ON THE THEORY OF THE AMPLIFICATION OF ULTRASOUND BY SEMI-METALS IN ELECTRIC AND MAGNETIC FIELDS

R. F. KAZARINOV and V. G. SKOBOV

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

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The passage of ultrasound through a semi-metal in an electric field is investigated. It is shown that the interaction of conduction electrons with the sound wave leads to amplification of the latter if the electron drift velocity in the direction of sound propagation exceeds the velocity of sound propagation. The influence of a magnetic field on this effect is considered. It is noted that in many cases the presence of a strong magnetic field perpendicular to an electric field leads to a significant increase in the amplification factor of the sound. Various cases are considered of the resonance dependence of the amplification factor on the magnitude of the electric and magnetic fields.

As is well known, the interaction of a sound wave with the conduction electrons affects its passage through a crystal. It is therefore evident that the presence of external electric and magnetic fields can have a significant effect on the intensity of the sound vibrations. In particular, amplification of ultrasound by the conduction electrons is possible for sufficient intensity of the electric field **E**.

In the work of Watson, McFee, and White [1], evidence is given for the amplification of ultrasound in piezoelectric CdS in an electric field. The experiment was carried out at room temperature, where there is appreciable absorption of ultrasound by the thermal vibrations of the lattice. However, in spite of this fact and in spite of the low concentration of conduction electrons, the strong piezoelectric coupling of the latter with the sound vibrations makes sound amplification possible.

In our previous note, [2] it was recorded that in principle the possibility of amplification of sound by conduction electrons in an electric field does not depend on the concrete character of the interaction and is brought about by the Cerenkov radiation of sound waves forced by the electrons. The latter takes place in the case in which the electron drift velocity in the direction of propagation of the sound wave exceeds its phase velocity s.

1. Let the system of electrons be in a stationary state which is characterized by a drift velocity that is small in comparison with the characteristic electron velocities. We shall assume that the electron distribution function f_0 depends on $\epsilon(\mathbf{p}-\mathbf{p}_0)$, where $\epsilon(\mathbf{p})$ is the energy of an electron with momentum **p**, and $\mathbf{p}_0$ is the shift in the distribution of the electrons in momentum space brought about by their drift. We emphasize that this drift can be caused both by an electric field **E** and by the temperature gradient ∇T.

The electron acquires an additional energy ϵ' in the field of the sound wave, which in first approximation is proportional to the deformation tensor of the crystal:

$$\varepsilon'(\mathbf{p}, \mathbf{r}, t) = U_0(\mathbf{p}) \cos(\boldsymbol{\varkappa}\mathbf{r} - \omega t),$$

where ω and $\boldsymbol{\kappa}$ are the frequency and wave vector of the sound, $U_0(\mathbf{p})$ is the amplitude value of the deforming potential with account of its screening by the electrons.

We shall seek the electron distribution function in the form

$$F(\mathbf{p}, \mathbf{r}, t) = f_0\{\varepsilon(\mathbf{p}-\mathbf{p}_0) + \varepsilon'(\mathbf{p}, \mathbf{r}, t)\} + f'(\mathbf{p}, \mathbf{r}, t),$$

where the function f′, in the linear approximation relative to U_0, is determined by an equation that is similar to the equation of Akhiezer, Kaganov, and Lyubarskiĭ [3]

$$(\partial/\partial t + \mathbf{v}\cdot\nabla + \nu) f'(\mathbf{p}, \mathbf{r}, t) = -df_0/dt$$
$$= -\frac{\partial f_0}{\partial \varepsilon}\left\{\frac{\partial}{\partial t} + \left[\mathbf{v} - \frac{\partial \varepsilon(\mathbf{p}-\mathbf{p}_0)}{\partial \mathbf{p}}\right]\nabla\right\}\varepsilon'(\mathbf{p}, \mathbf{r}, t). \qquad (1)$$

Here $\tau = \nu^{-1}$ is the relaxation time of the conduction electrons, $\mathbf{v} = \partial\epsilon(\mathbf{p})/\partial\mathbf{p}$ is the velocity of the electron with momentum **p**. We neglect the contribution of the transverse electric field, brought about by deformation of the crystal under the action of the sound wave. Furthermore, in the evaluation of the derivative of f_0 with respect to time in (1), we have taken it into account that the collision integral, which is preportional to f_0, cancels out the term leading to drift of the electrons.

Solution of Eq. (1) has the form

$$f'(\mathbf{p}, \mathbf{r}, t) = \frac{\partial f_0}{\partial \varepsilon}[\omega - \varkappa \mathbf{v}_0(\mathbf{p})] \operatorname{Re} \frac{U_0 \exp i(\varkappa \mathbf{r} - \omega t)}{(\omega - \varkappa \mathbf{v}) + i\nu},$$
$$\mathbf{v}_0(\mathbf{p}) = \frac{\partial}{\partial \mathbf{p}}\left(\mathbf{p}_0 \frac{\partial \varepsilon(\mathbf{p})}{\partial \mathbf{p}}\right). \tag{2}$$

The sound energy absorbed by the electrons per unit time and per unit volume is equal to

$$Q = \left\langle \frac{d}{dt} 2 \int \frac{d^3p}{(2\pi\hbar)^3} \{\varepsilon(\mathbf{p}) + \varepsilon'(\mathbf{p}, \mathbf{r}, t)\} F(\mathbf{p}, \mathbf{r}, t) \right\rangle,$$

where the angle brackets denote the time average. Taking it into account that the total derivative with respect to time of the energy of the electron ($\epsilon + \epsilon'$) is equal to the partial derivative, while the corresponding derivative of the distribution function F is zero, we can represent Q in the form

$$Q = \int \frac{d^3p}{(2\pi\hbar)^3} |U_0|^2 \frac{\partial f_0\{\varepsilon(\mathbf{p} - \mathbf{p}_0)\}}{\partial \varepsilon} \omega [\omega - \varkappa \mathbf{v}_0(\mathbf{p})]$$
$$\times \frac{\nu}{\nu^2 + (\omega - \varkappa \mathbf{v})^2} \tag{3}$$

or, with accuracy up to terms of second order in p_0/p_F (p_F is the Fermi momentum)

$$Q \approx (1 - \varkappa \mathbf{v}_{0s}/\omega) Q_0, \tag{4}$$

where Q_0 is the sound energy absorbed by the electrons at $p_0 = 0$. In the case $\kappa l \ll 1$ ($l = v_F\tau$ is the length of the electron mean free path, v_F is the Fermi velocity) v_{0s} is of the order of the mean electron drift velocity of the system v_0. In the opposite limiting case $\kappa l \gg 1$, the quantity v_{0s} characterizes the drift of the electron moving in phase with the sound wave.

Thus if $\omega < \boldsymbol{\kappa} \cdot \mathbf{v}_{0s}$, then the sound absorption coefficient is negative and amplification of the sound waves by the electrons takes place. This is a consequence of the inequality of the distribution of electrons in the presence of the drift. In the case $\omega < \boldsymbol{\kappa} \cdot \mathbf{v}_{0s}$, the probability of emission of a quantum by an electron is greater than the probability of absorption, and forced Cerenkov radiation of the sound takes place.

The most useful crystal for the amplification of sound is apparently bismuth, in which the electronic sound absorption dominates the lattice absorption at low temperatures. On the other hand, the Joule power scattered in bismuth

$$W = n_e m v_0^2/\tau, \qquad \mathbf{v}_0 = e\mathbf{E}\tau/m$$

is relatively small, inasmuch as the concentration of electrons n_e and their effective mass m in bismuth are small, while τ is large.

We now investigate the amplification of ultrasound in a semi-metal in crossed electric and magnetic fields, $\mathbf{E} \perp \mathbf{H}$. We consider the case of a strong magnetic field $\Omega\tau \gg 1$ ($\Omega = |e| H/mc$, c is the velocity of light) and of strong spatial dispersion $\kappa l \gg 1$. In particular, we shall be interested in a quantizing magnetic field $\hbar\Omega \gg T$, so that our considerations will have a quantum character from the very beginning.

2. As is well known, stationary states exist for an electron in crossed electric and magnetic fields **E** and **H**. Let $\mathbf{E} \parallel OX$, $\mathbf{H} = \operatorname{curl} \mathbf{A}$, $A_x = A_z = 0$, $A_y = Hx$. Under such circumstances, the stationary states of the electron are characterized by a magnetic quantum number n, a projection of the wave vector in the direction of the magnetic field k_z, and coordinate of the center of rotation X. For simplicity, the spectrum of the conduction electrons will be assumed to be quadratic and isotropic; we shall not take spin into account. Then the eigenfunctions and the energy eigenvalues have the form [4]

$$\Psi_{nk_zX}(\mathbf{r}) = \gamma^{1/4}(\pi L_y L_z)^{-1/2} \exp\{i[k_z z - \gamma(X - eE/m\Omega^2)y]$$
$$- \gamma(x - X)^2/2\} H_n[\sqrt{\gamma}(x - X)],$$
$$\varepsilon_{nk_zX} = \hbar\Omega(n + 1/2) + \hbar^2 k_z^2/2m - eEX, \tag{5}$$

$\gamma = |e| H/\hbar c$, $H_n(v)$ is the Hermite polynomial of n-th order, normalized to unity, L_y and L_z are the dimensions of the crystal along the directions of the y and z axes. In what follows, we shall denote the choice of the quantum numbers nk_zX by the Greek letters α, β, etc.

The stationary states (5) are characterized by a Hall electron drift along the y axis with velocity $-cE/H$. The mean electron velocity in the direction of **E** is equal to zero in the absence of scattering. Interaction with the scatterers leads to the appearance of a conduction current. In this case the concentration of the electrons in the stationary states is homogeneous and their distribution function does not depend on X. By describing this non-equilibrium stationary state by the distribution function F, we can represent it in the form of the sum $F = f + f_1$. Here f_1 is the part of the distribution function (which is non-symmetric in the electron velocity) connected with the conduction current, while f is the symmetric part, depending only on the kinetic energy of the electron

$$\varepsilon_\alpha^{(0)} = \hbar\Omega(n + 1/2) + \hbar^2 k_z^2/2m.$$

The symmetric part of the distribution function of the electrons in crossed electric and magnetic fields was found earlier by the authors [5] for the case of Boltzmann statistics. It was shown that in the scattering of electrons by neutral impurities and acoustic phonons, this function differs from the equilibrium function by the replacing of the lattice temperature T by an effective temperature

$$T_{eff} = T\{1 + \tfrac{1}{3}(cE/sH)^2(1 + \nu_i/\nu_{ph})\}, \qquad (6)$$

here ν_i and ν_{ph} are the collision frequencies of the electrons with impurities and with phonons.

With the help of a method similar to that used in [5], it can be shown that for Fermi statistics, under satisfaction of the condition

$$mv_F s \ll T$$

f_α is the Fermi function of argument $(\epsilon_\alpha^{(0)} - \zeta)/T_{eff}$[1], i.e.,

$$f_\alpha \equiv f(\varepsilon_\alpha^{(0)}) = [1 + \exp\{(\varepsilon_\alpha^{(0)} - \zeta)/T_{eff}\}]^{-1}, \qquad (7)$$

where ζ is the chemical potential and T_{eff} is determined by Eq. (6).

3. The Hamiltonian of the interaction of the electron with the sound wave has the form

$$H'(t) = \tfrac{1}{2}(Ue^{-i\omega t} + U^+e^{i\omega t}),$$
$$U = U_0 e^{i\varkappa r}, \qquad U_0 = \Lambda_{ik}u_{ik}^{(0)}; \qquad (8)$$

here $u_{ik} = (\partial u_i/\partial x_k + \partial u_k/\partial x_i)/2$ is the deformation tensor, $u_{ik}^{(0)}$ is its amplitude value; Λ_{ik} is the deformation-potential tensor, which can be taken as a constant for bismuth; use of the repeated indices i and k in (8) denotes summation; the sign + denotes the Hermitian adjoint.

In the case $\kappa l \gg 1$, and in the absence of any sort of resonance, the effect of electron scattering on the absorption and emission of sound quanta can be neglected; the sound energy Q absorbed by the electrons per unit time has the form [6]

$$Q = \frac{\pi}{2\hbar}\sum_{\alpha\beta}(\varepsilon_\beta - \varepsilon_\alpha)\{|U_{\beta\alpha}|^2\delta(\varepsilon_\beta - \varepsilon_\alpha - \hbar\omega)$$
$$+ |U_{\beta\alpha}^+|^2\delta(\varepsilon_\beta + \hbar\omega - \varepsilon_\alpha)\}f_\alpha(1 - f_\beta), \qquad (9)$$

here $U_{\beta\alpha}$ is the matrix element of U between the wave functions α and β; we have neglected the contribution of the small non-symmetric part of f_1 in Q. Reordering the summation indices α and β in the second component in the curly brackets, and taking the law of conservation of energy into account, we get (9) in the form

$$Q = \frac{\pi\omega}{2}\sum_{\alpha\beta}|U_{\beta\alpha}|^2\delta(\varepsilon_\beta - \varepsilon_\alpha - \hbar\omega)[f(\varepsilon_\alpha^{(0)}) - f(\varepsilon_\beta^{(0)})]. \quad (10)$$

[1] The authors are grateful to V. L. Gurevich who pointed out this circumstance to them.

Direct calculation of the matrix element $U_{\alpha'\alpha}$ leads to the expression

$$|U_{\alpha'\alpha}| = |U_0|\,Q_n^{(n'-n)}(\varkappa_\perp^2/2\gamma)\,\delta(k_z', k_z + \varkappa_z)\,\delta(X', X - \varkappa_y/\gamma),$$
$$Q_n^{(k)}(v) = e^{-v/2}v^{k/2}L_n^{(k)}(v), \qquad (11)$$

$L_n^{(k)}(v)$ is the associated Laguerre polynomial normalized to unity:

$$\varkappa_\perp^2 = \varkappa_x^2 + \varkappa_y^2.$$

It follows from the conservation laws that

$$\varepsilon_\beta^{(0)} - \varepsilon_\alpha^{(0)} = eE(X_\beta - X_\alpha) + \hbar\omega = \hbar(\omega - \varkappa_y v_y),$$
$$v_y = -cE/H. \qquad (12)$$

By assuming the difference in the energies (12) to be small in comparison with T_{eff}, we get

$$f(\varepsilon_\alpha^{(0)}) - f(\varepsilon_\beta^{(0)}) \approx \hbar(\omega - \varkappa_y v_y)\,\partial f(\varepsilon_\alpha^{(0)})/\partial\zeta. \qquad (13)$$

Thus, if the Hall electron drift velocity is larger than the sound velocity, and the direction of the wave vector κ is close to the direction of the Hall current, then the difference (13) is negative. This means that in the absorption of a sound quantum, the electron enters a state with higher population. In radiation of the same quantum, the kinetic energy of the electron is increased and it enters a state with smaller population. Therefore the probability of emission of similar phonons is shown to be greater than the probability of their absorption. This fact is a consequence of the non-equilibrium character of the system, which appears in the fact that the distribution function f_α does not depend on the coordinates of the center of rotation X_α, while the total energy of the electron possesses the component $-eEX_\alpha$.

4. We first consider the case $\kappa_z = 0$. We substitute (11) and (13) in (10), and carry out summation over the quantum indices k_z and X. Setting $\partial f(\epsilon)/\partial\zeta \approx \delta(\epsilon - \zeta)$, and taking it into account that the number of states of the electron with different X is equal to $\gamma L_x L_y/2\pi$ (L_x is the dimension of the crystal along the x axis, $V = L_x L_y L_z$), we get

$$Q = \omega(\omega - \varkappa_y v_y)|U_0|^2\frac{\gamma V}{8\pi\hbar}\sum_n\left[\frac{2m}{\zeta - \hbar\Omega(n + 1/2)}\right]^{1/2}$$
$$\times\sum_{k=-n}^{\infty}\left[Q_n^{(k)}\left(\frac{\varkappa_\perp^2}{2\gamma}\right)\right]^2\delta(\omega - \varkappa_y v_y - k\Omega), \qquad (14)$$

where the summation is carried out over all n for which the radicand in (14) is positive. If the values of E, H, and κ are such that the argument of the δ-function in (14) vanishes at any positive k, then resonance sound absorption is observed. If the vanishing of this argument takes place for any negative k, then resonance amplification of the sound wave takes place. This resonance is similar to the cyclotron resonance in sound absorption in metals in a magnetic field, which was predicted by Mikoshiba [7] and observed in Ga by Roberts. [8] However, owing to the presence of the electric field, not only resonance absorption is possible, but also resonance amplification. Moreover, it should be noted that the cyclotron resonance absorption (amplification) is also possible for $\omega\tau \ll 1$ in the presence of an electric field, provided that $\Omega\tau \gg 1$.

We shall now consider in more detail the case in which the argument of the δ-function in (14) vanishes for $k = -1$. In this case, the change in the potential energy of the electron in the electric field $\hbar\kappa_y v_y$ results in the increase of its kinetic energy by $\hbar$ and in the emission of a sound quantum with energy $\hbar\omega$.

Making use of the asymptotic form of the Laguerre function for $n \gg |k|$, $n \gg v$:

$$Q_n^{(k)}(v) \approx J_k\left[\sqrt{v(4n+2|k|+2)}\right], \qquad (15)$$

where J_k is the Bessel function of order k, and replacing the summation over n in (14) by integration, we put the coefficient Γ in the form

$$\Gamma \approx -\Omega^2\omega^{-1}A(\varkappa R)\,\delta(\omega - \varkappa_y v_y + \Omega)\,\Gamma_0;$$

$$A(v) = v\int_0^1 dx\, J_1^2\left(v\sqrt{1-x^2}\right), \qquad (16)$$

where

$$\Gamma_0 = m^2|U_0|^2/2\pi\hbar^3\rho u_0^2 s\varkappa$$

is the sound absorption coefficient for $E = H = 0$, [3] ρ is the density of the crystal, $R = v_F/\Omega$, for $v \gg 1$ the quantity $A(v) \to 1$.

The expression (16) becomes infinite if the argument of the δ-function vanishes. For all other values of the argument, $\Gamma = 0$. This result is a consequence of neglect of electron scattering ($\tau \to \infty$). Account of scattering leads to a finite relaxation time for the electrons τ and to a finite width of the energy levels, equal to $\hbar/\tau$. Therefore, the δ-function has the order of τ at resonance and the maximum value of the sound amplification coefficient is

$$(-\Gamma)_{max} \approx \Gamma_0\Omega^2\tau/\omega. \qquad (17)$$

5. We now consider the case of stronger magnetic fields H, satisfying the condition

$$|\varkappa_y v_y| \ll \Omega, \qquad (18)$$

and small but non-vanishing κ_z:

$$|\varkappa_z|R \ll 1, \qquad |\varkappa_z|l \gg 1. \qquad (19)$$

In this case, the argument of the δ-function vanishes in (10) only for $n = n'$, and the expression for Q takes the form

$$Q = \frac{\gamma V}{8\pi}\omega(\omega - \varkappa_y v_y)|U_0|^2\sum_{n=0}^{\infty}\left[Q_n^{(0)}(\varkappa_\perp^2/2\gamma)\right]^2 \times\int_{-\infty}^{+\infty} dk_z \frac{\partial f(\varepsilon_{nk_z}^{(0)})}{\partial\zeta}\,\delta\left(\frac{\hbar\varkappa_z k_z}{m} - \omega + \varkappa_y v_y\right). \qquad (20)$$

Carrying out the integration over k_z in (20), and using the asymptotic form (15), we write the coefficient Γ in the form

$$\Gamma = \Gamma_0\frac{w}{s}\frac{\hbar\Omega}{4T_{eff}}\sum_n J_0^2\left(\varkappa_\perp\sqrt{\frac{2n+1}{\gamma}}\right)\cosh^{-2}\xi_n; \qquad (21)$$

$$\xi_n = |\zeta - \hbar\Omega(n+\tfrac{1}{2}) - mw^2/2|/2T_{eff},$$

$$w = (\omega - \varkappa_y v_y)/|\varkappa_z|. \qquad (22)$$

Upon decrease in $|\kappa_z|$, the value of $|w|$ increases. Therefore, at sufficiently small $|\kappa_z|$, the absolute value of w becomes larger than v_F, and the coefficient Γ approaches zero exponentially. In what follows, we shall be interested in such κ_z for which $|w| < v_F$.

Let us investigate (21) in various limiting cases.

a) In the case $\hbar\Omega \ll T_{eff}$, summation over n in (21) can be replaced by integration. For not too high frequencies of the ultrasound, when

$$\varkappa_\perp R < 1, \qquad (23)$$

the value of the Bessel function in (21) is close to unity, and

$$\Gamma \approx \Gamma_0 w/s, \qquad (24)$$

i.e., the absorption (amplification) coefficient of sound in the presence of a magnetic field can be v_F/s times larger than in its absence.

b) In the case of a quantizing magnetic field $\hbar\Omega \gg T_{eff}$ and $\kappa_z^2 l^2 \gg N$ [9] the summation in (21) can be limited to a single term with $n = N$ for which the quantity ξ_n is minimum. Then

$$\Gamma = \Gamma_0\frac{w}{s}J_0^2(\varkappa_\perp R_N)\frac{\hbar\Omega}{4T_{eff}}\cosh^{-2}\xi_N, \qquad (25)$$

where $R_N = \sqrt{(2N+1)/\gamma}$ is the Larmor radius of the electron with magnetic quantum number N.

In the change of the magnetic field in the case (23), the value of Γ (25) undergoes strong oscillations, similar to those studied in the work of Gurevich et al. [10] The origin of these oscillations is the following. It follows from the laws of conservation of energy and momentum that only electrons with $v_z = w \operatorname{sign} \kappa_z$ can absorb and emit sound quanta. In the case $\hbar\Omega \gg T_{eff}$, such electrons can, upon change in the magnetic field, turn out to be in the region of smearing out of the Fermi distribution or outside of it. Here the argument of the hyperbolic cosine ξ_N varies over the range from zero to $\hbar\Omega/4T_{eff}$, which produces oscillations of Γ as a function of H. However, in the case under consideration, the value of w determined by Eq. (22) depends on H, as a consequence of which the period of these oscillations is different and is given by the formula

$$\Delta H/H \approx \hbar\Omega\,[\zeta - mw\,(\tfrac{3}{2}w - \omega/|\varkappa_z|)]^{-1}; \qquad (26)$$

here ΔH is the difference between neighboring maxima. Equation (26) for the period of oscillations is applicable when the absolute value of the right hand side of (26) is small in comparison with unity.

The value of w also depends on the electric field E. Therefore, upon a change in the latter, ξ_N also varies in the range from zero to $\hbar\Omega/4T_{eff}$. This leads to similar oscillations of Γ as a function of E.

In the case

$$\varkappa_{\perp} R_N > 1 \qquad (27)$$

the Bessel function in (25) oscillates upon variation of the magnetic field, and oscillations of "geometric resonance" are superimposed on the quantum oscillations of the coefficient Γ. The explanation of this resonance was first given by Pippard.[11] The resonance maxima and minima are also observed in the case in which a half-integral number of sound wavelengths is included in the orbit of the electron. In the case under consideration quantization of the electrons leads to the result that the amplitude, period and phase of these oscillations differs from the corresponding quantities in the case $\hbar\Omega \ll T$. In particular, the amplitude of the oscillations of the "geometric resonance" in (25) is not small. The period of the oscillations depends in a rather complicated way on the magnetic field, and we shall not write out their expression here.

Thus, if the Hall velocity of the electrons is larger than the sound velocity, and the direction of its propagation is close to the direction of the Hall current, then interaction with electrons leads to amplification of the sound waves. However, in passing through a crystal, the sound is attenuated as a result of the interaction with the thermal vibrations of the lattice and with defects. Therefore the resulting coefficient of amplification Γ_T is the difference of the electron amplification coefficient $-\Gamma$ and the coefficient of absorption due to other interactions Γ_1:

$$\Gamma_T = -(\Gamma + \Gamma_1).$$

We are interested in the case in which the electron coefficient Γ is negative and Γ_T is greater than zero. The coefficients Γ and Γ_1 again depend on the value and direction of the wave vector of the sound κ. In most cases, the coefficient Γ is proportional to κ, while Γ_1 increases more rapidly with increase in κ. Therefore, close to some definite value of κ, the amplification coefficient of the sound Γ_T reaches a maximum value. By changing the value of the electric and magnetic fields, one can change the value of this optimal κ_{opt}. Corresponding values of Γ_T can be very large in value which in principle makes it possible to amplify thermal phonons with $\kappa \sim \kappa_{opt}$, i.e., to use this effect for the generation of ultrasound of high frequencies. We note that the resonant dependence of the expressions (16) and (25) on the value of κ corresponds to amplification of a collection of definite discrete frequencies.

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253