SINGULARITIES CAUSED BY ELECTRON-PHONON INTERACTION IN THE PHONON DISPERSION LAW

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The singularities in the dispersion law of phonons, caused by the electron-phonon interaction, are analyzed by using the Green's function method. It is shown that the character of the singularity is essentially related to the shape of the Fermi Surface. If one of the radii of curvature is large (Fermi surfaces almost cylindrical, for instance) then a root-type singularity occurs instead of a logarithmic one [1] in the derivative of the frequency with respect to the wave vector. If both radii of curvature are large (almost flat Fermi surfaces) a singularity of the form $1/x$ appears. In the latter case a significant absolute rearrangement of the spectrum can also arise.

1. INTRODUCTION

SOME time ago Kohn [1], starting from model systems, called attention to the fact that for phonons with a wave vector of magnitude $q = 2p_0$ (p_0 is the limiting momentum of a spherical Fermi surface) the electron-phonon interaction brings about a logarithmic singularity in the derivative of the frequency ω with respect to q. Later the work of Brockhouse [2] appeared, in which a direct observation of such a singularity in the phonon spectrum of lead was reported, based on experiments with cold neutrons. The connection between the position of the singularities and the properties of the Fermi surface offers the attractive possibility of an independent determination of this surface by measuring the phonon spectrum. Since the Fermi surface of real metals has a complicated shape, the question arises of the character of the singularities occurring for such cases in the phonon dispersion law. As is shown in the present work, an abrupt intensification of the singularities and a noticeable rearrangement of the phonon spectrum arise for a certain class of surfaces. This appears especially clearly for Fermi surfaces which are nearly cylindrical or planar. These cases are considered in detail below.

In this work the double-time Green's function method is used for the consideration of the electron-phonon system at $T = 0$. This method was applied consistently for the first time by Migdal [3] to an isotropic model.

2. GENERAL FORMULAE

We write the Hamiltonian of the electron-phonon system in the usual form

$$H = \sum_{\mathbf{p}} \varepsilon^0_{\mathbf{p}} a^+_{\mathbf{p}} a_{\mathbf{p}} + \sum_{\mathbf{q}} \omega^0_{\mathbf{q}} b^+_{\mathbf{q}} b_{\mathbf{q}} + \sum_{\mathbf{p},\,\mathbf{q}} A_{\mathbf{q}} a^+_{\mathbf{p}+\mathbf{q}} a_{\mathbf{p}} (b_{\mathbf{q}} + b^+_{\mathbf{q}}). \qquad (1)$$

In this connection we assume that the electron and phonon branches of excitations in the crystal have the arbitrary laws $\epsilon^0_{\mathbf{p}}$ and $\omega^0_{\mathbf{q}}$. For simplicity we neglect the periodicity in the reciprocal lattice space. To find the phonon spectrum we look for the Green's function $D(\mathbf{q}, \omega)$, the poles of whose analytic continuation determine the dispersion law and the damping.

The Dyson equation for this function can be written down in the following form:

$$D^{-1}(\mathbf{q}, \omega) = D_0^{-1}(\mathbf{q}, \omega) + \Pi(\mathbf{q}, \omega), \qquad (2)$$

where the polarization operator Π is defined by the relation

$$\Pi(\mathbf{q}, \omega) = i \int G\left(\mathbf{p} + \frac{\mathbf{q}}{2}, \varepsilon + \frac{\omega}{2}\right) G\left(\mathbf{p} - \frac{\mathbf{q}}{2}, \varepsilon - \frac{\omega}{2}\right) \times \Gamma(\mathbf{p}, \varepsilon; \mathbf{q}, \omega) \frac{d\mathbf{p}\, d\varepsilon}{(2\pi)^4}. \qquad (3)$$

Here $G(\mathbf{p}, \epsilon)$ is the electron Green's function and $\Gamma(\mathbf{p}, \epsilon; \mathbf{q}, \omega)$ the vertex part.

In all the following calculations we are interested only in G_0, the Green's function in the absence of the electron-phonon interaction:

$$G_0(\mathbf{p}, \varepsilon) = (\varepsilon - \varepsilon^0_{\mathbf{p}} + i\Delta)^{-1}, \qquad \Delta = \begin{cases} +0 & \varepsilon^0_{\mathbf{p}} > \varepsilon_F \\ -0 & \varepsilon^0_{\mathbf{p}} < \varepsilon_F. \end{cases} \qquad (4)$$

The analogous expression for D_0 takes the form

$$D_0(\mathbf{q}, \omega) = A_{\mathbf{q}}^2 \left\{ \frac{1}{\omega - \omega_{\mathbf{q}}^0 + i\delta} - \frac{1}{\omega + \omega_{\mathbf{q}}^0 - i\delta} \right\}, \qquad \delta \to +0. \tag{5}$$

According to such a definition the vertex part in the first approximation is $\Gamma_0 = 1$.

In the usual analysis of the electron-phonon interaction we need consider only this simple vertex part, since the following graphs are of order $(m/M)^{1/2}$ [3]. However, for a certain class of Fermi surfaces, this is not the case and the problem of the renormalization of the vertex part can be essential. As is shown below such a situation arises for nearly planar surfaces. In all remaining cases we can replace Γ by 1 in (3).

It is well known that a noticeable change of the electron excitation spectrum due to the electron-phonon interaction occurs only in a very narrow shell of order $\omega_{\max}$ (maximum frequency of the phonon spectrum) around the Fermi surface. (The corresponding scale in momentum space is $\xi \sim p_0\omega/\epsilon_F$, where p_0 is a typical momentum of the Fermi surface). In the integration in (3) over all variables the four-dimensional volume corresponding to this shell plays a negligibly small role and we can replace G by G_0 in that expression. Then, putting $\Gamma = 1$, we find after integrating over the energy

$$\Pi(\mathbf{q}, \omega) = \frac{1}{(2\pi)^3} \int \frac{\theta(\varepsilon_F - \varepsilon_{\mathbf{p}-\mathbf{q}/2}^0) - \theta(\varepsilon_F - \varepsilon_{\mathbf{p}+\mathbf{q}/2}^0)}{\varepsilon_{\mathbf{p}-\mathbf{q}/2}^0 - \varepsilon_{\mathbf{p}+\mathbf{q}/2}^0 + \omega + i\delta\omega/|\omega|} d\mathbf{p}. \tag{6}$$

3. NEARLY CYLINDRICAL FERMI SURFACES

We begin by considering the cases where the Fermi surface is similar to the surface of a circular cylinder of radius p_0 whose axis we choose as the z axis. Then

$$\varepsilon_{\mathbf{p}}^0 = p_\perp^2 / 2m^*. \tag{7}$$

Transforming to a cylindrical coordinate system in (6) and bearing in mind that we are interested in values $q_\perp$ for which automatically $q_\perp^2/2m^* \gg \omega$, we find the following expression for the polarization operator

$$\operatorname{Re}\Pi(\mathbf{q}, \omega) = \frac{m^* p_{z0}}{2\pi^2} \operatorname{Re}\left\{1 - \frac{1}{2}\left(1 - \frac{4p_0^2}{q_\perp^2} - \frac{4m\omega}{q_\perp^2}\right)^{1/2} - \frac{1}{2}\left(1 - \frac{4p_0^2}{q_\perp^2} + \frac{4m\omega}{q_\perp^2}\right)^{1/2}\right\},$$

$$\operatorname{Im}\Pi(\mathbf{q}, \omega) = \frac{m^* p_{z0}}{4\pi^2} \operatorname{Im}\left\{\left(1 - \frac{4p_0^2}{q_\perp^2} - \frac{4m\omega}{q_\perp^2}\right)^{1/2} - \left(1 - \frac{4p_0^2}{q_\perp^2} + \frac{4m\omega}{q_\perp^2}\right)^{1/2}\right\}. \tag{8}$$

We now find the dispersion law and the damping of the phonon branch of excitations. We introduce the notation $\omega = \omega_{\mathbf{q}} - i\gamma_{\mathbf{q}}$; then we obtain from (2) and (5), assuming $\gamma_{\mathbf{q}}/\omega_{\mathbf{q}} \ll 1$

$$\omega_{\mathbf{q}}^2 = (\omega_{\mathbf{q}}^0)^2 - 2\omega_{\mathbf{q}}^0 A_{\mathbf{q}}^2 \operatorname{Re}\Pi(\mathbf{q}, \omega_{\mathbf{q}}),$$

$$\gamma_{\mathbf{q}} = \frac{\omega_{\mathbf{q}}^0}{\omega_{\mathbf{q}}} \operatorname{Im}\Pi(\mathbf{q}, \omega_{\mathbf{q}}) \Big/ \left[1 + \frac{\omega_{\mathbf{q}}^0}{\omega_{\mathbf{q}}} \frac{\partial \operatorname{Re}\Pi(\mathbf{q}, \omega_{\mathbf{q}})}{\partial \omega_{\mathbf{q}}}\right]. \tag{9}$$

From (8) and (9) follows the important result that in the case of a cylindrical Fermi surface $\partial\omega/\partial q_\perp$ has a root singularity at $q = 2p_0$ in contrast to the weak logarithmic singularity for a spherical Fermi surface [1]. This point is a point of discontinuity of the phonon spectrum: the derivative approaching from the side $q_\perp < 2p_0$ is finite, while on approaching from the side $q_\perp > 2p_0$ the derivative increases sharply by going to infinity. In addition there is no damping in the region $q_\perp > 2p_0 + 2m\omega_{\mathbf{q}}/p_0$. In the reverse case, $q_\perp < 2p_0 - 2m\omega_{\mathbf{q}}/p_0$, the ratio $\gamma_{\mathbf{q}}/\omega_{\mathbf{q}} \sim \omega_{\mathbf{q}}/\epsilon_F$ but directly in the shell $\gamma_{\mathbf{q}}/\omega_{\mathbf{q}} \sim (\omega_{\mathbf{q}}/\epsilon_F)^{1/2}$. The transition occurs in a narrow interval of wave vectors of the order of ξ and thus the quantity $\gamma_{\mathbf{q}}/\omega_{\mathbf{q}}$ has a sharp narrow peak for $q_\perp \approx 2p_0$. Such a dependence is strongly distinct from the case of a spherical surface, to which corresponds a smooth curve with a sharp discontinuity and where for $q \le 2p_0$ the ratio $\gamma_{\mathbf{q}}/\omega_{\mathbf{q}} \sim \omega_{\mathbf{q}}/\epsilon_F$.

We give here, for purposes of comparison, the value of the real part of the polarization operator for a spherical Fermi surface (obtained from (6) for $\epsilon_{\mathbf{p}}^0 = p^2/2m$):

$$\operatorname{Re}\Pi(\mathbf{q}, \omega) = \frac{mp_0}{4\pi^2}\left[1 - \frac{1 - (q/2p_0)^2}{q/p_0} \ln\left|\frac{1 + q/2p_0}{1 - q/2p_0}\right|\right]. \tag{10}$$

This expression is valid for $q > 2p_0$ as well as $q < 2p_0$.

Figure 1 shows the dispersion curves close to the point $q = 2p_0$, for the cases of a spherical (a) and a cylindrical (b) Fermi surface.

Since the transition from a spherical to a cylindrical surface implies a strong increase of the singularity, it is natural that the question about

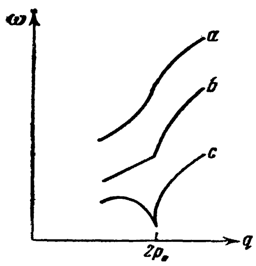

FIG. 1

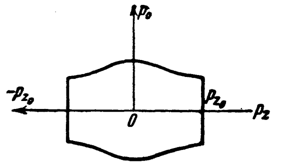

FIG. 2

the stability of the results against a deviation from strict cylindricality arises. For an analysis of this question we consider the axially symmetric case for which the dispersion law for the electrons is of the form

$$\varepsilon_{\mathbf{p}}^0 = p_{\perp}^2/2m^* + \varepsilon_1(p_z). \tag{11}$$

The Fermi surface corresponds, obviously, to a certain functional dependence $p_0(p_z)$ (Fig. 2). It is easy to check that the polarization operator (6) for the region $q_{\perp}/2 > p_{max}$, where p_{max} is the maximal value of $p_0(p_z)$, is determined by the formula

$$\Pi(\mathbf{q}, \omega) = \frac{m^*}{4\pi^2}\int dp_z \times \left[1 - \frac{1}{p_{max}}\left(2p_{max}q' + p_{max}^2 - p_0^2(p_z)\right)^{1/2}\right]. \tag{12}$$

Here $q' = q_{\perp}/2 - p_{max}$ and it is assumed that

$$\omega p_{max}/\varepsilon_F \ll q' \ll p_{max}.$$

From (12) and (9) we get

$$\frac{\partial\omega_{\mathbf{q}}}{\partial q_{\perp}} \approx \frac{\partial\omega_{\mathbf{q}}^0}{\partial q_{\perp}} + \frac{A_{\mathbf{q}}^2 m^*}{4\pi^2}\int dp_z\left(2p_{max}q' + p_{max}^2 - p_0^2(p_z)\right)^{-1/2}. \tag{13}$$

Assume, next, that within a certain interval

$$\varepsilon_1(p_z) = \alpha p_z^n.$$

It can easily be shown that here the second term in (13) contains a singularity of the form $(q'/p_{max})^{-(1/2-1/n)}$ $(n > 2)$. Thus if the expansion of the energy with respect to p_z around a certain point begins with a power higher than the second we arrive at a root singularity with the root exponent smaller than $1/2$.

If $n = 2$ then there occurs a factor of the form $\alpha^{-1/2}\ln(p_{max}/q')$ in the second term on the right of (13). The singularity now has a logarithmic character just as in the spherical case. However there appears a sharp increase due to the $\alpha^{-1/2}$ in the denominator in all cases where the deviation from cylindricality has a small amplitude (we note that if the energy surface forms an ellipsoid of revolution the singularity increases with an increase of the ratio of the semiaxes).

The whole situation with the singularities in the case considered can be most clearly traced in the example of a Fermi surface in the form of a corrugated cylinder. Let

$$\varepsilon_1(p_z) = a\sin^2(bp_z).$$

Hence from (13) we obtain

$$\frac{\partial\omega_{\mathbf{q}}}{\partial q_{\perp}} \approx \frac{\partial\omega_{\mathbf{q}}^0}{\partial q_{\perp}} + \frac{A_{\mathbf{q}}^2 m^*}{(2\pi)^3}\frac{p_{z0}}{\sqrt{2p_{max}q' + 4m^*a}}K\left[\left(\frac{2m^*a}{p_{max}q' + 2m^*a}\right)^{1/2}\right], \tag{13'}$$

where K is the complete elliptic integral of the second kind. If the corrugation of the cylinder is small, i.e., $(2m^*a)^{1/2} \ll p_{max}$, then the second term in (13′) as a function of q' has two special regions. For

$$2ma/p_{max}^2 < q'/p_{max} < 1$$

this term depends on q' as $(q')^{-1/2}$, just as in the reverse limiting case $q'/p_{max} < 2m^*a/p_{max}^2$ it becomes proportional to

$$p_{z0}(4m^*a)^{-1/2}\ln(16m^*a/p_{max}q').$$

Thus the root dependence is maintained in this case for the basic region of change of q' and the transition to a logarithmic dependence occurs in an interval of the order of the amplitude of corrugation.

4. NEARLY PLANE FERMI SURFACES

The increase in strength of the singularity on going over to nearly cylindrical Fermi surfaces is connected with the growth of the phase volume corresponding to the creation of an electron-hole pair with fixed total momentum. In fact an analogous situation will arise each time the Fermi surface has a significant region where one of the radii of curvature is large in comparison to p_0. It is clear that the maximal increase of the phase volume occurs with Fermi surfaces (or parts of a surface) approaching a plane, when both radii of curvature are large. (Similar parts of the Fermi surface always arise in pairs since in any crystal $\epsilon_{\mathbf{p}}^0 = \epsilon_{-\mathbf{p}}^0$.)

We consider the limiting case corresponding to a completely plane surface. However before calculating the polarization operator we have to analyze further the vertex part in view of the extreme character of the case considered. We determine the correction to Γ in first approximation described by the graph represented in Fig. 3. The expression for Γ_1 takes the form

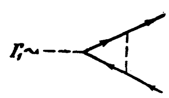

FIG. 3

$$\Gamma_1(\mathbf{p}, \varepsilon; \mathbf{q}, \omega) = -\frac{i}{(2\pi)^4}\int G_0\left(\mathbf{p}' + \frac{\mathbf{q}}{2};\ \varepsilon' + \frac{\omega}{2}\right) G_0\left(\mathbf{p}' - \frac{\mathbf{q}}{2},\ \varepsilon' - \frac{\omega}{2}\right) \times D_0(\mathbf{p} - \mathbf{p}', \varepsilon - \varepsilon')\, d\mathbf{p}'\, d\varepsilon'. \tag{14}$$

We shall be interested merely in values $q \sim 2p_0$. It is then easy to see that the small $\mathbf{p}'$ corresponding to the merging of the poles of both Green's functions give the main contribution in (14). With this in mind we neglect the dependence of $\omega^0_{\mathbf{p}-\mathbf{p}'}$ on $\mathbf{p}'$ in D_0.

For the planar case we have as the quadratic dispersion law

$$\varepsilon^0_{\mathbf{p}} = p_z^2/2m^*. \tag{15}$$

We introduce the notation

$$\varepsilon_F = p_0^2/2m^*, \qquad q_z/2 = p_0 + q', \qquad v_0 = p_0/m^*. \tag{16}$$

Then we have for G_0 in (14)

$$G_0(\mathbf{p} \pm \mathbf{q}/2,\ \varepsilon \pm \omega/2) = [\varepsilon \pm \omega/2 - v_0(q' \pm p_z) + i\Delta(q' \pm p_z)]^{-1}$$

(ϵ is reckoned from the Fermi energy ϵ_F). Integrating first over p_z' and then over ϵ' we finally find:

$$\operatorname{Re}\Gamma_1 = \frac{A^2_{\mathbf{q}}}{2(2\pi)^3}\left[\frac{1}{v_0q' + \omega^0_{\mathbf{q}} - \varepsilon}\ln\left|\frac{(v_0q')^2 - (\omega/2)^2}{(\varepsilon - \omega^0_{\mathbf{q}})^2 - (\omega/2)^2}\right| - \frac{1}{v_0q' - \omega^0_{\mathbf{q}} - \varepsilon}\ln\left|\frac{(v_0q')^2 - (\omega/2)^2}{(\varepsilon + \omega^0_{\mathbf{q}})^2 - (\omega/2)^2}\right|\right]\frac{m^*}{p_0}\int dp_x dp_y,$$

$$\operatorname{Im}\Gamma_1 = \frac{A^2_{\mathbf{q}}}{8\pi^2}\left\{\frac{\omega^0_{\mathbf{q}}}{(\omega^0_{\mathbf{q}})^2 - (\varepsilon - v_0q')^2}\left[\theta\left(v_0q' - \frac{\omega}{2}\right) + \theta\left(-v_0q' - \frac{\omega}{2}\right)\right] + \frac{1}{2}\frac{1}{v_0q' - \varepsilon + \omega^0_{\mathbf{q}}}\left[\theta\left(\varepsilon - \omega^0_{\mathbf{q}} - \frac{\omega}{2}\right) - \theta\left(-\varepsilon + \omega^0_{\mathbf{q}} - \frac{\omega}{2}\right)\right] + \frac{1}{2}\frac{1}{v_0q' - \varepsilon - \omega_0}\left[\theta\left(\varepsilon + \omega^0_{\mathbf{q}} - \frac{\omega}{2}\right) - \theta\left(-\varepsilon - \omega^0_{\mathbf{q}} - \frac{\omega}{2}\right)\right]\right\}\frac{m^*}{p_0}\int dp_x dp_y. \tag{17}$$

It follows from (17) that, as $q' \to \omega/2v_0$, Γ_1 goes logarithmically to infinity in the whole domain of $\mathbf{p}$ and ϵ, and that as a consequence it becomes necessary in this case to renormalize the vertex part. However, this situation occurs only within a narrow strip $\sim \xi$ close to the Fermi surface. Actually, for

$$\omega p_0/\varepsilon_F \ll q' \ll p_0 \tag{18}$$

we obtain for the domain of the variables essential to the determination of the polarization operator (3) directly from (17)

$$\Gamma_1 \sim \omega/v_0q'$$

Thus one can use the unrenormalized vertex Γ_0, and consequently the expression for the polarization operator (6), for the practically most interesting region of q' in the determination of the polarization operator for nearly plane Fermi surfaces. However the determination of Π for $q' \sim \omega/2v_0$ requires a consistent renormalization of the vertex part.

Having in mind the domain of q' corresponding to (18) we find a value of the polarization operator (6). Taking (15) and (16) into account, we obtain

$$\Pi(\mathbf{q}, \omega) = \frac{1}{2(2\pi)^3}\frac{m^*}{p_0}\left(\int dp_x dp_y\right)\ln\left(\frac{4\varepsilon_F}{v_0q'}\right)^2. \tag{19}$$

This expression is valid for both $q' < 0$ and $q' > 0$.

It follows from (9) and (19) that $\partial\omega/\partial q_z$ behaves like $1/q'$ for the most interesting domain (18) of the wave vector and that as a consequence the singularity in the phonon spectrum due to the electron-phonon interaction becomes very sharp (cf. the curve c in Fig. 1).

One can easily conclude from formula (19) that in the case considered there can also occur an essential absolute rearrangement of the spectrum in the region (18). Effectively, the presence of the large factor $\ln(4\epsilon_F/v_0q')^2$ in (19) in comparison with (8) and (10) makes such a renormalization quite real in the absence of a small parameter in the electron-phonon interaction.

In order to determine the character of the renormalization of the phonon energy we evaluate the quantity $A^2_{\mathbf{q}}$. Neglecting Umklapp processes, the usual calculation leads to the expression

$$A^2_{\mathbf{q}} \approx \frac{q^2U_0}{2M\omega_{\mathbf{q}}}\left|\frac{\mathbf{q}}{q}\,\mathbf{j}_{\mathbf{q}}\right|^2 C^2_{\mathbf{q}}. \tag{20}$$

Here $\mathbf{j}_{\mathbf{q}}$ is the polarization vector of the phonon, $C_{\mathbf{q}}$ the Fourier component of the electron-lattice interaction potential referred to the volume of the elementary cell U_0. We introduce the notation $C_{\mathbf{q}} = \beta_{\mathbf{q}}\epsilon_F$. Generally one takes $\beta_{\mathbf{q}} \sim 1$ in all calculations.

In the case of interest to us $q \approx 2p_0$. Then the relative change of the frequency due to electron-phonon interaction is determined by the expression

$$(\omega^2_{\mathbf{q}} - \omega^{02}_{\mathbf{q}})/\omega^{02}_{\mathbf{q}} = -B\ln(4\varepsilon_F/v_0q')^2, \tag{21}$$

$$B = 2\beta^2_{\mathbf{q}}\left(\frac{\mathbf{q}}{q}\,\mathbf{j}_{\mathbf{q}}\right)^2\frac{m^*}{M}\left(\frac{\varepsilon_F}{\omega^0_{\mathbf{q}}}\right)^2\frac{2p_0\left(\int dp_x dp_y\right)}{\tilde{V}_0}, \tag{21'}$$

where $\tilde{V}_0$ is the volume of the elementary cell of the reciprocal lattice (the integration is extended only over the first cell of the reciprocal lattice). An analysis of the expression (21') shows that even

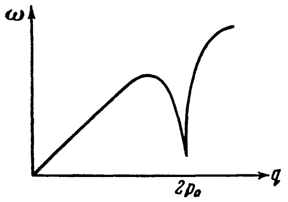

FIG. 4

though this coefficient is smaller than unity the absence of a small parameter leads under favorable conditions (in particular if an essential part of the surface area is nearly planar) to a considerable quantity. But in this case, as is directly seen from (21), a strong drop of ω_q occurs for $q_z \sim 2p_0$.

Figure 4 shows the shape of the dispersion curve of ω_q corresponding to the case considered. The depth of the dip in the dispersion curve is connected with the size of the coefficient (21′). We note that a very strong diminution of the frequency near $q_z = 2p_0$, which cannot be excluded in principle, could lead to a considerable change in the energy dependence of a whole series of quantities, since there could appear phonons with large wave vector but relatively small energy.

In order to analyze how the character of the singularity ceases to be planar at small deviations of the Fermi surface, we can proceed in analogy to what we did in the case of cylindrical surfaces. Let the Fermi surface be defined by the equation $p_z = p_z(p_x, p_y)$. For definiteness we consider $q' > 0$, whence we shall understand p_0 to be the maximum value of p_z for the part of the surface considered. Then, introducing the notation

$$\Delta p\,(p_x, p_y) = p_0 - p_z\,(p_x, p_y),$$

for the term in $\partial\omega/\partial q_z$ due to the electron-phonon interaction, we find an expression proportional to

$$\int \frac{dp_x dp_y}{q' + \Delta p\,(p_x, p_y)}\,.$$

Hence it is seen immediately that if, for example, the Fermi surface is a plane corrugated in one direction, then for q' larger than the corrugation amplitude $\partial\omega_q/\partial q \sim 1/q'$, and for $q' \ll \delta$, the singularity becomes rootlike. If $\Delta p \sim p_y^{2n}$ then $\partial\omega_q/\partial q_z$ will behave like $(q')^{-1+1/2n}$.

A similar analysis can easily be carried out for most different cases. It is clear that for small deviations of the surface from a plane, in the presence of arbitrary deviations in one direction and in the presence of a region of flatness—in all these cases the singularity in $\partial\omega_q/\partial q_z$ becomes considerably stronger than in the case of almost spherical Fermi surfaces.

We remark that in the case of a plane Fermi surface the phonon damping is zero for the whole domain of q_z except a narrow strip $\sim \xi$ near $q_z = 2p_0$.

5. CONCLUDING REMARKS

Inelastic neutron scattering experiments afford the possibility of detecting singularities in the phonon spectra of metals, and under favorable conditions even the phonon width. However the determination of the behavior of ω_q within a strip around the Fermi surface corresponding to $\Delta q \sim \omega p_0/\epsilon_F$ is made very difficult by the experimental setup. Therefore in the first place one has to wait for an investigation of the course of the dispersion curve for the wave vector interval determined by (18). However it follows from the results of the preceding section that an anomalous course of the ω_q curve clearly appears in that region, particularly if there are significant parts of the Fermi surface corresponding to one or both radii of curvature large compared to p_0. Obviously the latter situation can be very easily realized in the case of open surfaces. Thallium can be called an example of a metal with an almost plane Fermi surface, where the singularity should show up particularly strongly (N. E. Alekseevskiĭ, private communication).

The weak logarithmic singularity in the case of nearly spherical Fermi surfaces is in all probability very hard to detect experimentally. The dispersion curve found by Brockhouse for tin, if it is true, clearly shows a stronger singularity. One has to observe that the course of the curve corresponds to the case where at least one of the radii of curvature is large though the vagueness of the experimental resolution makes this statement conditional.

In those cases, where the damping due to the electron-phonon interaction represents a considerable part of the general phonon damping with $q \lesssim 2p_0$, a real possibility of getting information about the Fermi surface on the basis of measurements of the phonon width, particularly considering the sharp jump of γ_q at $q = 2p_0$, opens up. It is interesting that the character of the dependence of γ_q on q differs strongly for the various types of Fermi surfaces as shown in the previous considerations.

In conclusion we observe that the results obtained in the present work can be directly applied to an analysis of the singularities in the dispersion law of spin waves due to the electron-phonon interaction.

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