and becomes equal to 3.1 instead of 2.65 at 10^{11} eV. The differential spectrum is shown in Fig. 4.

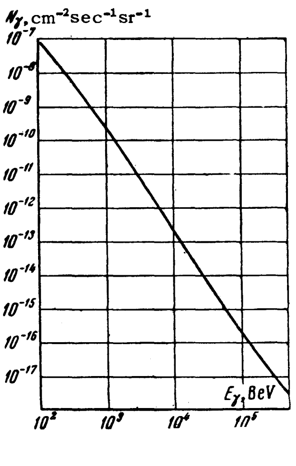

FIG. 4. Differential spectrum of photons at high altitudes calculated according to the hyperon hypothesis.

It should be noted that the experimental data of Duthie et al[21] indicate a considerable increase in the slope of the spectrum in this energy range. This fact is emphasized within the framework of the hyperon hypothesis, under the condition that the primary charge has a decisive influence on the charge distribution of the secondary hyperons.

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