ANOMALOUS DIFFUSION OF A LOW-DENSITY CURRENT-CARRYING PLASMA IN A MAGNETIC FIELD

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Submitted to JETP editor March 29, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1688-1696 (November, 1962)

It is shown that a low-density plasma in a strong magnetic field is unstable with respect to excitation of drift waves in the presence of a longitudinal current. This instability is analogous to the current convective instability of a plasma with finite conductivity; the sole difference is that Landau damping plays the role of collisions. Turbulent convection arising as a result of this instability leads to anomalous plasma diffusion characterized by a diffusion coefficient of the same order as the Bohm coefficient.

1. INTRODUCTION

It is now well known that discharges in strong longitudinal magnetic fields exhibit an anomalous rate of loss of charged particles; this observation holds for a wide range of discharge parameters, starting with the glow discharge [1] and going up to high-current discharges in the stellarator [2–4] and Tokomak. [5] As far as the glow discharge is concerned we find that the experimental data on anomalous diffusion [1] can be explained on the basis of the current convective instability in a plasma whose conductivity depends on density. [6,7] In work reported earlier [8] it has been shown that the current convective instability mechanism also operates in high-current discharges if there is appreciable cooling of electrons at the walls of the discharge chamber. At very high electron temperatures this instability must appear by virtue of the very high thermal conductivity along the magnetic field. However, this result applies only within the framework of the hydrodynamic analysis, which is valid only when the particle mean free path is small. At very high electron temperatures and low densities the mean free path of the charged particles can be greater than the wavelength of the perturbations along the magnetic field; in this case the instability must be investigated by means of the kinetic equation. It is the purpose of the present work to present an analysis of this kind.

It is shown below that the presence of a longitudinal current in a low-density plasma leads to a drift-wave instability such as that considered by Rudakov and Sagdeev. [9] This instability is similar to the current convective instability, differing from the former only in that Landau damping replaces the finite conductivity. On the other hand, since the drift waves considered here go over continuously to ion acoustic waves as the angle between the wave vector and the magnetic field is reduced, we can say that in the present analysis we have "joined" the current-convective instability and the plasma instability associated with the excitation of ion acoustic waves. [10,11] It will be shown below that the instability considered here can lead to turbulent diffusion with a diffusion coefficient of the order of the Bohm coefficient. [12]

2. INSTABILITY OF AN INHOMOGENEOUS CURRENT CARRYING PLASMA

We consider the following idealized problem. Suppose that in a plasma located in a uniform magnetic field **H** directed along the z axis there flows a longitudinal current so weak that the associated magnetic field can be neglected compared with **H**. We assume that the plasma density n and temperature T vary slowly along the x axis. If the Larmor radii of the electrons and ions are small compared with the characteristic length a, over which there are appreciable changes in n and t, the equilibrium distribution functions for the electrons f_e and ions f_i can be written in the form

$$f_j = f_{0j}(v_\perp^2,\ v_z,\ x) + \frac{v_y}{\Omega_j}\frac{\partial f_{0j}}{\partial x}, \qquad (1)$$

where $v_\perp^2 = v_x^2 + v_y^2$, $\Omega_j = e_j H/m_j c$, e_j is the charge and m_j is the mass of a particle of type j. (For convenience we use the coordinate system in which the electric field vanishes.)

We investigate the stability of the plasma with respect to longitudinal oscillations, in which case the electric field is derivable from a potential:

$\mathbf{E}' = -\nabla\varphi$. In the semiclassical approximation the dependence of the potential on time and coordinates can be written in the form $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$. If the transverse wavelength of the waves is much greater than the Larmor radius of the particles we can neglect the gyration so that the solution of the kinetic equation giving the perturbation of the distribution function f_j' for particles of type j is written in the form of an integral over a rectilinear trajectory:

$$f_j' = \frac{e_j}{m_j} ik\varphi \frac{\partial f_j}{\partial \mathbf{v}} \int_{-\infty}^{0} \exp(-i\omega t + ik_z v_z t + \nu_j t), \qquad (2)$$

where ν_j, the collision frequency for particles of type j, takes account of collision damping.

At wavelengths appreciably greater than the Debye radius the dispersion equation determining ω can be obtained from the neutrality condition

$$\sum_j e_j \int f_j' \, d\mathbf{v} = 0.$$

Using (1) and (2) we write this equation in the form (cf. [9])

$$\sum_j \int \left\{ \frac{k_z}{m_j} \frac{\partial f_{0j}}{\partial v_z} + \frac{k_y}{m_j \Omega_j} \frac{\partial f_{0j}}{\partial x} \right\} \frac{d\mathbf{v}}{\omega - k_z v_z + i\nu_j} = 0. \qquad (3)$$

We can carry out the integration over $v_\perp$; then f_{0j} depends only on v_z. We assume that f_{0j} is a Maxwellian at temperature T_j.

We shall be interested in waves with phase velocity $v_p = \omega/k_z$ much greater than the ion thermal velocity $v_i = \sqrt{2T_i/m_i}$ but much smaller than the electron thermal velocity $v_e = \sqrt{2T_e/m_e}$, i.e., $k_z v_i \ll \omega \ll k_z v_e$. We also assume that $\nu_e \lesssim k_z v_e$; consequently, if $T_i \sim T_e$ ion collisions can be neglected $\nu_i \sim \sqrt{m_e/m_i}\, \nu_e < k_z v_i \ll \omega$. Under these conditions we can make an asymptotic expansion of the integrals with respect to velocity in Eq. (3); this equation then assumes the form

$$\frac{n}{T_e} + \frac{k_y c}{eH\omega}\frac{dn}{dx} - \frac{\omega^2 m_e n}{k_z^2 T_e^2} - \frac{k_z^2 n}{\omega^2 m_i} - i\frac{\nu_e c k_y m_e}{2eHk_z^2}\frac{d}{dx}\left(\frac{n}{T_e}\right)$$
$$- i\pi\left\{\frac{1}{m_e}\frac{\partial f_{0e}}{\partial v_z} + \frac{1}{m_i}\frac{\partial f_{0i}}{\partial v_z} + \frac{k_y c}{neHk_z}\frac{dn}{dx}(f_{0i} - f_{0e})\right.$$
$$\left. + \frac{1}{2}\frac{k_y c}{eHk_z}\frac{1}{T_e}\frac{dT_e}{dx} f_{0e}\right\}_{v_z = \omega/k_z} = 0, \qquad (4)$$

where the values of the functions of velocity in the curly brackets are taken at the resonance point $v_z = \omega k_z$.

The wave frequency $\mathrm{Re}(\omega)$ is given by the real part of (4). If the inhomogeneity is small and if the ratio k_z/k_y appreciable it is evident that the waves are ion sound waves. As the ratio k_z/k_y is reduced the ion sound waves go over continuously to drift waves, with velocity given by

$$\omega = k_y v_0 = -k_y \frac{cT_e}{eHn}\frac{dn}{dx}, \qquad (5)$$

where $v_0 \sim v_i \rho_i / a$ is the drift velocity, ρ_i is the mean ion Larmor radius and $1/a = n^{-1} dn/dx$.

When $k_z/k_y < \rho_i/a$ the phase velocity of these waves along the magnetic field can be appreciably greater than the ion thermal velocity; consequently these waves are weakly damped even when the ion temperature is greater than the electron temperature, that is to say, these drift waves can propagate even when ordinary ion sound waves do not. We note that the frequency of the drift waves given in (5) applies for the coordinate system in which the electric field disappears. In the coordinate system in which the ions are at rest the expression in (5) must be supplemented by the term $-k_y(c/eHn)\, d(nT_i)/dx$ so that the wave frequency becomes

$$\mathrm{Re}(\omega) = -\frac{k_y c}{eHn}\left[T_e \frac{dn}{dx} + \frac{d}{dx}(nT_i)\right].$$

Thus, when $T_e = \text{const}$, $T_i = \text{const}$ the wave frequency vanishes in the coordinate system that moves with the unperturbed drift velocity of the electrons $v_{0e} = -(c/eHn)\, dp/dx$ where $p = n(T_e + T_i)$ is the plasma pressure. In other words the wave is at rest in the coordinate system that moves with the electrons.

Assuming that the growth rate of the drift waves $\nu = \mathrm{Im}(\omega)$ is small compared with $\mathrm{Re}(\omega)$, we have from (4)

$$\frac{\nu}{k_y v_0} = \pi\left\{\frac{T_e}{m_e}\frac{\partial f_{0e}}{\partial v_z} + \frac{T_e}{m_i}\frac{\partial f_{0i}}{\partial v_z} - v_z f_{0i} + v_z f_{0e}\left(1 - \frac{1}{2}\alpha\right)\right\}_{v_z = v_p}$$
$$- \frac{k_y v_0 \nu_e (1-\alpha)}{k_z^2 v_e^2}, \qquad (6)$$

wher $v_p = \omega/k_z = k_y v_0/k_z$ is the phase velocity of the wave and $\alpha = d\ln T/d\ln n$.

We assume that the electron velocity distribution in v_z is Maxwellian and shifted with respect to the ions by an amount $u_0 \ll v_e$. We then have from (6)

$$\frac{\nu}{k_y v_0} = \frac{\sqrt{\pi}}{v_i}\left\{\frac{v_i}{v_e}\left(u_0 - \frac{1}{2}\alpha v_p\right) - \frac{T_e + T_i}{T_i} v_p \exp(-v_p^2/v_i^2)\right\}$$
$$- \frac{k_y v_0 \nu_e (1-\alpha)}{k_z^2 v_e^2}. \qquad (7)$$

We now consider this expression in somewhat greater detail. The first term in the curly brackets corresponds to the electron Landau damping. It is evident that this term vanishes when $u_0 = 0$ and the temperature is constant ($\alpha = 0$). This is a result of the fact that the wave is at rest in the coordinate system that moves with the electrons.

When $\alpha > 0$, as is usually the case, the first term causes excitation ($\nu > 0$) only in the presence of a directed electron velocity u_0. Since this term contains the small factor $v_i/v_e \sim 10^{-2}$, an instability arises only when the phase velocity of the wave along the magnetic field is appreciably greater than the ion thermal velocity, in which case the wave damping on the ions is exponentially small. When $\alpha \sim 1$ and $T_i \sim T_e$ the minimum value of the expression in the curly brackets (as a function of v_p) is reached when $v_p \approx 3v_i$ so that oscillations can be excited only when $u_0 > \alpha v_p/2 \approx 1.5 v_i \alpha$.

When $\alpha < 1$ the electron-ion collisions, given by the last term in (6), contribute additional damping; this damping becomes important (compared with the first term) only when $\lambda_e k_z < 1$, where $\lambda_e = v_e/\nu_e$ is the mean free path of the electrons. When $\lambda_e k_z > 1$ collisions are unimportant and the possibility of wave excitation is determined by the collisionless wave-particle interaction.

3. TURBULENT DIFFUSION

We now examine the nonlinear motion of the plasma that arises as a consequence of the instability. We first consider the case in which electron collisions are rare enough so that they need not be considered in the expression for the growth rate, but frequent enough to maintain the original electron longitudinal velocity distribution.

We start with small perturbations. As indicated above, the only perturbations that can grow are those whose phase velocities along z are greater than $3v_i$. To find the upper limit for the phase velocities for growing waves we consider the third term in (4). We have

$$\omega \cong k_y v_0 \left(1 - 2k_y^2 v_0^2 / k_z^2 v_e^2\right), \tag{8}$$

$$\nu = \pi k_y v_0 \left\{\frac{T_e}{m_e}\frac{\partial f_{0e}}{\partial v_z} + f_{0e} v_z \left(1 - \frac{2v_z^2}{v_e^2}\right)\right\}_{v_z = v_p}, \tag{9}$$

where we have taken $\alpha = 0$ and neglected collisions and ion wave damping in the expression for ν.

If the electron distribution function is approximated by a Maxwellian whose peak is shifted with respect to the peak of the ion distribution by an amount u_0, (9) assumes the form

$$\nu = \sqrt{\pi} k_y \frac{v_0}{v_e}\left(u_0 - \frac{2v_p^3}{v_e^2}\right), \tag{10}$$

where $v_p = k_y v_0/k_z$. It is thus obvious that the only growing waves are those for which $v_p < v_m = (u_0 v_e^2/2)^{1/3}$. When $u_0/v_e \ll 1$ the frequency of these waves (8) is still very close to $k_y v_0$.

Since the phase velocities of the growing waves are much smaller than v_e, to a first approximation the electrons may be assumed to have a Boltzmann distribution, that is to say, we can take the perturbation of the potential to be $\varphi \cong T_e n'/en$ where n' is the density perturbation. This perturbation, as has been noted above (and as can be shown by hydrodynamic methods) is displaced in the y direction with the electron drift velocity v_0. If Landau damping is neglected perturbations of this kind can not cause diffusion; the electrons do not drift along the x axis since the electric field component along y is exactly balanced by the perturbation of the pressure gradient, i.e., $k_y T_e n' - enk_y\varphi = 0$; also, since the perturbation E_y' is shifted in phase by an amount $\pi/2$ with respect to the density perturbation the mean displacement of the ions along x, $\langle n'cE_y'/H\rangle$, also vanishes.

When Landau damping is taken into account the electric field E_z' acquires an additional component in phase with the density perturbation n'. It can be shown that with a directed electron current the number of electrons overtaking the wave (consequently, capable of losing energy to the wave) is greater than the number of electrons falling behind the wave. In other words, in the region in which $n' > 0$ there is a retarding field E_z' while in the region characterized by $n' < 0$ there is an accelerating field; hence, on the average the electrons are retarded by the wave. In an oblique wave, however, in addition to the field E_z' there is a field $E_y' = E_z' k_y/k_z$, which causes drift of charged particles across the magnetic field. With the appropriate sign of k_y/k_z this drift causes a growth of the initial perturbation, as in the current convective instability of the plasma in a positive column.[6,7]

This interpretation of nonlinear plasma motion can also be extended to a glow discharge. As already indicated, all the perturbations we have considered are essentially at rest in the coordinate system that moves with the electron drift velocity v_0, i.e., the waves are purely aperiodic in this coordinate system [$\mathrm{Re}(\omega) = 0$]. It thus follows that perturbations with different wavelengths will maintain their relative phase shifts over long periods of time and interact strongly with each other: the density gradients of the large scale fluctuations then serve as a mechanism for the development of smaller scale fluctuations (due to the same instability) at other angles with respect to the magnetic field. Hence one expects that a broad fluctuation spectrum will develop in a plasma carrying a longitudinal current, that is to say, turbulent convection will arise.

We now establish the qualitative nature of the fluctuation spectrum. Since the growth rate is proportional to the wave number k_y the short-wave perturbations develop first and it may be assumed that the short-wave part of the spectrum reaches a steady-state. Since the longitudinal motion of the ions can be neglected while the motion across the magnetic field with velocity $\mathbf{v}' = cH^{-2}\mathbf{H} \times [\nabla\varphi - T_i\nabla(\ln n)]$ is incompressible, the square of the density satisfies the continuity equation

$$\partial n^2/\partial t + \operatorname{div}(\mathbf{v}' n^2) = 0. \tag{11}$$

This means that the quantity n'^2 is conserved in the breakup of the fine-scale local density fluctuations and that under steady-state conditions there is a steady transfer of density fluctuations in the spectrum. This transfer flow ϵ can be written in the form $\epsilon = n_\lambda'^2/\tau_\lambda$ where τ_λ is the lifetime of a fluctuation with transverse scale factor λ against breaking up into smaller fluctuations. The quantity $1/\tau_\lambda$ is evidently determined by the growth rate $\nu_{\lambda'}$ of the smaller scale fluctuations λ' times the fluctuations in the density gradient $n_{\lambda'}'/\lambda$ of scale λ. The basic contribution to the damping of the fluctuations at n_λ' is due to perturbations characterized by $\lambda' \sim \lambda$ while the growth rate ν_λ is proportional to λ^{-1}; hence, the condition that ϵ the transfer flow of fluctuations in the spectrum be a constant can be written in the form $\epsilon \sim n_\lambda'^3/\lambda^2 = \text{const}$, when $n_\lambda' \sim \lambda^{2/3}$.

It is thus evident that the amplitude of the fluctuations n_λ' increases with λ, that is to say, the strongest fluctuations are those with the greatest scale length l. Furthermore, since the nonlinear interaction means that the large-scale fluctuations are rapidly transformed into small-scale fluctuations one expects that even the fluctuations associated with the largest scale size n_l' will be small compared with the mean density n while the scale size l itself will be small compared with the characteristic length a over which there is a significant change in mean density. Thus, we can write as an approximation

$$n_\lambda'/n = A_1 (\lambda/a)^{2/3}, \tag{12}$$

where A_1 is a numerical factor of order unity.

We thus find the fluctuation of potential

$$\varphi_\lambda' = A_1 (T_e/e) (\lambda / a)^{2/3}. \tag{13}$$

Fluctuations of the largest scale size l develop by virtue of the gradient in mean density. It is these fluctuations which make the largest contribution to the turbulent flow of plasma across the magnetic field $q = \langle n_l' v_l' \rangle$ where v_l' is the fluctuation in velocity on the largest scale l, which plays the role of a displacement length. Since fluctuations of the largest scale size develop in the gradient in mean density as a result of the interchange of tubes with plasma over a length of approximately l, the velocity fluctuation v_l' can be estimated to be $v_l' \approx l\nu_l$ where ν_l is the growth rate for a perturbation of scale l. Thus, writing $k_y = 2\pi/l$, we find $q = -D_T\, dn/dx$, where the coefficient of turbulent diffusion is given by

$$D_T = A u_0 c T_e / v_e e H. \tag{14}$$

Here, $A = 2\pi^{3/2} A_1 (l/a)^{2/3}$ is a numerical factor of order unity (under the single assumption that $l \sim 10^{-1}a$). When $u_0/v_e \sim 10^{-1}$, as is usually the case, the diffusion coefficient given by (14) is of the same order as the Bohm coefficient $D_B = (^1/_{16})\, cT_e/eH$.[12]

4. EFFECT OF COLLISIONS

The expression for the turbulent diffusion coefficient given in (14) applies only when collisions are capable of restoring the electron distribution function but do not have an effect on wave damping. We now consider the effect of collisions on the diffusion coefficient in two limiting cases.

At high collision rates we must consider the last term in the growth rate expression (7). Since this term increases rapidly as k_z is reduced, the first waves to grow are those with the maximum possible k_z, that is, phase velocities v_p. But, as we have indicated above, the phase velocity of the growing perturbations must be greater than κv_i, where κ is a numerical factor approximately equal to three; if this requirement is not satisfied there is strong ion damping. Taking this situation into account, assuming $\alpha = 0$, and neglecting ion damping, we can write an approximate expression for (7):

$$\nu \cong \frac{\sqrt{\pi} u_0}{v_e} k_y v_0 - \varkappa^2 \frac{v_i^2}{v_e^2} \nu_e. \tag{15}$$

It is thus evident that an increase in ν_e leads to a reduction in the maximum scale size $l \sim 2\pi/k_x \sim 2\pi/k_y$ for which small perturbations can grow

($\nu > 0$); consequently the diffusion coefficient is reduced and can be written approximately as $D_T = l^2\nu_l$. Determining l from the condition $\nu \sim 0$, we have

$$D_T = B\frac{u_0^2 v_0^2}{v_i^2 \nu_e} = B\frac{u_0^2}{v_i^2}\frac{c^2T_e^2}{e^2H^2\nu_e}\frac{1}{n^2}\left(\frac{dn}{dx}\right)^2, \qquad (16)$$

where B is a numerical factor approximately equal to ten. The expression in (16) is $B\lambda_e^2/a^2$ times greater than the classical diffusion coefficient $D_C = T_e\nu_e/m_e\Omega_e^2$ (binary collisions) and reaches the order given by (14) when

$$\lambda_e\rho_i/a^2 > Av_iT_i/Bu_0T_e \sim 10^{-2}.$$

In other words, the anomalous diffusion effect considered here is of importance only in a low-density plasma, in which case $\lambda_e \gtrsim a$.

We now consider the second limiting case, in which collisions are so rare that account must be taken of the effect of fluctuations of the electric field on the electron distribution function. The kinetic equation for the averaged electron distribution f_e is of the form

$$\mathbf{v}\nabla f_e - \frac{e}{m_e}\mathbf{E}\frac{\partial f_e}{\partial v} - \frac{e}{m_e}[\mathbf{vH}]\frac{\partial f_e}{\partial \mathbf{v}} = \sum_i S_{ej} + S_E, \qquad (17)^*$$

where $\mathbf{E}$ is the average field, S_{ej} is the Landau collision term for electrons colliding with particles of type j

$$S_{ej} = -\frac{2\pi Le^4}{m_e}\sum_{\beta,\gamma}\frac{\partial}{\partial v_\beta}\int\left\{\frac{f_e(\mathbf{v})}{m_j}\frac{\partial f_j(\mathbf{v}')}{\partial v'_\gamma} - \frac{f_j(\mathbf{v}')}{m_e}\frac{\partial f_e(\mathbf{v})}{\partial v_\gamma}\right\}U_{\beta\gamma}\,d\mathbf{v}', \qquad (18)$$

L is the Coulomb logarithm, $U_{\beta\gamma} = (u^2\delta_{\beta\gamma} - u_\beta u_\gamma)/u^3$, $u_\beta = v_\beta - v'_\beta$. The term S_E on the right side of (17), which describes the wave-particle interaction, is given by

$$S_E = -\frac{e}{m_e}\frac{\partial}{\partial \mathbf{v}}\langle\nabla\varphi f'_e\rangle,$$

where the angle brackets denote time averages. To find S_E we must relate the fluctuations in the distribution function f'_e and φ. Strictly speaking, since we are treating highly developed oscillations the relation between f'_e and φ is nonlinear. Qualitatively, however, the effect can be described in the quasi-linear approximation (cf. [13]), that is, by taking account of quadratic terms only; this corresponds to neglecting in (2) the deviation of the particle trajectory from a rectilinear trajectory because of drift in the fluctuating field. Expanding φ in a Fourier integral and taking account of (2) we obtain the following expression for the steady-state spectrum of drift waves

$$S_E = \sum_\beta \frac{\pi e^2}{m^2}\frac{\partial}{\partial v_\beta}\int\left\{k_z\frac{\partial f_{0e}}{\partial v_z} + \frac{k_y}{\Omega_e}\frac{\partial f_{0e}}{\partial x}\right\}k_\beta \times \Phi_0(\mathbf{k})\,\delta(\omega - k_yv_0)\,\delta(\omega - k_zv_z)\,d\omega d\mathbf{k}, \qquad (19)$$

where $\Phi_0(\mathbf{k})\,\delta(\omega - k_yv_0)$ is the spectral function for the fluctuations of potential φ so that

$$\langle\varphi(\mathbf{r}, t)\,\varphi(0, 0)\rangle = \int e^{-i\omega t + i\mathbf{kr}}\Phi_0(\mathbf{k})\,\delta(\omega - k_yv_0)\,d\omega\,d\mathbf{k}. \qquad (20)$$

We integrate (17) over transverse velocity $v_\perp$ assuming that a Maxwellian distribution is maintained in $v_\perp$. Then, the following expression is obtained for the electron-ion collision term in the region $v_i \ll v_z \ll v_e$:

$$S_{ei} \cong \frac{2\pi Le^4 n\sqrt{\pi}}{m_e^2 v_e}\frac{\partial}{\partial v_z}\left\{\frac{\partial f_{0e}}{\partial v_z} + \frac{m_e v_z}{T_e}f_{0e}\right\}, \qquad (21)$$

where we have neglected a term of order m_e/m_i that takes account of heat exchange between electrons and ions.

We now simplify the electron-electron collision term, approximating $f_{0e}(\mathbf{v}')$ by a Maxwellian function shifted by an amount u_s with respect to the ions (this simplification is valid because there are few resonance electrons). After integration over $\mathbf{v}'$ and $v_\perp$ we have

$$S_{ee} \cong \frac{2\pi(\pi - 2)Le^4 n}{m_e^2 v_e}\frac{\partial}{\partial v_z}\left\{\frac{\partial f_{0e}}{\partial v_z} + \frac{m_e(v_z - v_s)}{T_e}f_{0e}\right\}. \qquad (22)$$

Inasmuch as the only electrons that interact with waves are those whose longitudinal velocities lie in the relatively narrow range $3v_i < v_z < v_m = (u_0v_e^2/2)^{1/3}$ the term S_E does not have a large effect on the longitudinal conductivity; hence the electric field E_z in (17) can be expressed in terms of the flow velocity of the electrons u_s by means of the usual formula (cf. [14])

$$E_z = -\frac{8}{3}\frac{m_e}{e}\frac{\sqrt{\pi}Le^4 n}{T_e m_e v_e}u_s.$$

Using this substitution, taking account of (19), (21), and (22), and carrying out a single integration over v_z, we reduce (17) to the form ($\alpha = 0$)

$$\frac{\partial f_{0e}}{\partial v_z} + v_z\frac{m_e}{T_e}f_{0e} - \frac{m_e u_0}{T_e}f_{0e} + F(v_z)\left\{\frac{\partial f_{0e}}{\partial v_z} + \frac{m_e v_z}{T_e}f_{0e}\right\} = 0. \qquad (23)$$

Here, $u_0 = 0.65u_s$ is the velocity at which the function f_{0e} reaches a maximum in the absence of oscillations (a more accurate calculation in [15] gives $u_0 = 0.5u_s$) while the function $F(v_z)$ is

$$F(v_z) = \frac{v_e}{2(\pi - 2 + \sqrt{\pi})\,e^2Ln}\int k_z^2\Phi_0(\mathbf{k})\,\delta(k_yv_0 - k_zv_z)\,d\mathbf{k}. \qquad (24)$$

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.

From (9) and (23) we find the growth rate for small oscillations taking account of the effect of waves on the electron longitudinal velocity distribution function:

$$\frac{\nu}{k_y v_0} = \frac{1}{\sqrt{\pi} v_e} \frac{u_0 - 2v_p^2/v_e^2}{1 + F(v_p)}. \quad (25)$$

As expected, the distortion of the electron distribution function reduces the growth rate for small perturbations, thereby retarding the entire process of excitation and interaction of fluctuations. The quantitative influence of this effect on transverse diffusion can be seen by introducing $(1 + F_0)^{-1}$ in (14) where F_0 is some mean value of the function $F(v_p)$.

The value of F_0 for developed fluctuations can be estimated as follows. We first carry out the integration over k_x in (24). Only those waves grow whose phase velocities lie in the range $3v_i < v_p < v_m = (u_0 v_e^2/2)^{1/3}$, in which the growth rate ν does not change greatly with v_p, and to a first approximation we may assume that $\Phi_0(k_y, k_z)$ (as a function of k_z) is constant and equal to $\Phi_0(k_y/\Delta k_z)$ within the interval $\Delta k_z \sim k_y v_0/v_m$ and zero outside this interval. With this assumption it is evident from (24) that $F(v_p)$ is a decreasing function of v_p; that is to say, according to (25), when $F(v_p) > 1$ the greatest growth rates will be those for waves with the greatest phase velocities $v_p \sim v_m$. In other words, the mean value of the function $F(v_p)$ may be taken as $F(v_m)$. Substituting $v_z \sim v_m$ in (24) and carrying out the integration over k_z we have

$$F_0 \cong \frac{v_0 v_e}{6e^2 L n v_m^2} \int k_y \Phi_0(k_y)\, dk_y. \quad (26)$$

Here, the function $\Phi_0(k_y)$, which is the spectral function of the transverse fluctuations, can, in accordance with (13), be given in the form

$$\Phi_0(k_y) = \frac{C_1}{k_y}\left(\frac{T_e}{e}\right)^2 \left(\frac{2\pi}{k_y a}\right)^{4/3}. \quad (27)$$

where C_1 is a numerical factor of order unity. Substituting this expression in (26) and carrying out the integration over k_y between the limits $2\pi/l \sim 10 \cdot 2\pi/a$ and infinity, we have

$$F_0 = C \frac{\lambda_e}{a} \frac{v_0}{(u_0^2 v_e)^{1/3}}, \quad (28)$$

where C is a numerical factor of order unity.

With $T_i \sim T_e$ and $u_0 \sim v_e/10$ we find $F_0 \sim 10^{-1} \lambda_e \rho_i/a^2$ that is, when $\lambda_e \rho_i/a^2 > 10$ the diffusion coefficient must be F_0 times smaller than the Bohm value. Thus, according to the mechanism considered here the turbulent diffusion coefficient must be of the order of the Bohm value if the parameter $S = \lambda_e \rho_i/a^2$ lies in the range $10^{-2} < S < 10$. For values of S lying outside this range the diffusion coefficient can be appreciably smaller: for small values of λ_e it falls off as λ_e^2 and for large values of λ_e as λ_e^{-1}.

6. CONCLUSION

The anomalous diffusion mechanism considered is based on only one kind of instability; hence, a comparison of the results obtained here with experimental data would be premature since we have not evaluated other possibilities for explaining enhanced diffusion. However, one must note the qualitative agreement between a number of features of the anomalous diffusion considered here and the experimental data on plasma loss from the stellarator.[2-4] For example the fact that the instability considered here is associated with a longitudinal current and that it disappears if the directed velocity of the electrons $u_s \approx 2u_0 < 3v_i d \ln T/d \ln n$ is in qualitative agreement with the experimental results given by Motley,[3] who showed that enhanced diffusion occurs only in the presence of a longitudinal current and only when the directed electron velocity is of order v_i. Further support for this mechanism is indicated by the results of reference 4, in which it has been shown that plasma loss is described satisfactorily by the Bohm diffusion coefficient ($S = \lambda_e \rho_i/a^2$ in these experiments varied approximately in the limits $10^{-2} < S < 10$ so that $D_T \cong D_B$ in accordance with the estimate given above). However, in order to verify this turbulent convection mechanism it will be necessary to carry out experiments designed expressly for this purpose, in particular, investigation of the correlation functions for the fluctuations in the density and in the electric field.

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Translated by H. Lashinsky
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