CONVERSION OF ELECTRON-POSITRON PAIRS INTO μ-MESON PAIRS

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The cross section for the $e^- + e^+ \to \mu^- + \mu^+$ process is calculated taking the invariant "smeared-out structure" of the electrons and muons into account.

1. In connection with the feasibility of experiments employing colliding beams of high-energy electrons and positrons, a number of articles have recently been devoted to processes, occurring in the collision of electrons and positrons, which could serve as test of the validity of quantum electrodynamics at small distances.[1,2]

In the present article we consider the conversion of an electron-positron pair into a μ-meson pair. Such a conversion has been first discussed by Berestetskiĭ and Pomeranchuk;[3] Baĭer and Synakh have studied the production of a bound $\mu^+\mu^-$ system (bimuonium) in electron-positron collisions.[4] This process requires a c.m.s. energy of the colliding particles greater than the threshold value of 106 MeV. At such energies one can expect a discrepancy between the experimental results and calculations based on quantum electrodynamics. The reason for this lies in a required modification of the equations at small distances, nonlocality of interactions, the existence of a structure of the electrons (positrons) and muons and, finally, nonelectromagnetic interaction between these particles.

A modification of the equations of quantum electrodynamics is expressed mathematically by the change of the photon propagation function[5] $1/q^2 \to C(q^2)/q^2$ (where q^2 is the square of the four-momentum of the virtual photon produced in the conversion of the colliding electron and positron).

An infringement of locality leads to the appearance of a certain form factor $F(q^2)$ in the expression for the current matrix element,[6] and is thus mathematically equivalent to the former case.

In order to take the structure of the π^0 particles involved in the $e^- + e^+ \to \mu^- + \mu^+$ process into account, we have to write in the expression for the particle density current $e\bar{u}(p_2)\gamma_\nu u(p_1)$ the following operator instead of γ_ν:

$$\Gamma_\nu(q) = a(q^2)\gamma_\nu + \frac{ib(q^2)}{4M}[\gamma_\nu, \hat{q}], \quad (1)$$

where $q = p_2 - p_1$; $a(q^2)$ and $b(q^2)$ are invariant functions, and M is the particle mass.

It can be easily seen that the description of deviations from quantum electrodynamics by means of the vertex operator (1) contains the two variants discussed above as special cases $[b(q^2) = 0]$.

2. Furlan and Peressutti[2] have calculated the cross section for the process $e^- + e^+ \to \mu^- + \mu^+$ taking the μ-meson structure into account. At energies necessary for the conversion to occur, the structure of the particles taking part in the process may influence the outcome. It is therefore necessary to obtain a general formula that takes the "smeared-out structure" of both muons and electrons (positrons) into account.

The matrix element of the process is[1)]

$$S = (2\pi)^4 \frac{e^2}{q^2}\delta(p_- + p_+ - P_- - P_+)\Big\{\bar{v}(-p_+)\Big[a(q^2)\gamma_\nu - \frac{ib(q^2)}{4m}(\gamma_\nu\hat{q} - \hat{q}\gamma_\nu)\Big]u(p_-)\Big\}\Big\{\overline{U}(P_-)\Big[A(q^2)\gamma_\nu + \frac{iB(q^2)}{4M}(\gamma_\nu\hat{q} - \hat{q}\gamma_\nu)\Big]V(-P_+)\Big\}, \quad (2)$$

where $q = p_+ + p_- = P_+ + P_-$; lower-case letters refer to electrons and positrons, and capitals to muons. Calculations are carried out in the c.m.s.

The differential cross section for the process is

$$\begin{aligned} d\sigma/d\Omega = {} & \tfrac{1}{16} r_0^2\lambda^2\sqrt{1-\Lambda^2}\,\{|a(q^2)|^2|A(q^2)|^2 \\ & \times [1 + \Lambda^2 + (1-\Lambda^2)\cos^2\vartheta] + 12\,\mathrm{Re}\,[a(q^2)b^*(q^2)] \\ & \times \mathrm{Re}\,[A(q^2)B^*(q^2)] + 4|a(q^2)|^2\,\mathrm{Re}\,[A(q^2)B^*(q^2)] \\ & + 2\,\mathrm{Re}\,[a(q^2)b^*(q^2)]\,|A(q^2)|^2(2+\Lambda^2) \\ & + |a(q^2)|^2|B(q^2)|^2\Lambda^{-2}[1+\Lambda^2-(1-\Lambda^2)\cos^2\vartheta] \\ & + |b(q^2)|^2|A(q^2)|^2\lambda^{-2}[1-(1-\Lambda^2)\cos^2\vartheta] \\ & + 2\,\mathrm{Re}\,[a(q^2)b^*(q^2)]\,|B(q^2)|^2\Lambda^{-2}(1+2\Lambda^2) \\ & + |b(q^2)|^2|B(q^2)|^2\lambda^{-2}\Lambda^{-2}[\Lambda^2+(1-\Lambda^2)\cos^2\vartheta] \\ & + 2|b(q^2)|^2\,\mathrm{Re}\,[A(q^2)B^*(q^2)]\,\lambda^{-2}\}, \end{aligned} \quad (3)$$

1)We are using a system of units in which $\hbar = c = 1$, $e^2/4\pi = \varphi = 1/137$.

where $r_0 = \alpha/m$ is the classical electron radius, $\lambda = m/E$, and $\Lambda = M/E$.

The functions $a(q^2)$ and $b(q^2)$ enter the cross section for electron-electron scattering.[7] However, different domains of the argument q^2 correspond to the scattering annihilation processes: for scattering

$$q^2 = 4(E^2 - m^2)\sin^2(\vartheta/2) > 0,$$

and for annihilation

$$q^2 = -4E^2 < 0.$$

Information concerning the electron form factors $a(q^2)$ and $b(q^2)$ in the range where $q^2 < 0$ can be obtained from experiments on elastic scattering of positrons on electrons, since the annihilation part of the cross section for this process will contain form factors with negative arguments. Since in that case the form factors a, b, A, and B are independent of ϑ, we can integrate (3) over the angles and obtain the total cross section:

$$\begin{aligned}\sigma = \tfrac{1}{4}\pi r_0^2\lambda^2\sqrt{1-\Lambda^2}\{&\tfrac{2}{3}|a|^2|A|^2(2+\Lambda^2)\\ &+12\,\mathrm{Re}(ab^*)\,\mathrm{Re}(AB^*)+4|a|^2\,\mathrm{Re}(AB^*)\\ &+2|A|^2\,\mathrm{Re}(ab^*)(2+\Lambda^2)\\ &+\tfrac{2}{3}|a|^2|B|^2\Lambda^{-2}(1+2\Lambda^2)\\ &+\tfrac{1}{3}|b|^2|A|^2\lambda^{-2}(2+\Lambda^2)\\ &+2\,\mathrm{Re}(ab^*)|B|^2\Lambda^{-2}(1+2\Lambda^2)\\ &+2\,\mathrm{Re}(AB^*)|b|^2\lambda^{-2}\\ &+\tfrac{1}{3}|b|^2|B|^2\lambda^{-2}\Lambda^{-2}(1+2\Lambda^2)\}. \end{aligned} \quad (4)$$

If we put here $a = A = 1$, $b = B = 0$, and go over to the frame in which the electron or positron is at rest, we obtain the Berestetskiĭ-Pomeranchuk formula.[3]

Form factors a, b, A, and B also contain radiative corrections corresponding to reducible diagrams.[7] Irreducible two-photon diagrams do not contribute to the cross section.[8,9]

3. In order to test the validity of quantum electrodynamics it is necessary to determine experimentally the angular dependence of the differential cross section $d\sigma(\vartheta)$ for a given energy, and to plot the function

$$S(\vartheta) = [d\sigma(\vartheta) - d\sigma_1(\vartheta)]/d\sigma_0(\vartheta). \quad (5)$$

Here $d\sigma_0(\vartheta)$ is the differential cross section, which we obtain putting $a = A = 1$ and $b = B = 0$ in Eq. (3); $d\sigma_1(\vartheta)$ is the differential cross section with radiative corrections calculated on the basis of "pure" quantum electrodynamics.[9] If that calculation were absolutely correct, we would obtain a straight line $S = 0$. In the case of either the first or the second violation of quantum electrodynamics mentioned above, we would obtain $S = f(E)$.

The dependence of S on ϑ denotes a violation of electrodynamics which requires introducing the vertex operator (1). As in the case considered in [2] (only the muon is smeared), this dependence will be of the form

$$S(\vartheta) = f_1(E) + f_2(E)\cos^2\vartheta.$$

We neglect here the effect of the violation of quantum electrodynamics on radiative corrections as an effect of a higher order.

Equations (3) and (4) are of course applicable to any electromagnetic process of the type[8]

$$e^- + e^+ \to f + \bar{f},$$

where f and $\bar{f}$ are a particle and an antiparticle described by spinors (but not an electron and a positron). However, in the case of strongly-interacting fermions (nucleons, hyperons), the equations are less suitable for a test of quantum electrodynamics.

4. Cabibbo and Gatto[8] have discussed the role of a nonelectromagnetic interaction in the $e^- + e^+ \to \mu^- + \mu^+$ process. They have shown that the strong interaction contribution can be neglected at energies not greater than 10 BeV. This contribution becomes, however, considerable if we assume that weak interactions propagate through an intermediate boson field. The term in the differential cross section corresponding to the weak interaction will not be then an even function of $\cos\vartheta$ [in contrast to Eq. (3)].

We cannot exclude the possibility that there exists an anomalous interaction of μ mesons capable of causing an increase in the cross section for the process under consideration. As an example let us mention the hypothetic anomalous interaction between electrons and muons responsible for the difference between the mass of the electron and of the muon,[10] and which should be felt in the $e \to \mu$ conversion.

[1] G. Furlan and G. Peressutti, Nuovo cimento **14**, 758 (1959).

[2] G. Furlan and G. Peressutti, Nuovo cimento **16**, 1144 (1960).

[3] V. B. Berestetskiĭ and I. Ya. Pomeranchuk, JETP **29**, 864 (1955), Soviet Phys. JETP **2**, 580 (1956); A. I. Akhiezer and V. B. Berestetskiĭ,

Kvantovaya elektrodinamika (Quantum Electrodynamics), 2d ed., Fizmatgiz, 1959.

[4] V. I. Baĭer and V. S. Synakh, JETP **41**, 1576 (1961), Soviet Phys. JETP **14**, 1122 (1962).

[5] S. D. Drell, Ann. Phys. **4**, 75 (1958).

[6] A. I. Nikishov, JETP **36**, 1323 (1959), Soviet Phys. JETP **9**, 937 (1959).

[7] V. I. Baĭer, JETP **37**, 1490 (1959), Soviet Phys. JETP **10**, 1057 (1960).

[8] N. Cabibbo and R. Gatto, Phys. Rev. **124**, 1577 (1961).

[9] A. I. Nikishov, JETP **39**, 757 (1960), Soviet Phys. JETP **12**, 529 (1960).

[10] Yu. A. Gol'fand, JETP **37**, 1493 (1959), Soviet Phys. JETP **10**, 1059 (1960).

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