FERMION REGGE POLES AND THE ASYMPTOTIC BEHAVIOR OF MESON-NUCLEON LARGE-ANGLE SCATTERING

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It is shown that the Regge pole trajectories, describing a family of fermions (which have direct physical meaning for half-integral values of the angular momentum), possess a number of properties that are substantially different from those discussed previously and from those present in nonrelativistic quantum mechanics. Namely, it is shown that the poles of the scattering amplitudes $f_+^j(u)$ and $f_-^j(u)$ in states with angular momentum j and parity $(-1)^{j\pm 1/2}$ must coincide when the square of the energy in the barycentric frame u tends to zero, and must become complex conjugate when $u < 0$. This leads to a quite specific character of the amplitudes for elastic scattering of mesons on nucleons in the angular region near 180°.

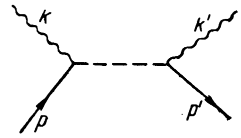
1. INTRODUCTION

In recent months a most interesting and promising development has come about in the theory of strong interactions based on the concept of moving poles of scattering amplitudes as functions of the angular momentum—the so called Regge poles. [1] The main attraction of this concept lies in the way Regge poles unify the spectrum of particles and resonances with the asymptotic behavior of high energy scattering. At that the asymptotic scattering behavior, particularly in the region of small angles, turns out to be comparatively simple and universal, although not subject to a simple classical description.

From the point of view of the Regge-pole hypothesis the fundamental objects to be studied in the theory of strong interactions are the trajectories of these poles, and not elementary particles. The trajectories of boson Regge poles, and in particular the trajectory of the pole having the vacuum quantum numbers (the Pomeranchuk trajectory), have been discussed in some detail. [2-5]

In this note we shall show that the Regge-pole trajectories describing a family of fermions (having direct physical meaning for half-integral values of the angular momentum), have a number of properties substantially different from those discussed previously and from those present in nonrelativistic quantum mechanics. Namely, we shall show that the poles of the scattering amplitudes $f_+^j(u)$ and $f_-^j(u)$ in states with angular momentum j and parity $(-1)^{j\pm 1/2}$ should coincide when the square of the energy in the barycentric frame u tends to zero, and should become the complex conjugates of each other for $u < 0$. This leads to a quite specific character of the amplitudes for elastic scattering of mesons on nucleons in the angular region near 180°.

The fact that the poles of $f_+^j(u)$ and $f_-^j(u)$ should coincide at $u = 0$ can be understood almost without any calculations. Let us suppose that the pole of some one amplitude at $u = 0$ has $j = 1/2$; then corresponding to it we have a particle with zero mass and spin $1/2$—a "neutrino." The contribution to the amplitude for the scattering of a meson on a nucleon due to such a particle is described by the Feynman diagram shown in the figure, and is equal for nonderivative coupling to $(\hat{p} + \hat{k})^{-1}$ or $i\gamma_5(\hat{p} + \hat{k})^{-1} i\gamma_5$, depending on the parity of the neutrino relative to the nucleon-meson system. In the first case the pole should be present in the amplitude $f_-^j(u)$ ($s_{1/2}$ state),



in the second case in the amplitude $f_+^j(u)$ ($p_{1/2}$ state).

However, since

$$i\gamma_5 (\hat{p} + \hat{k})^{-1} i\gamma_5 = (\hat{p} + \hat{k})^{-1}, \quad (1a)$$

the interaction is in both cases the same and, consequently, the pole should be present in both amplitudes $f_-^j(u)$ and $f_+^j(u)$. This is a manifestation of the we l-known fact that as a consequence of the γ_5 invariance of the Dirac equation for a massless particle the concept of a neutrino parity makes no sense, even in a theory with parity conservation. The fact that the poles of $f^j_+(u)$ and $f^j_-(u)$ become each other's complex conjugates for negative u testifies to a close connection between these amplitudes. This connection is due to the kinematic singularity $\sqrt{u}$ present in spinor amplitudes. $\sqrt{u}$ enters into the expressions for $f^j_\pm(u)$ in such a way that a function $f^j(\sqrt{u})$ can be introduced for which[1)]

$$f^j_+(u) = f^j(\sqrt{u}), \qquad f^j_-(u) = f^j(-\sqrt{u}). \tag{1}$$

At that each pole $j = j(\sqrt{u})$ of the function $f^j(\sqrt{u})$ appears in both amplitudes and has as a function of $\sqrt{u}$ rather simple properties. The explicit dependence on $\sqrt{u}$ of the equation determining the pole trajectory gives rise to the result that for negative u the angular momentum becomes necessarily complex.

Thus, on the basis of purely kinematic considerations we arrive at the conclusion that there are no fermion Regge poles corresponding to states with real angular momentum and imaginary mass.

The question arises whether states with imaginary mass should not have complex angular momentum also in the case of bosons, in spite of the absence of the corresponding kinematic reason. At this time we do not know the answer to this question. We shall only remark that if such a behavior is assumed for the Pomeranchuk trajectory then the partial wave with $l = 0$ will, generally speaking, have complex unphysical poles as a function of energy.

2. FERMION REGGE POLES NEAR u = 0

To prove the above assertions we consider scattering of mesons on nucleons. The Regge poles in the amplitudes for this process have been considered in a number of papers,[3-5] however the above-mentioned situation was not noticed. The scattering amplitude in states of definite isotopic spin has the form

$$F = a(u, t) + \tfrac{1}{2} b(u, t)(\hat{k} + \hat{k}'), \tag{2}$$

where $u = (p + k)^2$ is the square of the energy in the barycentric frame, and $-t = -(p' - p)^2$ is the square of the momentum transfer.

By considering matrix elements $\langle \lambda' | F | \lambda \rangle$ between states of nucleons with definite helicity λ and λ' it is easy to obtain relations between $a(u,t)$, $b(u,t)$ and the partial amplitudes $\varphi_{\lambda'\lambda}(u)$:[6]

$$A(u, t) = 2ma(u, t) + (u - m^2 - \mu^2)\, b(u, t) = 2\sum_j \varphi^j_{\lambda\lambda}(u)\,[P'_{j+1/2}(z) - P'_{j-1/2}(z)],$$

$$B(u, t) = (u + m^2 - \mu^2)\, a(u, t) + (u - m^2 + \mu^2)\, b(u, t) = 2\sqrt{u}\sum_j \varphi^j_{-\lambda\lambda}(u)\,[P'_{j+1/2}(z) + P'_{j-1/2}(z)], \tag{3}$$

where z is the cosine of the scattering angle:

$$z = 1 + 2ut\,[u^2 - 2u(m^2 + \mu^2) + (m^2 - \mu^2)^2]^{-1}.$$

The helicity amplitudes $\varphi^j_{\lambda'\lambda}(u)$ are related to the above introduced amplitudes with definite angular momentum and parity $f^j_\pm(u)$ by

$$\varphi^j_{\pm\lambda\lambda}(u) = \tfrac{1}{2}[f^j_-(u) \pm f^j_+(u)]. \tag{4}$$

If use is made of the dispersion relations for $A(u,t)$ and $B(u,t)$ in the momentum transfer t at fixed u, then, analogously to what was done in [2,7], one can introduce analytic functions $\varphi^{\pm j}_{\lambda'\lambda}(u)$ of j with decreasing asymptotic behavior as $j \to \infty$ satisfying the unitarity condition and coinciding with the physical partial amplitudes for even and odd values of $j + 1/2$ respectively.

By expressing $\varphi^j_{\lambda'\lambda}(u)$ with the help of Eq. (3) in terms of $A(u,t)$ and $B(u,t)$ and continuing the resultant expressions to the point $u = 0$, one concludes from the fact that $A(u,t)$ and $B(u,t)$ have no singularities at $u = 0$ that, as is obvious from the point of view of (3), for any j the function $\varphi^j_{-\lambda\lambda}(u)$ tends to infinity or to zero as $u \to 0$, while the function $\varphi^j_{\lambda\lambda}(u)$ remains finite. This is possible only if the singularities of $f^j_\pm(u)$ as a function of j coincide at $u = 0$. This conclusion is valid no matter what the nature of the singularities of $\varphi^j_{\lambda'\lambda}$.

If the conjecture is made that in any event the nearest singularities from the side of large j are poles, then their trajectories may be studied in more detail. In order to do so we consider the asymptotic behavior of $A(u,t)$ and $B(u,t)$ for $u > 0$ and $t \to -\infty$, i.e., $s \to \infty$ [$s = (p - k')^2$, $s + t + u = 2m^2 + 2\mu^2$]. Passing in the usual way from summation to integration, and taking into account that for $u > 0$ the poles are on the real axis and considering only the nearest pole in the amplitudes $\varphi^{\pm j}_{\lambda'\lambda}$, we obtain

$$A(u, s) = r^{\pm}_{\lambda\lambda}\,[s^{j-1/2} \mp (-s)^{j-1/2}]\,\frac{1}{\cos\pi j},$$

$$B(u, s) = \sqrt{u}\, r^{\pm}_{-\lambda\lambda}\,[s^{j'-1/2} \mp (-s)^{j'-1/2}]\,\frac{1}{\cos\pi j'}. \tag{5}$$

where $j = j(u)$, $j' = j'(u)$ are the positions of poles in $\varphi^j_{\pm\lambda\lambda}(u)$; $r^{\pm}_{\lambda'\lambda}$ are residues of the partial amplitudes multiplied by

[1)]The author is grateful to V. I. Shekhter, who has called his attention to this circumstance.

$$2^{j+1}\sqrt{2\pi}\frac{\Gamma(j+1)}{\Gamma(j+1/2)}\gamma^{j-1/2},$$

$$\gamma = u\,[u^2 - 2u\,(m^2+\mu^2) + (m^2-\mu^2)^2]^{-1}.$$

It is easy to verify [2] that the absorptive parts $A_1(u,s)$ and $B_1(u,s)$ are

$$A_1(u,s) = \pm r^{\pm}_{\lambda\lambda}(u)\,s^{j-1/2},$$
$$B_1(u,s) = \pm\sqrt{u}\,r^{\pm}_{-\lambda\lambda}(u)\,s^{j'-1/2}. \qquad (6)$$

If no parity degeneracy is assumed then the poles of $f^j_+(u)$ and $f^j_-(u)$ do not coincide for $u > 0$. Therefore only the pole of one of the amplitudes, the one which for the given u has the larger j, contributes to the asymptotic behavior of A and B. In that case, according to Eq. (4), $j = j'$,

$$r_{\lambda\lambda} = \pm r_{-\lambda\lambda} \qquad (7)$$

and consequently

$$B_1(u,s) = \pm\sqrt{u}\,A_1(u,s). \qquad (8)$$

At $u = 0$ the equality of (8) loses meaning since the poles of $f^j_+(u)$ and $f^j_-(u)$ coincide. If we suppose that for $u < 0$ the poles of $f^j_+(u)$ and $f^j_-(u)$ remain, as before, on the real axis then again one of the poles will dominate and Eq. (8) is reestablished. However in that case, since $\sqrt{u}$ is pure imaginary, Eq. (8) is in contradiction with the reality of the functions $A_1(u,s)$ and $B_1(u,s)$ for $u < (m+\mu)^2$. If the poles of $f^j_+(u)$ and $f^j_-(u)$ go off into the complex plane but are not each other's complex conjugates we again arrive at a contradiction because of the reality of A_1 and B_1.

We thus reach the conclusion that the poles of the amplitudes $f^j_+(u)$ and $f^j_-(u)$ must be each other's complex conjugates for $u < 0$. For the same reason the residues at these poles must be each other's complex conjugates.

3. THE ASYMPTOTIC BEHAVIOR OF BACKWARD SCATTERING

Denoting the residues of $f^j_+(u)$ and $f^j_-(u)$, multiplied by

$$2^{j+1}\sqrt{2\pi}\frac{\Gamma(j+1/2)}{\Gamma(j+1)}\gamma^{j-1/2},$$

by $\rho e^{\pm i\varphi}$ we obtain

$$A_1(u,s) = \pm\rho^{\pm}(u)\,s^{j'-1/2}\cos(j''\xi+\varphi),$$
$$B_1(u,s) = \pm\sqrt{-u}\,\rho^{\pm}(u)\,s^{j'-1/2}\sin(j''\xi+\varphi), \qquad (9)$$

where $\xi = \ln s$, $j' = j'(u)$, $j'' = j''(u)$ are the real and imaginary parts of the function $j = j(u)$ which determines the location of the pole.

Equation (9) goes over into Eq. (6) for positive u in such a way that A_1 and B_1 have no singularities at $u = 0$, provided that $j''(u) = \alpha\sqrt{-u}$ and $\varphi(u) = \beta\sqrt{-u}$ for small u, where α and β have no singularities at $u = 0$.

The real parts of the amplitudes A and B have the form

$$\operatorname{Re} A = \alpha_{\pm}\rho^{\pm}(u)\cos(j''\xi+\varphi\mp\beta)\,s^{j'-1/2},$$
$$\operatorname{Re} B = \alpha_{\pm}\rho^{\pm}(u)\sqrt{-u}\sin(j''\xi+\varphi\mp\beta)\,s^{j'-1/2};$$
$$\alpha^2_{\pm} = (\operatorname{ch}\pi j'' \mp \sin\pi j')/(\operatorname{ch}\pi j'' \pm \sin\pi j'),$$
$$\operatorname{tg}\beta = \operatorname{sh}\pi j''/\cos\pi j'. \qquad (10)^*$$

Equations (9) and (10) determine the asymptotic behavior of meson-nucleon scattering in the region of angles close to 180°, in the channel where s is the energy. Analogous formulas are valid for the asymptotic behavior of two-meson annihilation (where t is the energy).

The differential cross section for elastic scattering in the region of angles close to 180° is of the form

$$d\sigma/d\Omega = -\gamma[\,|A|^2 - |B|^2/u] = c(u)s^{2j'-1},$$
$$c(u) = -\gamma\rho^2(1+\alpha^2). \qquad (11)$$

It does not oscillate as a function of energy in spite of the oscillatory character of the amplitudes A and B. All other characteristics of the scattering are oscillatory. Thus, for example, the nucleon polarization ζ, arising from scattering off an unpolarized target, does not fall off but oscillates with increasing energy:

$$\zeta = (1 - \sin^2\pi j'/\operatorname{ch}^2\pi j'')^{1/2}\sin(2j''\xi + 2\varphi \mp \beta). \qquad (12)$$

As was pointed out to the author by I. Ya. Pomeranchuk, the asymptotic behavior, Eqs. (9) and (10), corresponds to an effective radius proportional to ln s if $j''(u) \sim \sqrt{-u}$, which according to Froissart [8] is the maximum possible growth of the effective radius. Let us recall that the effective radius of the interaction responsible for diffraction scattering is proportional to $\ln^{1/2} s$. [2]

It is necessary to emphasize that since the amplitude for backward scattering does not increase with increasing energy, as is the case for forward scattering, the poles that determine the asymptotic behavior can lie, for small u, in the region $\operatorname{Re} j < 0$. When the poles do lie in the region $\operatorname{Re} j < 0$ they might not dominate the asymptotic behavior, both because of the presence of other types of singularities and because for $\operatorname{Re} j < 0$ the Legendre functions again start to increase with increasing z and, generally speaking, Eqs. (9)—(12) no longer follow from Eq. (3). In order to obtain the appropriate formulas in this case the procedure must

*tg = tan, sh = sinh, ch = cosh.

be modified in the way described by Mandelstam for spinless particles.[9]

The Mandelstam method consists of the following: when going over from the summation (3) to the integral and opening up the contour of integration it is necessary to replace $P'_{j\pm1/2}(-z)$ according to the formula

$$-\frac{P'_{j\pm1/2}(-z)}{\cos\pi j}=\frac{1}{\pi\sin\pi j}\left[Q'_{j\pm1/2}(-z)-Q'_{-(j\pm1/2)-1}(-z)\right]. \tag{13}$$

The behavior of $\varphi^{j}_{\lambda'\lambda}(u)$ at large j is such that when calculating the contribution from the first term the contour of integration may be closed in the right half-plane and reduced to the sum of residues at the poles $1/\sin\pi j$. The second term behaves for large z like $z^{j\pm1/2}$ and therefore, when calculating the contributions due to it, it is appropriate to deform the contour of integration into the left half-plane. At that one encounters the poles $1/\sin\pi j$. As a result, if the contour is deformed sufficiently far to the left, the amplitudes A and B will contain in addition to the usual contributions from the poles of $\varphi^{j}_{\lambda'\lambda}$ also terms of the form

$$\sum\left[\varphi^{n}_{\lambda'\lambda}(u)+\varphi^{-n}_{\lambda'\lambda}(u)\right]Q'_{n\pm1/2}(z),$$

which for large z behave like $z^{-(3/2+n)}$.

It can be shown that the amplitudes $\varphi^{j}_{\lambda'\lambda}$ in the case of the scattering of spin ½ particles by an external field are such that the contributions from these terms are equal to zero. Whether this is also true in the real case we do not know. We can only assert that if the partial waves have no singularities other than poles then for $u < 0$ the cross section $d\sigma/d\Omega$ has at high energies either the form, Eq. (11), or

$$d\sigma/d\Omega=c(u)\,s^{-(n+3)},$$

where n is an integer or zero.

Let us remark that the Eqs. (5) and (9)–(11) might, generally speaking, also be inapplicable in the region of u so small that $us/(m^2-\mu^2)^2 \lesssim 1$, since in that region $z \lesssim 1$, and there is no justification for keeping one pole only.

4. POLE TRAJECTORIES

In conclusion let us discuss in some detail the trajectories of the poles of $f^j_+(u)$ and $f^j_-(u)$.

As was already remarked at the beginning of this note, the close connection between states with the same total angular momentum and opposite parities manifests itself as follows: according to Eq. (3) and (4) the formal substitution of $\sqrt{u}$ by $-\sqrt{u}$ is equivalent to the substitution of $f^j_+(u)$ by $f^j_-(u)$. Should the functions $f^j_\pm(u)$ have no singularities at $u = 0$, aside from those due to the presence of $\sqrt{u}$ in Eq. (3), then Eq. (1) would determine an analytic function $f^j(\sqrt{u})$, whose values in the right and left half-planes coincide with $f^j_+(u)$ and $f^j_-(u)$ respectively.

Such singularities actually exist. They are due to the so called left cuts of $f^j_\pm(u)$. As was shown in [7], however, the discontinuities across the left cuts have no singularities in j. Therefore the equations determining the locations of the singularities in the j plane are not related to these cuts and should be analytic functions which, for $\mathrm{Re}\sqrt{u} > 0$ determine the singularities of $f^j_+(u)$, and for $\mathrm{Re}\sqrt{u} < 0$ the singularities of $f^j_-(u)$.

The function $j = j(\sqrt{u})$, determining the location of the pole of $f^j(\sqrt{u})$, should have the following character: $j(\sqrt{u})$ is complex for $\sqrt{u} > m+\mu$ and $\sqrt{u} < -(m+\mu)$ in view of the unitarity condition for the amplitudes $f^j_\pm(u)$. If we consider that the poles of $f^j_\pm$ lie on the real axis when $0 < u < (m+\mu)^2$, then $j(\sqrt{u})$ is real in the interval $-(m+\mu) < \sqrt{u} < m+\mu$. Thus $j(\sqrt{u})$ should be a function with two cuts and, provided that the poles of $f(\sqrt{u})$ do not intersect, it should satisfy a dispersion relation of the type

$$j(\sqrt{u})=\frac{1}{\pi}\int_{m+\mu}^{\infty}\frac{\mathrm{Im}\,j^+(u')}{\sqrt{u'}-\sqrt{u}}\,d\sqrt{u'}+\frac{1}{\pi}\int_{m+\mu}^{\infty}\frac{\mathrm{Im}\,j^-(u')}{\sqrt{u'}+\sqrt{u}}\,d\sqrt{u'}, \tag{14}$$

where $\mathrm{Im}\,j^\pm(u)$ are the imaginary parts of the positions of the poles of $f^j_\pm(u)$.

In conclusion I wish to express my deep gratitude to I. Ya. Pomeranchuk and V. M. Shekhter for numerous and exceptionally useful discussions.

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