$L/K = 6$, and $K = 0.9$[9], which yields as a result $b_0 = 2.56$. The cross section (6) should be averaged over the neutrino spectrum $\rho_\nu(E)$ from the reactor. Assuming as a lower estimate[2)] $\rho_\nu(E) \approx \rho_e(E)$ and taking $\rho_e(E)$ as given by Carter et al[8], we obtain

$$\sigma'(\mathrm{Li}^7) \geqslant \int_{\Delta E}^{\infty} \rho_e(E)\,\sigma(E)\,dE \approx 2\cdot 10^{-42}\,\mathrm{cm}^2. \qquad (7)$$

(For the reaction $\bar{\nu} + p \to e^+ + n$, the cross section for neutrinos from a reactor is 6.7×10^{-43} cm^2.) It must be noted that the excited nuclei during the process (3) will, generally speaking, be polarized in the direction of motion of the antineutrino, so that the succeeding gamma radiation will also have definite polarization properties. In principle, this fact could be used to separate the process (3) from the background.

We note that the interaction (1) which we use follows from the Bludman scheme, which presupposes the identity of the muon and the electron neutrino. It is not difficult to generalize the Bludman scheme to include the case $\nu_\mu \neq \nu_e$. In this case the interaction constant in the product of the neutral currents (1) turns out to be equal to G/2, and the cross sections given above are decreased to one-quarter. At the same time, the process $\nu_e + e \to \nu_e + e$ arises, which is forbidden by the Bludman scheme (see also [5]), but unlike the Feynman–Gell-Mann scheme it should be characterized by a constant $G/\sqrt{2}$ (and not G). Analogously, the process $\nu_\mu + e \to \nu_\mu + e$ should have a constant $-G/2$. From this point of view, it is quite interesting to study experimentally the scattering $\nu + e \to \nu + e$ both on reactor neutrinos and on neutrinos from the decay $\pi \to \mu + \nu$.

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LIMITATION ON THE RATE OF DECREASE OF AMPLITUDES FOR VARIOUS PROCESSES

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It has become clear recently that the asymptotic behavior of the amplitudes $A(s,t)$ for the transition of two particles into two particles at high energies and at fixed momentum transfer t is governed by the singularities of the partial wave amplitudes $f_l(t)$ as a function of the angular momentum l in the channel where t is the energy.[1-5] If the singularity of $f_l(t)$ farthest to the right is a Regge pole at $l = l(t)$ then the invariant amplitude behaves like $s^{l(t)}$. In the case of elastic processes for small t such a pole is the vacuum pole, which for $t = 0$ has $l(0) = 1$. As the momentum transfer $\sqrt{-t}$ increases $l(t)$ can become negative. And so the impression is created that for sufficiently large negative t the amplitude may decrease arbitrarily fast with increasing s.

We now show that in the relativistic theory the partial wave amplitudes $f_l(t)$ for any t have singularities when $\mathrm{Re}\, l \geq -1$, consequently, that the amplitude $A(s,t)$ cannot decrease faster than $1/s$ for any value of t. This conclusion is valid for the amplitudes of any two-particle processes. The reason for the existence of these singularities is due to the fact that the relativistic amplitude has three Mandelstam spectral functions, which give rise to the appearance of singularities near negative integer l. These singularities are, apparently, poles that accumulate at these points, i.e., the points themselves become essential singular points.

To prove these assertions we consider the expression for the partial wave amplitude:[2]

$$f_l(t) = \frac{2}{\pi}\int_{z_0}^{\infty} Q_l(z)\,A_1(s,t)\,dz,$$

$$z_0 = 1 + 8\mu^2/(t - 4\mu^2), \quad z = 1 + 2s/(t - 4\mu^2); \qquad (1)$$

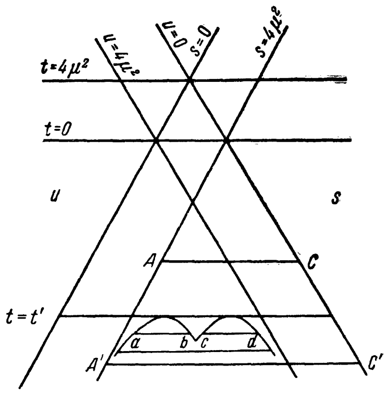
for simplicity we consider the case of identical particles of mass μ. If $\mathrm{Re}\, l > l_0$, where l_0 is determined by the maximum number of subtractions, then, as was shown in[6], the quantity $\Phi_l(t) = f_l(t)(t - 4\mu^2)^{-l}$ satisfied as a function of t a dispersion relation of the form

$$\Phi_l(t) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{\mathrm{Im}\,\Phi_l(t')\,dt'}{t' - t} + \frac{1}{\pi}\int_{-\infty}^{0} \frac{\Delta\Phi_l(t')}{t' - t}\,dt', \qquad (2)$$

where

$$\Delta\Phi_l(t) = 4\int_{4\mu^2}^{4\mu^2-t} P_l\left(\frac{2s}{4\mu^2-t-i\varepsilon}-1\right) A_1(s,t-i\varepsilon)\frac{ds}{(4\mu^2-t)^{l+1}}$$
$$+ \frac{4}{\pi}\int_{s_1(t)}^{s_2(t)} Q_l\left(\frac{2s}{4\mu^2-t-i\varepsilon}-1\right)\rho(s,u)\frac{ds}{(4\mu^2-t)^{l+1}}. \qquad (3)$$

The first integral in Eq. (3) is taken along lines of the type AC, A′C′ (see figure); the second integral, which exists only in the relativistic theory, is taken along lines of the type abcd, a′d′ in the region where the Mandelstam spectral function $\rho(s,u)$ is different from zero. The dispersion relation (2) is to be understood in the sense that the necessary number of subtractions has been carried out. All of the remaining argument is based on the fact that the Legendre functions $Q_l(s)$ have poles at all negative integer values of l.



In spite of the presence of these poles in $Q_l(z)$ it does not follow from Eq. (1) that the partial wave amplitudes $f_l(t)$ have poles at these points. This is because the representation of $f_l(t)$ in the form (1) is valid only for Re $l > m$, where m is the exponent of the power that determines the behavior of $A(s,t)$ at large s. If $m > -1$ then the integral is meaningless for negative integer l and the problem does not arise. If $m < -1$ then the residue of the pole at $l = -n-1$ is equal to $(4/\pi)\int P_n(z)A_1(s,t)dz$. If $n = 2k+1$ (k integer) then this residue vanishes as a consequence of Cauchy's theorem for the function $P_n(z)A$ for $n > -(m+1)$. If $n = 2k$ then, generally speaking, the residue does not vanish, but for such l the pole in the partial wave amplitude is exactly compensated by the factor $Q_{-l-1}(-z) + Q_{-l-1}(z)$ that is present in the amplitude $A(s,t)$ expressed in terms of partial waves.[7] If we were discussing the antisymmetric amplitude, then even and odd l would exchange places.

In the relativistic theory the situation is different in that $Q_l(z)$ enters, according to Eq. (3), also in the expression for the jump in $\Phi_l(t)$ across the left cut. As was remarked previously,[6] the expression (3) for $\Delta\Phi_l(t)$ is meaningful for arbitrary complex l since it is determined by integrals over a finite region of analytic functions P_l and Q_l. Therefore in the relativistic theory $\Delta\Phi_l$ has, generally speaking, poles at negative integer l.

Let us analyze the possibility that the residues at these poles vanish. Since the residue of $Q_l(z)$ at the pole $l = -n-1$ is equal to $\pi P_n(z)$, in order for the residues at all poles to vanish it is necessary that the integral

$$\int_{-z_0}^{z_0} P_n(z)\rho(s,u)dz$$

be equal to zero for all n. Since $z_0 < 1$, this is possible only if $\rho(s,u) = 0$. It is easy to see that the residue at the pole $l = -1$ also cannot vanish, at least not in a certain region of t for which the line abcd (see the figure) lies in a region where the Mandelstam spectral function is positive [such a region always exists near the border of existence of $\rho(s,u)$]. We note that in the case of scattering of identical particles $\Delta\Phi_l$ has no singularities for even negative l since $\rho(s,u)$ is a symmetric function of z. When the dispersion relation (2) and the unitarity condition are used as the equations determining $\Phi_l(t)$, the jump across the left cut $\Delta\Phi_l$ plays the role of the inhomogeneity in the problem (equivalent to a potential). It therefore follows from the above considerations that the amplitude $\Phi_l(t)$ will have singularities at negative integer l, at the very least in some interval of values of t.

In order to make clear precisely what happens with $\Phi_l(t)$ at these l we turn to the dispersion relation, Eq. (2). If we continue this equation into the region Re $l < l_0$ along the real axis, then it will change if singularities for some values of t are encountered while moving in the l plane. Let us suppose first that these singularities are moving Regge poles. Then, if for some $t = t'$ we first encounter a pole of $\Phi_l(t)$ on the real axis at $l = l'$, Re $l < l_0$, a pole will appear in the t plane for $l = l'$ on the physical sheet at $t = t'$. Then a term of the form $r/(t-t')$ is added to the dispersion relation, Eq. (2). If on further changing l we meet with several poles for various t, then a number of terms are added to the dispersion relation, Eq. (2), which takes the form

$$\Phi_l(t) = \frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}\,\Phi_l(t')}{t'-t}dt' + \frac{1}{\pi}\int_{-\infty}^{0}\frac{\Delta\Phi_l(t)}{t'-t}dt' + \sum_{n=1}^{N}\frac{r_n(l)}{t_n-t}. \qquad (4)$$

As was remarked previously,[6] the poles can appear only on the right cut since $\Delta\Phi_l(t)$ is an analytic function of l. In fact, because of the unitarity condition they can appear for $t > 4\mu^2$ only if their residues vanish at the same time.

Let now $l \to -1$. Then $\Delta\Phi_l \to \infty$ and if the sum over the poles contains a finite number of terms then $\Phi_l(t) \to \infty$ for any t. However, as a consequence of the unitarity condition $\Phi_l(t)$ has its modulus bounded for $t > 4\mu^2$. Therefore the number of poles must be infinite in order for them to compensate the contribution from the left cut. Moreover, the poles must fill the whole of the real axis for $t < t'$ (see the figure), so that the distance between the poles must tend to zero. Otherwise the contributions from the poles and from the left cut would have different analytic properties and would not be able to compensate each other. This means that for given $t < t'$ as $l \to -1$ we should encounter in the l plane arbitrarily many poles in a small neighborhood of $l = -1$. It therefore follows that $l = -1$ is an essential singular point of $\Phi_l(t)$ for $t < t'$. It is obvious that this essential singularity exists for any t since its position is independent of t for $t < t'$.

Let us see now whether the situation is changed if there exist in the l plane in the interval $-1 < l < l_0$ of the real axis singularities other than Regge poles. If such a singularity is a branch point, whose position is independent of the energy t, then the restrictions on the asymptotic behavior of A(s, t) become even stronger. The analytic properties of $\Phi_l(t)$ as a function of t are in that case unchanged. Only the unitarity condition for l to the left of the branch point is changed. The unitarity condition written in the form [2]

$$\frac{1}{2i}\left[\Phi_l - \Phi^*_{l^*}\right] = \frac{k}{\omega}\,\Phi_l(t)\Phi^*_{l^*}(t-4\mu^2)^l, \tag{5}$$

is valid for any l, but to the left of the branch point it does not mean that Φ_l has a restricted modulus, because Φ_{l^*} is not equal to Φ_l in view of the presence of a cut in the l plane. From the unitarity condition, Eq. (5), it follows, however, that Φ_l cannot be unbounded on both sides of the cut. If $\Phi_l \to \infty$ on one side of the cut then it must equal $\pm(\omega/2ik)(t-4\mu^2)^{-l}$ on the other side. If we consider l on that side of the cut where Φ_l is finite then all of the above considerations remain unchanged including the conclusion about the existence of an essential singularity near $l = -1$. If in the indicated interval we meet a branch point whose position depends on t then, at least for $t < t'$, the restriction on the asymptotic behavior of A(s, t) becomes even stronger. This problem will not be discussed here in detail because we do not understand how there can appear in the l plane moving cuts. [6]

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COMBINATION RADIATION IN UNIFORM MOTION OF A CHARGE IN A HOMOGENEOUS MEDIUM

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1. In the case when the velocity v of a uniformly moving charge and the dielectric constant $\epsilon(\omega)$ of a homogeneous medium satisfy the inequality $v^2\epsilon(\omega) < 1$ ($\hbar = c = 1$), no Cerenkov radiation is possible. It is shown below that under such conditions, in real media, a different type of radiation of a uniformly moving charge is possible, one which can naturally be called combination radiation. As is well known, the possibility of Cerenkov radiation in a medium is connected with the coherent scattering of a photon by atomic electrons, something accounted for by introducing the dielectric constant $\epsilon(\omega)$. Yet in addition to coherent photon scattering there exists also Raman scattering, wherein the quantum frequency changes upon scattering as a result of the transition of the atom