THE EFFECT OF CONDUCTING ELECTRONS ON THE KAPITZA TEMPERATURE JUMP

A. F. ANDREEV

Institute of Physics Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor May 31, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1535-1542 (October, 1962)

The contribution of conducting electrons to the thermal flux generated between a metal and a fluid at different temperatures is calculated. When the electron mean free path is much greater than the phonon wavelength, the additional thermal flux has the order of magnitude of the total flux and is proportional to the cube of the temperature. It is small in the inverse limiting case.

1. It is known[1-3] that when heat is released in a solid in contact with a liquid (liquid He^4 or He^3), a temperature jump is produced on the boundary between the solid and the liquid. This phenomenon was discovered by Kapitza[1] in 1941 and explained theoretically by Khalatnikov[2], who found the connection between the temperature jump ΔT and the heat flux Q through the boundary

$$Q = \frac{\rho}{D} c \frac{16\pi^5}{15} \frac{T^3 \Delta T}{(2\pi\hbar c_t)^3} F\left(\frac{c_t}{c_l}\right). \tag{1}$$

Here ρ and c are the density and velocity of sound in the liquid; D, c_l, and c_t are the density and the longitudinal and transverse velocities of sound in the solid, while F(x) is some function of the order of unity.

We note that the flux $Q(\Delta T)$ in (1) must be distinguished, generally speaking, from the heat flux which arises upon contact between two bodies in equilibrium but having different temperatures T and $T+\Delta T$. The latter flux remains finite in the case when the bodies are identical, something that cannot be said of the former[1)]. Both fluxes, however, coincide in the case considered below, when one body (liquid) has an appreciably lower acoustic impedance than the second (solid).

In the case when the solid is a metal, it was observed[5] that the value and temperature dependence of the jump change noticeably when the metal goes over into the superconducting state. This clearly indicates that the conducting electrons make an appreciable contribution to the heat flux, something not accounted for by formula (1).

The mechanism of heat exchange between the solid and liquid helium consists[2] of absorption and emission of phonons of the liquid by the surface of the solid. The heat flux is therefore determined essentially by the coefficient of reflection of the sound from the boundary of the solid. It will be shown below that for several angles of incidence the conducting electrons change very strongly the reflection coefficient of sound. They consequently make a noticeable contribution to the heat flux between the metal and the liquid[2)].

2. Assume that a plane sound wave is incident on the surface of a metal from a liquid filling the half space $z > 0$. The velocity field in the liquid is determined by specifying the scalar potential φ, namely $\mathbf{V} = \text{grad}\,\varphi$; in the solid we specify a scalar potential Φ and a vector potential $\boldsymbol{\Psi}$, such that

$$\dot{\mathbf{u}} = \text{grad}\,\Phi + \text{rot}\,\boldsymbol{\Psi}.$$

Here $\mathbf{u}$ is the displacement vector.

If the wave vector $\mathbf{k}$ incident on the wave lies in the xz plane, we can choose $\boldsymbol{\Psi}$ such that only its y component is different from zero; this component will be denoted Ψ. Let the angle of incidence of the sound be θ and the frequency ω; then

$$\begin{aligned} \varphi &= \{A_0 \exp[ik(x\sin\theta - z\cos\theta)] \\ &\quad + A\exp[ik(x\sin\theta + z\cos\theta)]\}\, e^{-i\omega t}, \\ \Phi &= A_l \exp\{ik_l(x\sin\theta_l - z\cos\theta_l) - i\omega t\}, \\ \Psi &= A_t \exp\{ik_t(x\sin\theta_t - z\cos\theta_t) - i\omega t\}, \end{aligned} \tag{2}$$

where

$$k = \frac{\omega}{c}, \quad k_l = \frac{\omega}{c_l}, \quad k_t = \frac{\omega}{c_t}; \quad \frac{\sin\theta}{c} = \frac{\sin\theta_t}{c_t} = \frac{\sin\theta_l}{c_l}.$$

The coefficients A, A_l, and A_t are determined

[1)]This circumstance was not taken into account by Little[4]. As a result he obtained for the temperature jump on the boundary between two bodies formulas in which the jump does not vanish if the bodies are identical.

[2)]An attempt to take into account the influence of the electrons on the temperature jump was made by Little[6].

from the system of boundary conditions when z = 0, which stipulate that the normal displacements and stresses on both sides of the boundary be equal. Neglecting the absorption of sound in the metal, this system has the known solutions [7]

$$\frac{A}{A_0}=\frac{Z_l\cos^2 2\theta_t+Z_t\sin^2 2\theta_t-Z}{Z_l\cos^2 2\theta_t+Z_t\sin^2 2\theta_t+Z},$$

$$\frac{A_l}{A_0}=\frac{\rho}{D}\frac{2Z_l\cos 2\theta_t}{Z_l\cos^2 2\theta_t+Z_t\sin^2 2\theta_t+Z},$$

$$\frac{A_t}{A_0}=-\frac{\rho}{D}\frac{2Z_t\sin 2\theta_t}{Z_l\cos^2 2\theta_t+Z_t\sin^2 2\theta_t+Z}, \qquad (3)$$

where

$$Z_l=\frac{Dc_l}{\cos\theta_l},\quad Z_t=\frac{Dc_t}{\cos\theta_t},\quad Z=\frac{\rho c}{\cos\theta}.$$

If $\sin\theta > c/c_t$, then it is seen from (3) that $|A/A_0| = 1$, that is, total internal reflection occurs. This is true only to the extent that we neglect absorption of sound in the metal. The presence of absorption, of course, decreases the reflection coefficient. It is also seen from (3) that the amplitude of the oscillations in the metal has a sharp and narrow maximum when the angle of incidence is such that

$$Z_l\cos^2 2\theta_t+Z_t\sin^2 2\theta_t=0. \qquad (4)$$

Condition (4) corresponds to the propagation of Rayleigh surface waves in the metal. The corresponding angle of incidence lies in the region of total internal reflection. It is clear that the absorption will particularly strongly influence the reflection coefficient in the vicinity of this angle of incidence. We shall consider this region below.

In determining the absorption of sound in a metal it is necessary to distinguish between the two cases [8] $\omega\tau \ll 1$ (τ is the electron relaxation time) and $\omega\tau \gg 1$. The values $\omega \sim T/\hbar$ are those of interest to us. On the other hand, near the boundary the metal lattice is usually strongly disrupted. We therefore assume here that

$$\omega\tau \leqslant 1. \qquad (5)$$

At helium temperature this means that the mean free path of the electrons is less than 10^{-3} cm. When relation (5) is satisfied, the electron mean free path l can be either smaller or larger than the wavelength of sound λ, because $v_0 \gg c_t$, where v_0 is the electron velocity on the Fermi boundary. Let us consider each case separately.

3. If $l \ll \lambda$, we can use the concept of electron viscosity. The energy absorbed per unit time is determined in this case by the well known relation

$$\dot{E}=2\eta\int\left(\dot{u}_{ik}-\frac{1}{3}\delta_{ik}\dot{u}_{ll}\right)^2 dV+\tilde{\zeta}\int \dot{u}_{ll}^2 dV. \qquad (6)$$

In general it is necessary to include in (6) an additional term connected with the heat conduction, but it is shown in [8] that the heat conduction does not influence the absorption of sound in metal at low temperatures.

Substituting the solutions (2) and (3) in (6) we obtain for the sound transmission coefficient w, defined as the ratio of E to the energy incident on the surface of the metal per unit time with the incident wave,

$$w(s)=\frac{\eta\omega}{\rho c c_t}\frac{f(\xi)}{(Dc_t/\rho c)^2(s-s_0)^2+1}. \qquad (7)$$

We introduce here in place of θ a new variable $s = \sin^2\theta_t$, where s_0 is the solution of the cubic equation

$$16(1-\xi^2)s^3-(24-16\xi^2)s^2+8s-1=0,$$

which expresses the equality (4), $\xi = c_t/c_l$ and $f(\xi)$ is a certain function of order unity.

Thus, w is proportional in our case to the frequency of sound and has a sharp maximum with a width on the order of $\rho c/Dc_t$ when $s = s_0$.

According to the calculations of Akhiezer, Kaganov, and Lyubarskiĭ [8], the electron viscosity is $\eta \sim p_0^4(\pi\hbar)^{-3}l$, where p_0 is the momentum on the Fermi boundary. Substituting this value of the viscosity, and also the values of ρ and c for liquid helium, we obtain for the maximum value of w

$$w(s_0) \sim l/\lambda \ll 1,$$

that is, the transmission coefficient is small, but increases with increasing range l. We can therefore expect values $w \sim 1$ in the opposite limiting case $l \gg \lambda$.

4. We now consider the case $l \gg \lambda$, where we can no longer use the concept of viscosity coefficient. To calculate the transmission coefficient we must solve the kinetic equation for the electron distribution function $f(\mathbf{p},\mathbf{r},t)$:

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\dot{\mathbf{p}}\frac{\partial f}{\partial \mathbf{p}}+\frac{f-\bar{f}}{\tau}=0, \qquad (8)$$

where $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$, $\epsilon(\mathbf{p})$ is the electron energy, and the bar denotes averaging over the equal-energy surface.

In the presence of a sound field, the electron energy is determined by the relation [3)]

$$\varepsilon(\mathbf{p})=\varepsilon_0(\mathbf{p})+\lambda_{ik}(\mathbf{p})u_{ik}+v_ip_k\frac{\partial u_k}{\partial x_i}+(\mathbf{p}-m\mathbf{v})\dot{\mathbf{u}}, \qquad (9)$$

and the equilibrium distribution function has the form

[3)] L. D. Landau, private communication.

$$f_0 = n_0\,[\varepsilon_0(\mathbf{p}) + \Lambda_{ik}u_{ik} + v_l p_k\,\partial u_k/\partial x_l - m\mathbf{v}\dot{\mathbf{u}}],$$
$$n_0(\varepsilon) = \left(\exp\frac{\varepsilon-\mu_0}{T}+1\right)^{-1}, \qquad (10)$$

where $\epsilon_0(\mathbf{p})$ is the electron energy in the absence of sound, $\lambda_{ik}(\mathbf{p})$ is a certain symmetrical tensor of the order of the Fermi energy; μ_0 is the chemical potential of the electrons in the absence of sound; $\Lambda_{ik} = \lambda_{ik} - \bar{\lambda}_{ik}$. Putting $f = f_0 + \chi\,\partial n_0/\partial\epsilon$ and linearizing Eq. (8), we obtain

$$\frac{\partial\chi}{\partial t} + \mathbf{v}\frac{\partial\chi}{\partial\mathbf{r}} + \frac{\chi}{\tau} = -\Lambda_{ik}\dot{u}_{ik} + \mathbf{v}\nabla\bar{\lambda}_{ik}u_{ik} + e\mathbf{v}\mathbf{E}, \qquad (11)$$

where account is taken of the fact that $\dot{\mathbf{p}} = -e\mathbf{E} - \nabla\epsilon$; $\mathbf{E}$ is the electric field resulting from the presence of the sound wave. Since all the characteristic velocities entering into the problem are small compared with the velocity of light, we can assume that $\operatorname{curl}\mathbf{E} = 0$. We put $\nabla\psi = e\mathbf{E} + \nabla\bar{\lambda}_{ik}u_{ik}$.

The frequency of sound ω is much smaller than the plasma frequency ω_{pl}, and therefore the charge density must be regarded as equal to zero. From this condition we determine ψ.

In the isotropic case considered by us we have, obviously, on the Fermi boundary,

$$\lambda_{ik} = \gamma\delta_{ik} + \beta n_i n_k, \qquad (12)$$

where $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$, while γ and β are constants of order μ_0. With the aid of (12) we can readily find that

$$\Lambda_{ik} = \beta\left(n_i n_k - \frac{1}{3}\delta_{ik}\right). \qquad (13)$$

It is necessary to add to (11) the boundary conditions for $z = 0$. We assume diffuse reflection of the electrons from the boundary. Then the distribution function for the electrons moving from the boundary at $z = 0$ will be equal to the equilibrium function, but with a somewhat modified value of the chemical potential, the addition ζ to which is determined by equating to zero the electron flux through the boundary. Thus, for $z = 0$ we have

$$\chi(n_z < 0) = \zeta. \qquad (14)$$

It is obvious that all the quantities are independent of the coordinate y. The dependence on x reduces to the factor e^{iqx}, where $q = k\sin\theta$.

A solution of (11) satisfying the condition (14) has the form

$$\chi = \psi(z) + \begin{cases} -\dfrac{1}{v_z}e^{-z\alpha/v_z}\displaystyle\int_{-\infty}^{z} e^{z'\alpha/v_z}\Lambda_{ik}\dot{u}_{ik}(z')\,dz' & \text{for } n_z > 0 \\ (\zeta-\psi_0)\,e^{-z\alpha/v_z} + \dfrac{1}{v_z}e^{-z\alpha/v_z}\displaystyle\int_{z}^{0} e^{z'\alpha/v_z}\Lambda_{ik}\dot{u}_{ik}\,dz' & \text{for } n_z < 0 \end{cases}, \qquad (15)$$

where $\alpha = iqv_x + 1/\tau$ and ψ_0 is the value of ψ when $z = 0$.

The sound field $\mathbf{u}$ is the sum of a longitudinal wave $\mathbf{u}_l$, which depends on z via the factor $\exp(\varkappa_l z)$, and a transverse wave $\mathbf{u}_t$, proportional to $\exp(\varkappa_t z)$ $(\varkappa_{l,t} = k_{l,t}|\cos\theta_{l,t}|)$. By virtue of the linearity of the problem, χ can be sought in the form

$$\chi = \chi^{(l)} + \chi^{(t)},$$

where $\chi^{(l)}$ is the solution corresponding to the propagation of a longitudinal wave only and $\chi^{(t)}$ corresponds to a transverse wave. Analogously

$$\psi = \psi^{(l)} + \psi^{(t)}, \qquad \zeta = \zeta^{(l)} + \zeta^{(t)}.$$

Separating the real part in (15), carrying out elementary integration, we obtain, neglecting small terms proportional to $1/\tau$,

$$\chi^{(a)} = \psi^{(a)}(z) - \frac{\varkappa_a v_z}{(\varkappa_a v_z)^2 + (qv_x)^2}\Lambda_{ik}\dot{u}_{ik}^{(a)} + \begin{cases} 0 & \text{for } n_z > 0, \\ [\zeta^{(a)} - \psi_0^{(a)}]\cos\left(q\dfrac{n_x}{n_z}z\right) + R_a\Lambda_{ik}\dot{U}_{ik}^{(a)} & \text{for } n_z < 0, \end{cases} \qquad (16)$$

where $a = l, t$; $\dot{U}_{ik}^{(a)}$ is the value of $\dot{u}_{ik}^{(a)}$ when $z = 0$;

$$R_a = \frac{\left[\varkappa_a v_z\cos\left(q\dfrac{n_x}{n_z}z\right) - kv_x\sin\left(q\dfrac{n_x}{n_z}z\right)\right]}{[(\varkappa_a v_z)^2 + (qv_x)^2]}.$$

The condition for the vanishing of the electron flux through the boundary

$$\int \mathbf{v}\frac{\partial n_0}{\partial\varepsilon}\chi d^3\mathbf{p} = 0, \qquad \text{for } z = 0$$

leads to the equation

$$\psi_0^{(a)} - \zeta^{(a)} = \frac{1}{\pi}\int_{n_z>0} do\,\frac{\varkappa_a v_z n_z}{(\varkappa_a v_z)^2 + (qv_x)^2}\Lambda_{ik}\dot{U}_{ik}^{(a)}, \qquad (17)$$

where do is the element of solid angle in momentum space.

Finally, we obtain $\psi^{(a)}$ from the condition that the charge density vanish, which reduces to the equation

$$\int\chi\,do = 0. \qquad (18)$$

We obtain

$$\psi^{(a)} = \frac{\psi_0^{(a)} - \zeta^{(a)}}{4\pi}\int_{n_z>0} do\cos\left(q\frac{n_x}{n_z}z\right) - \frac{1}{4\pi}\int_{n_z<0} do\,R_a\Lambda_{ik}\dot{U}_{ik}^{(a)}. \qquad (19)$$

We are interested in the additional force acting on the boundary and resulting from the presence of the electrons. This force is equal to the electron momentum flux through the boundary

$$\sigma_{iz} = \int 2\, p_i v_z \frac{\partial n_0}{\partial \varepsilon} \chi \frac{d\mathbf{p}}{(2\pi\hbar)^3}. \qquad (20)$$

Putting z = 0, we obtain from (20), using (16), (17), (19), and (13)

$$\sigma_{xz} = \sigma_{xz}^{(l)} + \sigma_{xz}^{(t)}, \qquad \sigma_{zz} = \sigma_{zz}^{(l)} + \sigma_{zz}^{(t)}, \qquad (21)$$

where

$$\sigma_{xz}^{(a)} = \left(\frac{p_0}{\pi\hbar}\right)^3 \frac{\beta}{2v_0} \frac{1}{\varkappa_a} \dot{U}_{xz}^{(a)} f_1^{(a)}(\xi),$$

$$\sigma_{zz}^{(t)} = \left(\frac{p_0}{\pi\hbar}\right)^3 \frac{\beta}{4v_0} \frac{1}{\varkappa_t} \dot{U}_{zz}^{(t)} f_2(\xi),$$

$$\sigma_{zz}^{(l)} = \left(\frac{p_0}{\pi\hbar}\right)^3 \frac{\beta}{4v_0} \frac{1}{\varkappa_l} \{\dot{U}_{zz}^{(l)} f_3(\xi) + \dot{U}_{xx}^{(l)} f_4(\xi)\},$$

$f_1^{(a)}(\xi)$, $f_2(\xi)$, $f_3(\xi)$, $f_4(\xi)$ are functions of order of unity:

$$f_1^{(a)} = \int_0^{2\pi} d\varphi \int_0^1 (1 - t^2)\, t^3 \cos^2\varphi \frac{dt}{g(t;\varkappa_a)},$$

$$f_3 = \int_0^{2\pi} d\varphi \int_0^1 \left(t^5 - \frac{2}{3} t^2 + \frac{1}{9} t\right) \frac{dt}{g(t;\varkappa_l)},$$

$$f_2 = \int_0^{2\pi} d\varphi \int_0^1 \left[t^5 - \frac{1}{3} t^3 + \left(t^5 - \frac{4}{3} t^3 + \frac{1}{3} t\right)\cos^2\varphi\right] \frac{dt}{g(t;\varkappa_t)},$$

$$f_4 = \int_0^{2\pi} d\varphi \int_0^1 \left[\cos^2\varphi\left(\frac{4}{3} t^3 - t^5 - \frac{1}{3} t\right) - \frac{1}{3} t^3 + \frac{1}{9} t\right] \frac{dt}{g(t;\varkappa_l)},$$

$$g(t;\varkappa) = t^2 + (q/\varkappa)^2 (1 - t^2)\cos^2\varphi.$$

In order to determine the transmission coefficient of the sound, it is now necessary to set up an ordinary system of boundary conditions for the functions (2), but in writing down the conditions for the equality of the stresses at z = 0 it is necessary to take into account the presence of the forces (21). This system has the form

$$-k_l \cos\theta_l A_l + k_t \sin\theta_t A_t = k\cos\theta\,(A - A_0),$$

$$k_t^2 \cos 2\theta_t A_t + k_l^2 \sin 2\theta_l A_l - \left(\frac{p_0}{\pi\hbar}\right)^3 \frac{\beta}{2v_0} \frac{i\omega}{2Dc_t^2}$$

$$\times \left\{\frac{k_t^2}{\varkappa_t} \cos 2\theta_t f_1^{(t)} A_t + \frac{k_l^2}{\varkappa_l} \sin 2\theta_l f_1^{(l)} A_l\right\} = 0,$$

$$A_t\left(1 - 2\frac{c_t^2}{c_l^2}\sin^2\theta_l\right) - \sin 2\theta_t A_t - \left(\frac{p_0}{\pi\hbar}\right)^3 \frac{\beta}{4v_0} \frac{i}{\omega D}$$

$$\times \left\{A_l \frac{k_l^2}{\varkappa_l}(f_4 \sin^2\theta_l + f_3\cos^2\theta_l) - A_t \frac{k_t^2}{2\varkappa_t} f_2 \sin 2\theta_t\right\}$$

$$= \frac{\rho}{D}(A + A_0). \qquad (22)$$

Solving the system (22) with respect to A, we obtain

$$\frac{A}{A_0} = \frac{Z_l \cos^2 2\theta_t + Z_t \sin^2 2\theta_t + Y - Z}{Z_l \cos^2 2\theta_t + Z_t \sin^2 2\theta_t + Y + Z}, \qquad (23)$$

where

$$Y = -i\left(\frac{p_0}{\pi\hbar}\right)^3 \frac{\beta}{v_0}\left\{\frac{k_l}{2\varkappa_l}\sin\theta_l \sin 2\theta_t f_1^{(l)} + \frac{k_t}{4\varkappa_t}\sin\theta_t \sin 2\theta_t f_2\right.$$

$$+ \frac{k_t}{4\varkappa_t}\frac{c_l}{c_t}\frac{\cos 2\theta_t}{\cos\theta_l}\left(1 - 2\frac{c_t^2}{c_l^2}\sin^2\theta_l\right) f_1^{(t)}$$

$$\left. + \frac{k_l}{4\varkappa_l}\frac{\cos 2\theta_t}{\cos\theta_l}(f_4 \sin^2\theta_l + f_3\cos^2\theta_l)\right\}.$$

It is now easy to determine the transmission coefficient w:

$$w(s) = 1 - \left|\frac{A}{A_0}\right|^2 = \frac{4B}{H^2 (Dc_t/\rho c)^2 (s - s_0)^2 + (B + 1)^2}, \qquad (24)$$

where

$$B = \frac{|Y(s_0)|}{\rho c},$$

$$H = \frac{1}{\sqrt{s_0 - \xi^2}} \frac{d}{ds}\left\{(1 - 2s)^2 - 4s\sqrt{(s-1)(s-\xi^2)}\right\}_{s=s_0}.$$

Thus, in the case $l \gg \lambda$, the transmission coefficient w is found to be independent of the sound frequency. Since $B \sim 1$ for the case of liquid helium, the maximum is $w(s_0) \sim 1$.

5. Knowing w, we can readily find the additional heat flux from the liquid to the metal

$$W_{\text{el}}(T) = \int \nu\left(\frac{\hbar\omega}{T}\right) c\cos\theta\, \hbar\omega w \frac{d\mathbf{k}}{(2\pi)^3}, \qquad (25)$$

where T is the temperature of the liquid, $\nu(x) = (e^x - 1)^{-1}$.

In the case $l \ll \lambda$, substituting w from (7) and going over from integration with respect to θ to integration with respect to s, we obtain

$$W_{\text{el}}(T) = 24\pi^2 \frac{T^5}{(2\pi\hbar c_t)^3} \frac{\eta}{Dc_t\hbar} f(\xi). \qquad (26)$$

Analogously, when $l \gg \lambda$, we obtain on the basis of (24)

$$W_{\text{el}}(T) = \frac{\rho}{D} c \frac{4\pi^5}{15} \frac{T^4}{(2\pi\hbar c_t)^3} \frac{B}{(B+1)|H|}. \qquad (27)$$

If the temperature of the liquid is equal to the temperature of the metal, then the flux W_{el} is offset by a flux of the same magnitude from the metal to the liquid. If there is a small difference in temperatures ΔT, then the resultant heat flux is

$$Q_{\text{el}} = \frac{\partial W_{\text{el}}}{\partial T}\Delta T.$$

Differentiating (26) and (27) we obtain

$$Q_{\text{el}} = 120\pi^2 \frac{T^4 \Delta T}{(2\pi\hbar c_t)^3} \frac{\eta}{Dc_t\hbar} f(\xi) \qquad (28)$$

when $l \ll \lambda$ and

$$Q_{el} = \frac{\rho}{D} c \frac{16\pi^5}{15} \frac{T^3 \Delta T}{(2\pi\hbar c_t)^3} \frac{B}{(B+1)|H|} \quad (29)$$

when $l \gg \lambda$.

Comparing (28) with (1), we see that when $l \ll \lambda$ the electrons barely influence the size of the temperature jump.

To the contrary, when $l \gg \lambda$ the flux Q_{el} has the same order of magnitude as (1), and the temperature dependences of both fluxes coincide. We note that Q_{el} as given by (29) depends rather weakly on the density of the liquid. Consequently, the temperature jump depends on the helium density less than predicted by formula (1). This is in qualitative agreement with the experimental data of Dransfeld and Wilks [9].

Finally, let us dwell briefly on the case of a superconducting metal. At temperatures much below critical, the number of electronic excitations is exponentially small. Therefore the contribution of the electrons to the heat flow will also be small. Near the critical temperature, Q_{el} will obviously be of the same order in the superconductor as in the normal metal. It is therefore clear that in the case of a superconductor the temperature jump ΔT is larger than in the case of a normal metal at the same value of heat flux, and its temperature dependence is stronger. This agrees qualitatively with the experimental results of Challis [5].

In conclusion, I am grateful to A. A. Abrikosov, L. P. Gor'kov, I. E. Dzyaloshinskiĭ, and I. M. Khalatnikov for a discussion and for valuable remarks.

[1] P. L. Kapitza, JETP **11**, 1 (1941).

[2] I. M. Khalatnikov, JETP **22**, 687 (1952).

[3] I. L. Bekarevich and I. M. Khalatnikov, JETP **39**, 1699 (1960), Soviet Phys. JETP **12**, 1187 (1961).

[4] W. A. Little, Can. J. Phys. **37**, 334 (1959).

[5] L. J. Challis, Proc. of Eighth Int. Conf. on Low Temp. Phys. Toronto, (1960).

[6] W. A. Little, Phys. Rev. **123**, 435 (1961).

[7] L. M. Brekhovskikh, Volny v sloistykh sredakh (Waves in Layered Media), AN SSSR, 1957, p. 31 [Academic Press, 1959].

[8] Akhiezer, Kaganov, and Lyubarskiĭ, JETP **32**, 837 (1957), Soviet Phys. JETP **5**, 685 (1957).

[9] K. Dransfeld and J. Wilks, Low Temp. Phys. and Chem., Madison (1958).

Translated by J. G. Adashko
260