# *DEPOLARIZATION OF $\mu^+$ MESONS IN SOLIDS*

V. G. NOSOV and I. V. YAKOVLEVA

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A theory is developed of depolarization of $\mu^+$ mesons in matter, taking into account the presence of an external magnetic field. It is shown that for the process under consideration chemical reactions involving the $\mu^+$ mesons in emulsions are important. The parameters that enter into the formulas of phenomenological theory are determined, and the asymptotic trend of the polarization in large fields is predicted. Certain possible mechanisms of the depolarization of an electron of muonium are discussed, and the effect produced by them is evaluated. The case of media with high dielectric constants with a slight frequency dispersion is considered.

## 1. INTRODUCTION

ACCORDING to present notions regarding the nature of weak interactions [1-6], the angular distribution of the positrons from the $\mu^+ - e^+$ decay has the form $1 - (1/3) P \cos\vartheta$, where $\vartheta$ is the angle between the positron momentum and the negative direction of the $\mu^+$ meson polarization vector, and P is the absolute value of the polarization. However, in the preceding $\pi^+ - \mu^+$ decay, owing to the longitudinal nature of the neutrino, the $\mu^+$ mesons are completely polarized in the direction opposite to their motion [3,7]. One can thus expect the positrons to have an angular distribution in the form $1 - (1/3)\cos\vartheta$ relative to the direction of emission of the $\mu^+$ meson.

Experimentally, however, a much smaller anisotropy of the $\mu^+ - e^+$ decay is observed in many substances [8,9]. It is obvious that a noticeable depolarization of the $\mu^+$ mesons can occur within a lifetime $\tau_\mu = 2.2 \times 10^{-6}$ sec, owing to the interaction with the medium. Usually the most effective $\mu^+$ meson depolarization mechanism in a substance is the formation of muonium—a bound system comprising a $\mu^+$ meson and an electron [9-11]. Actually, estimates of the depolarization due to other mechanisms make the effect negligibly small[9]. On the other hand, it is well known that in the ground state of muonium there takes place the so-called contact interaction between the $\mu^+$ meson and electron spins [12,13].

A simple quantum-mechanical calculation[14] shows that the magnetic field produced by an electron at the origin is equal to

$$\mathbf{H}_c = -\frac{16\pi}{3}\beta|\psi(0)|^2\mathbf{s}^{(e)}, \qquad (1)$$

where $|\psi(0)|^2 = 1/\pi a_0^3$ is the value of the electron density at this point; $a_0 = \hbar^2/me^2$ is the Bohr radius; $\beta = e\hbar/2mc$ is the Bohr magneton, $\mathbf{s}^{(e)} = (1/2)\,\boldsymbol{\sigma}^{(e)}$ is the electron-spin operator. The Hamiltonian of the contact interaction has the form

$$\hat{\mathcal{H}} = -\boldsymbol{\mu}\mathbf{H}_c = \tfrac{1}{4}\hbar\omega_0\boldsymbol{\sigma}^{(\mu)}\boldsymbol{\sigma}^{(e)}, \qquad (2)$$

where $\boldsymbol{\mu} = \beta^{(\mu)}\boldsymbol{\sigma}^{(\mu)}$ is the positive-muon magnetic-moment operator, $\beta^{(\mu)} = e\hbar/2m_\mu c$, and $\hbar\omega_0 = 32\pi\beta\beta^{(\mu)}/3a_0^3$ is the hyperfine splitting of the muonium ground level.

It is easy to verify that owing to the contact interaction the value of the $\mu^+$-meson polarization, averaged over the times $t \gg 1/\omega_0$, turns out to be equal to half the initial polarization. We actually have $1/\omega_0 = 3.6 \times 10^{-11}$ sec, $\omega_0\tau_\mu \gg 1$, and $P = 1/2$. Physically this means that during the lifetime of the $\mu^+$ meson it continuously exchanges angular momentum with the muonium electron, and half of the initial value of the $\mu^+$ meson is "transferred" on the average to the electron.

The mechanism described above is best verified by measuring the angular distribution of the $\mu^+ - e^+$ decay positrons in a longitudinal magnetic field H. Because of the presence of the field, the function P(H) turns out to depend on the dimensionless ratio

$$x = \hbar\omega'/\hbar\omega_0 = 2\beta H/\hbar\omega_0 = H/H_0,$$

where $\omega' = 2\beta H/\hbar$ is the frequency of the Larmor precession of the electron spin, and $H_0 = 1580$ G. We obtain for the polarization of the $\mu^+$ mesons, averaged over the times $\sqrt{\omega_0^2 + \omega'^2}\,t \gg 1$ the formula[11,15]

$$P = (1 + 2x^2)/2(1 + x^2). \qquad (3)$$

Experiment does not confirm this formula, gen-

erally speaking, and the polarization P(0) in the absence of the field also turns out to be different in different substances [8,9]. The point is that the depolarization of $\mu^+$ mesons in condensed media is much more complicated than the scheme described above and, at any rate, does not reduce to the mere production of muonium.

Account must be taken first of the interaction between the muonium electron and the medium. As the spin of the $\mu^+$ meson is partially "transferred" to the electron by the contact interaction, the latter, in turn, becomes depolarized because of irreversible interaction with the medium. As a result, the depolarization of the $\mu^+$ meson does not stop even after the lapse of a time $t \gg (\omega_0^2 + \omega'^2)^{-1/2}$. The mechanism whereby the muonium electron becomes depolarized is usually thought to be exchange with the electrons of the medium, which have opposite spins [9], but this apparently is not the only possibility. Incidentally, the form of the formulas obtained in this paper for the function P(x) does not depend on any specific assumptions concerning the mechanism whereby the muonium electron is depolarized.

To understand the processes involved in the $\mu^+$-meson depolarization it is necessary to bear in mind also the main features of their slowing down in matter. In the region $v \gg v_{at}$ (v is the velocity of the $\mu^+$ meson and $v_{at}$ is the order of magnitude of the velocity of atomic electrons) slowing down is the result of ionization losses and the production of muonium has extremely low probability. In the opposite limiting case, $v \ll v_{at}$, the $\mu^+$ meson is a very deep "trap" for the electrons (its depth is equal to the hydrogen-atom ionization potential, $E_0 = 13.5$ eV), a trap which at each given instant can be regarded as immobile.

It is obvious that equilibrium with respect to the electronic degrees of freedom necessitates the filling of so deep a trap, and multiple successive acts of muonium production and its ionization (charge exchange), which occur in the Bohr velocity region $v \sim v_{at}$, are a powerful mechanism for the establishment of such an equilibrium. Therefore when $v \ll v_{at}$ the $\mu^+$ mesons accumulate in the form of muonium atoms, which continue to slow down and approach rapidly the thermal-velocity region.

However, the establishment of thermal equilibrium still does not mean the occurrence of chemical equilibrium. By chemical equilibrium we mean, in the broadest sense of the word, a situation whereby the $\mu^+$ meson, owing to the chemical bond between the muonium electron and the valence electrons of the medium, occupies a certain definite place in the crystal lattice and will subsequently experience only small vibrations relative to this position. By its chemical properties, muonium is prefectly analogous to hydrogen, but it is to be expected that $\mu^+$ mesons will occupy in the crystal the same positions as were occupied by the protons that have slowed down in it.

We emphasize that we are dealing exclusively with a real crystal, in which, in particular, a chemical bond is possible between the muonium and the impurity atoms. The presence of defects of the crystal structure can also contribute to the formation of a chemical bond for the $\mu^+$ meson. When it comes to an idealized crystal of the substance under consideration, free of impurities and defects, it may turn out that the muonium does not enter at all in any chemical reactions with its atoms, i.e., in this crystal the $\mu^+$ meson has no equilibrium positions that are stable relative to small oscillations.

It is obvious that any solid can be characterized by a certain average time $\tau$ during which this final stopping of the $\mu^+$ meson takes place. The number of muonium atoms that do not interact chemically decreases with time as $\exp[-t/\tau]$. We shall assume that $\tau_\mu \gg \tau$, although as can be seen from the foregoing it is easy to obtain formulas that are free of this limitation.

It is interesting that at the instant when the muonium enters into a chemical reaction with the atoms of the medium, the depolarization of the $\mu^+$ meson ceases, and later its polarization remains constant up to the instant of the $\mu - e$ decay (for some exceptions to this rule see Sec. 5). If, as is the case with the ordinary chemical bond, all the electrons form closed diamagnetic shells and their spins are pairwise compensated, then the magnetic field produced by the electrons at the point where the $\mu^+$ meson is situated is equal to zero (we do not consider substances with magnetic structure or paramagnets). Under these conditions, of course, the $\mu^+$-meson spin can no longer be "transferred" to any of the electrons of the medium. It can be stated that the chemical interaction of the muonium electron with the medium electrons disrupts as it were its coupling with the $\mu^+$-meson spin. The experiment fully confirms this point of view. Thus, for example, experiments using electronic circuitry [8,9], carried out with different substances, have shown that no $\mu^+$-meson polarization is observed within times on the order of $10^{-7} - 10^{-6}$ sec: the polarization occurred earlier, within a shorter time, which could not be determined in these experiments.

In addition, it was found that the frequency of rotation of the angular distribution of the decay positrons in a weak magnetic field corresponds to the gyromagnetic ratio of the $\mu^+$ meson, and not at all to the muonium triplet ratio. We have here an analogy with experiments on nuclear magnetic resonance in solids, for example on proton resonance[1)]. In the latter case the resonance occurs at a frequency corresponding to the gyromagnetic ratio of the proton, but no resonance is ever observed at a frequency corresponding, say, to the gyromagnetic ratio of the hydrogen atom in the triplet state [16].

## 2. PHENOMENOLOGICAL THEORY OF $\mu^+$-MESON DEPOLARIZATION

We start from the premise that, independently of its interaction with the $\mu^+$-meson spin (and with the external magnetic field if the latter is present), the muonium electron becomes depolarized in the medium. In other words, there exists a mechanism by which the electron spin "flips" from time to time: the average expectation time of this event will be denoted by $1/\nu$. Then, as can be readily verified, in the absence of interaction with the spin of the $\mu^+$ meson and with the external magnetic field the electron polarization would vary as $\exp[-2\nu t]$.

The spin density matrix of muonium

$$\rho = \sum_{\varkappa, k=0}^{3} \rho_{\varkappa k}\, U_\varkappa\, U_k \qquad (4)$$

obeys the Wangsness-Bloch equation [17-19]:

$$\begin{aligned}
d\rho_{\varkappa 0}/dt &= -\tfrac{1}{2}\omega_0 e_{\varkappa\lambda l}\,\rho_{\lambda l}(t) - \zeta e_{\varkappa\mu\lambda}\,\omega'_\mu\,\rho_{\lambda 0}(t),\\
d\rho_{0k}/dt &= \tfrac{1}{2}\omega_0 e_{k\lambda l}\,\rho_{\lambda l}(t) + e_{kml}\,\omega'_m\,\rho_{0l}(t) - 2\nu\rho_{0k}(t),\\
d\rho_{\varkappa k}/dt &= \tfrac{1}{2}\omega_0\,[e_{\varkappa k\lambda}\,\rho_{\lambda 0}(t) - e_{\varkappa kl}\,\rho_{0l}(t)] - \zeta e_{\varkappa\mu\lambda}\,\omega'_\mu\,\rho_{\lambda k}(t)\\
&\quad + e_{kml}\,\omega'_m\,\rho_{\varkappa l}(t) - 2\nu\rho_{\varkappa k}(t). \qquad (5)
\end{aligned}$$

Here

$$U_0 = \chi/\sqrt{2},\quad \mathbf{U} = \boldsymbol{\sigma}/\sqrt{2} \qquad (6)$$

are orthogonal and normalized spin operators, $\chi$ is a unit two-row matrix, $\boldsymbol{\sigma}$ the Pauli operator, $\boldsymbol{\omega}'$ a vector directed along the magnetic field $\mathbf{H}$ with magnitude $\omega'$, $\zeta = \beta^{(\mu)}/\beta = m/m_\mu$ is the ratio of the muon and electron magnetic moments, and $e_{kml}$ is a unit antisymmetrical tensor. In the density matrix components $\rho_{\varkappa k}$, the first subscript (Greek letter) pertains to the $\mu^+$ meson and the second (Latin letter) to the electron. In formula (5) and henceforth the letter designations for the indices are used only for the "vector" values 1, 2, and 3, while the "scalar" value zero is written out explicitly. In expressions containing two identical subscripts, summation is understood over its "vector" values.

It is required to find the particular solutions of the system (5)

$$\rho_{\varkappa 0}(t) = \rho_{\varkappa 0}\,e^{-\Omega t},\qquad \rho_{0k}(t) = \rho_{0k}\,e^{-\Omega t},\qquad \rho_{\varkappa k}(t) = \rho_{\varkappa k}\,e^{-\Omega t}, \qquad (7)$$

satisfying different eigenvalues $\Omega$, and to set up a linear combination of these solutions, satisfying the initial conditions

$$\rho_{\varkappa 0}(0) = \rho^{(0)}_{\varkappa 0},\qquad \rho_{0k}(0) = 0,\qquad \rho_{\varkappa k}(0) = 0. \qquad (8)$$

We consider the following two limiting cases.

a) $\nu \gg \sqrt{\omega_0^2 + \omega'^2}$ (fast electron-spin relaxation). This case is best considered in most general form, without making any specific assumptions regarding the mutual orientation of the vectors $\boldsymbol{\omega}'$ and $\boldsymbol{\rho}^{(0)}$.

Let us substitute (7) in (5). As a triplet of linearly-independent vectors we choose the aforementioned $\omega'_k$ and $\rho^{(0)}_{K0}$, and also their vector product $e_{klm}\omega'_l\rho^{(0)}_\lambda$. We expand the sought vectors $\rho_{K0}$ and $\rho_{0k}$ in terms of these vectors; this leads to the appearance of six unknown expansion coefficients. In addition, we list nine linearly-independent tensors, in which one can expand the $\rho_{Kk}$:

$$e_{\varkappa kl}\quad \omega'_l,\ e_{\varkappa k\lambda}\omega'_\lambda\quad \rho^{(0)}_{\lambda 0},\quad \rho^{(0)}_{\varkappa 0}\omega'_k,\quad \omega'_\varkappa\delta_{k\lambda}\rho^{(0)}_{\lambda 0},$$
$$\omega'_\varkappa\omega'_k,\quad \rho^{(0)}_{\varkappa 0}\delta_{k\lambda}\rho^{(0)}_{\lambda 0},\quad \omega'_\varkappa e_{kl\lambda}\omega'_l\rho^{(0)}_{\lambda 0},\quad \rho^{(0)}_{\varkappa 0}e_{kl\lambda}\omega'_l\rho^{(0)}_{\lambda 0},\quad \delta_{\varkappa k}.$$

As a result we obtain for the expansion coefficients a system of linear equations which are used also to determine the eigenvalues $\Omega$. In the case considered here we solve this system by series expansion in the reciprocal powers of $\nu$. It is easy to see that for the 12 eigenvalues this expansion begins with the term $2\nu$. Such solutions will not be considered, since they attenuate rapidly with time, as $\exp[-2\nu t]$, and in addition, they enter in the $\mu^+$-meson polarization of interest to us with a small coefficient.

We are left with three eigenvalues $\Omega$, for which the foregoing expansion begins with the zero-order terms; in the same zeroth approximation the electron polarization and the polarization correlation vanish in all three cases. One of the considered eigenvalues is zero, and for the other two we have $\Omega = \pm i\zeta\omega'$. The first-order correction to the value of $\Omega$ in all three cases is $\omega_0^2/4\nu$.

1) This analogy was pointed out to us by Yu. M. Kagan.

Satisfying the initial condition (8), we obtain

$$\rho_{x0}(t) = \left\{\frac{\omega'\rho^{(0)}}{\omega'^2}\left[1 - \frac{1}{2}(e^{-i\zeta\omega' t} + e^{i\zeta\omega' t})\right]\omega'_x + \frac{1}{2}[e^{-i\zeta\omega' t} + e^{i\zeta\omega' t}]\rho^{(0)}_{x0} - \frac{i}{2\omega'}[e^{-i\zeta\omega' t} - e^{i\zeta\omega' t}]\, e_{xl\lambda}\omega'_l\, \rho^{(0)}_{\lambda 0}\right\} \exp(-\omega_0^2 t/4\nu). \quad (9)$$

The physical meaning of this formula is simple: the relaxation time of the $\mu^+$-meson spin is $4\nu/\omega_0^2 \gg 1/\omega_0$. It cannot be less than $1/\omega_0$, for in any mechanism of electron spin relaxation in the medium, the spin of the $\mu^+$ meson can "go over" to the electron only after the characteristic time of the hyperfine splitting.

It is easy to verify that the expression in the curly brackets describes the precession of the muon spin with its characteristic Larmor frequency $\xi\omega'$. Frequent "flipping" of the electron spin in the medium weakens its coupling with the $\mu^+$-meson spin, and the latter precesses in the magnetic field as if there were no electron there.[2)]

In a longitudinal field there is precession, and the polarization of the $\mu^+$ meson varies as

$$P(t) = e^{-\omega_0^2 t/4\nu}, \quad (10)$$

which is independent of the magnitude of the field.

At the instant when chemical equilibrium sets in, the depolarization of the $\mu^+$ meson stops (see the introduction). The probability that the chemical reaction occurs within a time interval dt is equal to $e^{-t/\tau}dt/\tau$. For the observable average value of the polarization we obtain

$$P = \int_0^\infty P(t)\, e^{-t/\tau}\frac{dt}{\tau} = \frac{1}{1 + \omega_0^2\tau/4\nu}. \quad (11)$$

b) $\nu \ll \sqrt{\omega_0^2 + \omega'^2}$ (slow relaxation of the electron spin). In this case, which is of practically greatest significance, we assume the field to be longitudinal: the axis z = 3 is directed along the vector $\boldsymbol{\omega}'$ (i.e., along the vector of the external magnetic field $\mathbf{H}$), while $\boldsymbol{\rho}^{(0)}$ is assumed directed either along this axis or opposite to it. Then the system (5) assumes the form

$$\begin{aligned}
d\rho_{30}/dt &= -\tfrac{1}{2}\omega_0[\rho_{12}(t) - \rho_{21}(t)],\\
d\rho_{03}/dt &= \tfrac{1}{2}\omega_0[\rho_{12}(t) - \rho_{21}(t)] - 2\nu\rho_{03}(t),\\
d\rho_{11}/dt &= -\omega'\rho_{12}(t) - 2\nu\rho_{11}(t),\\
d\rho_{12}/dt &= \tfrac{1}{2}\omega_0[\rho_{30}(t) - \rho_{03}(t)] + \omega'\rho_{11}(t) - 2\nu\rho_{12}(t),\\
d\rho_{22}/dt &= \omega'\rho_{21}(t) - 2\nu\rho_{22}(t),\\
d\rho_{21}/dt &= -\tfrac{1}{2}\omega_0[\rho_{30}(t) - \rho_{03}(t)] - \omega'\rho_{22}(t) - 2\nu\rho_{21}(t)
\end{aligned} \quad (12)$$

(in the case under consideration the magnetic moment of the $\mu^+$ meson is small enough to be neglected, to that we put $\xi = 0$).

The particular solutions of (7) are sought by expansion in powers of $\nu$; the initial conditions (a) are satisfied in the same way. As a result, confining ourselves everywhere (including the eigenvalues $\Omega$) to quantities in the first order of $\nu$ inclusive, we obtain for the nonzero components of the density matrix the following solution:

$$\begin{aligned}
\rho_{30}(t) &= \left\langle \frac{\omega_0^2 + 2\omega'^2}{2(\omega_0^2 + \omega'^2)}A + \frac{\omega_0^2}{4(\omega_0^2 + \omega'^2)}B\left\{\left[1 - \frac{i}{2}\frac{5\omega_0^2 + 8\omega'^2}{(\omega_0^2 + \omega'^2)^{3/2}}\nu\right]C + \text{c.c.}\right\}\right\rangle \rho^{(0)}_{30},\\
\rho_{03}(t) &= \left\langle \frac{\omega_0^2}{2(\omega_0^2 + \omega'^2)}A - \frac{\omega_0^2}{4(\omega_0^2 + \omega'^2)}B\left\{\left[1 - \frac{i}{2}\frac{\omega_0^2 + 4\omega'^2}{(\omega_0^2 + \omega'^2)^{3/2}}\nu\right]C + \text{c.c.}\right\}\right\rangle \rho^{(0)}_{30},\\
\rho_{11}(t) &= \rho_{22}(t) = \left\langle -\frac{\omega_0\omega'}{2(\omega_0^2 + \omega'^2)}A + \frac{\omega_0\omega'}{4(\omega_0^2 + \omega'^2)} \times B\left\{\left[1 - \frac{i}{2}\frac{\omega_0^2 + 4\omega'^2}{(\omega_0^2 + \omega'^2)^{3/2}}\nu\right]C + \text{c.c.}\right\}\right\rangle \rho^{(0)}_{30},\\
\rho_{12}(t) &= -\rho_{21}(t) = \left\langle \frac{\omega_0(\omega_0^2 + 2\omega'^2)}{2(\omega_0^2 + \omega'^2)^2}\nu A - \frac{\omega_0}{4\sqrt{\omega_0^2 + \omega'^2}}B\left\{i\left[1 - i\frac{\omega_0^2 + 2\omega'^2}{(\omega_0^2 + \omega'^2)^{3/2}}\nu\right]C + \text{c.c.}\right\}\right\rangle \rho^{(0)}_{30};\\
A &= \exp\left(-\frac{\omega_0^2}{\omega_0^2 + \omega'^2}\nu t\right),\\
B &= \exp\left(-\frac{3\omega_0^2 + 4\omega'^2}{2(\omega_0^2 + \omega'^2)}\nu t\right), \qquad C = \exp(i\sqrt{\omega_0^2 + \omega'^2}\, t).
\end{aligned} \quad (13)$$

The first formula in (13) determines the dependence of the instantaneous value of the $\mu^+$-meson polarization P(t) on the time. It must be integrated over the distribution $e^{-t/\tau}dt/\tau$ of the chemical-interaction events. It is useful to take into account here that the consideration of the cases $\nu\tau \ll 1$ and $\sqrt{\omega_0^2 + \omega'^2}\,\tau \gg 1$ actually covers the entire region $\nu \ll \sqrt{\omega_0^2 + \omega'^2}$ of the slow relaxation of the electron spin.

In the first case all the exponentials which contain $\nu t$ in the exponent can be replaced by unities. Introducing the dimensionless parameter

[2)] The physical aspect of this situation was already emphasized in an earlier paper[19], but the behavior of the muon in a magnetic field was not considered there.

$x = \omega'/\omega_0 = H/H_0$ (see the introduction) we obtain, in the required approximation

$$P = \frac{1}{2(1+x^2)}\left\{1 + 2x^2 + \frac{1}{1+(1+x^2)\omega_0^2\tau^2}\right\}, \qquad \nu\tau \ll 1. \tag{14}$$

In the second case it is convenient to deal with an expression for the polarization, averaged over the time intervals $1/\nu \gg \Delta t \gg (\omega_0^2 + \omega'^2)^{-1/2}$:

$$\overline{P(t)} = \frac{1+2x^2}{2(1+x^2)} \exp\left(-\frac{\nu t}{1+x^2}\right). \tag{15}$$

For the observed value of the polarization we have

$$P = \int_0^\infty \overline{P(t)}\, e^{-t/\tau}\frac{dt}{\tau} = \frac{1+2x^2}{2(1+x^2+\nu\tau)}, \qquad \sqrt{\omega_0^2+\omega'^2}\,\tau \gg 1. \tag{16}$$

If the conditions $\sqrt{\omega_0^2+\omega'^2}\,\tau \gg 1$ and $\nu\tau \ll 1$ are satisfied simultaneously, then both (14) and (16) go over into (3). Thus, in spite of the clearly unsatisfactory character of the initial physical picture, which has led to formula (3) (see the introduction), the latter turns out to be valid in the region of slow electron-spin relaxation for a sufficiently broad interval of possible values of the parameter $\tau$. Figure 1, which illustrates this clearly, is a schematic plot of the function $P(\tau)$ in the absence of a magnetic field.

It is desirable to compare the formulas given for the function $P(x)$ with the experimental data. Particularly numerous and reliable data are available on the depolarization of $\mu^+$ mesons in emulsion for various values of the magnetic field. It turns out that in emulsion $P(0) < 1/2$; consequently, at least for one of the two components (for the gelatin or for the silver bromide), formula (16) is valid. A more detailed examination of the experimental data [20,21] shows that formula (16) is applicable to AgBr, and in the case of gelatin $\omega_0\tau \gg 1 \gg \nu\tau$, i.e., it is necessary to use formula (3).

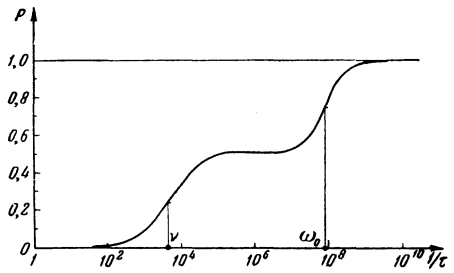


FIG. 1. Dependence of the observed polarization P of $\mu^+$-mesons on the time of chemical relaxation $\tau$ in the absence of a magnetic field. The abscissas represent the probability $1/\tau$ on a logarithmic scale (arbitrary units). The vertical lines correspond to the respective points $1/\tau = \nu$, $P = 1/4$ and $1/\tau = \omega_0$, $P = 3/4$. In the region of the plateau $P = 1/2$ formula (3) is applicable.

For the emulsion as a whole we have

$$P = f\frac{1+2x^2}{2(1+x^2)} + (1-f)\frac{1+2x^2}{2(1+x^2+\nu\tau)}, \tag{17}$$

where f is the fraction of the $\mu^+$ mesons slowed down in the gelatin. The values of the parameters f and $\nu\tau$ were determined by the least-squares method, by minimizing the expression

$$\sum_i [(P_{i\,\text{theor}} - P_{i\,\text{exp}})/\delta P_{i\,\text{exp}}]^2,$$

where the summation is over all the experimental data of various authors, which are gathered in [20]. As a result we obtain $f = 0.63$ and $\nu\tau = 80$. As can be seen from Fig. 2, the theoretical form of the $P(x)$ curve is in good agreement with the experimental data.

It would be interesting, of course, to determine the value of the chemical-relaxation time $\tau$, but unfortunately formula (17) contains only the product $\nu\tau$. However, the agreement between the theory and experiment offers evidence that the criterion $\omega_0 \gg \nu$ is satisfied sufficiently well; in the opposite limiting case the polarization is gen-

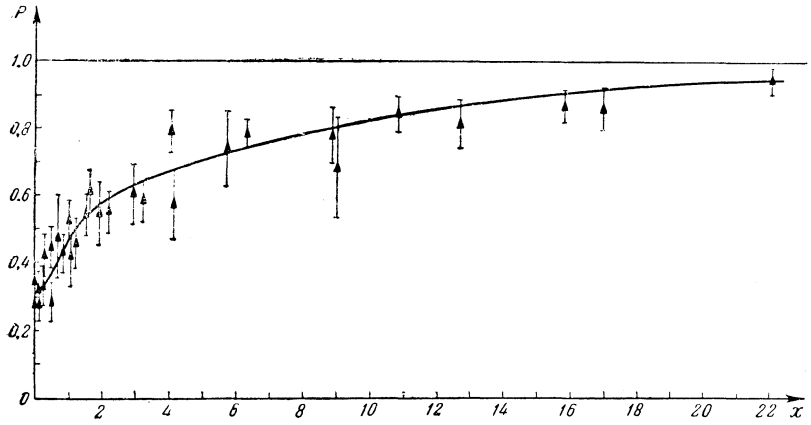


FIG. 2

erally independent of the field [ see formula (11) ]. Substituting the values of $\omega_0$ and $\nu\tau$, we obtain the inequality

$$\tau \gg 3 \cdot 10^{-9} \text{ sec} \qquad (18)$$

We note that in nuclear photoemulsion the average dimension of the AgBr crystallite does not exceed $3 \times 10^{-5}$ cm [22]. We can therefore not exclude the possibility that the muonium, slowed down in the silver bromide, fails to react chemically in it, and on leaving the crystallite rapidly enters into chemical reaction in the gelatin. In other words, it may turn out that the parameter $\tau$, contained in formula (17), actually represents the average time necessary for the muonium atom slowed down in the AgBr to go beyond the boundaries of the crystallite.

It is also of interest to ascertain the asymptotic behavior of polarization at large fields, when $x^2 \gg \nu\tau \gg 1$. In this case (17) goes over into

$$P = 1 - (1 - f)\nu\tau/x^2 = 1 - 30/x^2. \qquad (19)$$

## 3. MECHANISM OF DEPOLARIZATION OF THE MUONIUM ELECTRON

Let us discuss now some possible mechanisms whereby the muonium electron can become depolarized in a medium.

1. Relativistic interaction between moving muonium and the crystalline field. In ionic crystals, the internal electric fields are always appreciable. Inasmuch as the velocity v of the muonium remains constant in the thermal-equilibrium state, the question arises whether the observed value of $\nu$ can be attributed to the relativistic interaction between the muonium electron spin and the crystalline field.

A rough estimate of this effect can be obtained from a classical consideration of the motion of the muonium as a whole. As is well known [23], a transition to the non-relativistic approximation in the Dirac equation yields for the corresponding Hamiltonian the expression

$$\hat{\mathcal{H}} = -(\beta/2c)\,\sigma[\mathbf{vE}], \qquad (20)^*$$

where $\mathbf{E}$ is the intensity of the electric field. The Schrödinger equation $d\rho/dt = -i\hbar^{-1}[\hat{\mathcal{H}}, \rho]$ for the electron-spin density matrix assumes the form

$$d\rho_i/dt = (\beta/\hbar c)\,(v_i E_k - v_k E_i)\,\rho_k, \qquad (21)$$

where the vector part of the density matrix $\rho_i$ coincides, apart from a trivial normalization factor, with the electron polarization $\bar{\sigma}$.

*$[\mathbf{vE}] = \mathbf{v} \times \mathbf{E}$.

The electrostatic potential of the crystalline field will be written in the form of a Fourier expansion

$$\varphi = \sum_{\mathbf{b}} \varphi_{\mathbf{b}}\, e^{i2\pi b_n x_n}, \qquad (22)$$

where the summation is carried out over all the reciprocal lattice vectors $\mathbf{b}$. Taking into consideration the connection between the intensity and the potential and putting $x_n = v_n t$, we obtain

$$\frac{d\rho_i}{dt} = \frac{2\pi i\beta}{\hbar c} \sum_{\mathbf{b}}{}' \varphi_{\mathbf{b}}\,(b_i v_k - b_k v_i)\, e^{i2\pi b_n v_n t}\, \rho_k. \qquad (23)$$

In the perturbation-theory approximation it is easy to express the solution of Eq. (23) in terms of the initial value of the density matrix $\rho_i^{(0)}$. The result must be averaged over the muonium velocities, and also over all possible orientations of the crystal. In this case the linear approximation disappears, and in the quadratic approximation an important role is played as $t \to \infty$ only by the terms that increase linearly with the time. Because of this fact, the specific value of the mean free time $\Delta t$ drops out from the equation $d\bar{\rho}_i/dt = \Delta\rho_i/\Delta t$ for the density $\bar{\rho}_i$ which is averaged over many collisions ($\Delta\rho_i$ is the average increment in the vector $\rho$ over the time between two collisions $\Delta t$).

On the other hand, we have $d\bar{\rho}_i/dt = -2\nu\bar{\rho}_i$ (see start of Sec. 2). Leaving out the intermediate steps, we give only the final result

$$\nu = \frac{\pi^2}{6}\frac{e^2\bar{v}}{m^2c^4} \sum_{\mathbf{b}} b\varphi_{\mathbf{b}} \sum_{\mathbf{b}'=-g\mathbf{b}} \left(1 - \frac{\delta_{g0}}{2}\right) \varphi_{-(1+g)\mathbf{b}}, \qquad g \geqslant 0. \qquad (24)$$

Here $\bar{v}$ is a certain mean value of the muonium velocities; the reciprocal-lattice vector $\mathbf{b}'$ is antiparallel to $\mathbf{b}$ (or is equal to zero). At small values of b in ionic crystals we have $\varphi_{\mathbf{b}} \sim eb$. If we substitute in (24) in place of $\bar{v}$ the thermal velocity of the muon at room temperature, we obtain $1/\nu \sim 3 \times 10^{-4}$ sec, i.e., the mechanism under consideration does not have the observed magnitude of the effect.

2. Exchange collisions between the electrons of the medium and muonium (see the introduction). For a rough estimate we can put $m^* = m$, where $m^*$ is the effective mass of the carriers (electrons and holes). Then the statistical distribution of the free electrons assumes the form

$$\bar{n}_v = \exp\left[-(\Delta + mv^2)/2T\right], \qquad (25)$$

where $\Delta$ is the width of the energy gap, separating the conduction band from the valence band; T is the temperature.

The number of exchange collisions per unit time is

$$\nu = \int \sigma v \bar{n}_v \, d\Gamma, \qquad (26)$$

where $\sigma$ is the cross section of the exchange collision;

$$d\Gamma = (2\pi\hbar)^{-3} dp = m^3 v^2 dv / 2\pi^2 \hbar^3$$

is the number of the quantum states per unit volume. As a result we obtain

$$\nu = (mT^2/\pi^2\hbar^3)\, \sigma e^{-\Delta/2T}. \qquad (27)$$

To estimate the value of $\sigma$ let us use the results of the theoretical calculations on the scattering of slow electrons by a hydrogen atom [24]. It is easy to see that the exchange-scattering amplitude is equal to $(a_s - a_t)/2$, where $a_s$ and $a_t$ are respectively the scattering amplitudes in the singlet and triplet states of the incident electron and of the electron bound in the scattering system (in the hydrogen or muonium atom). Therefore

$$\sigma = \pi (a_s - a_t)^2 = 17.6\pi a_0^2. \qquad (28)$$

In ionic crystals the value of $\Delta$ is usually several electron volts, and because of the smallness of the factor $\exp[-\Delta/2T]$ the contribution of the electrons in the conduction band is extremely insignificant. In the particular case of silver bromide $\Delta = 1.35$ eV [25]; substituting $\sigma \sim 1.5 \times 10^{-15}$ cm$^2$, we obtain $1/\nu \sim 3$ sec at room temperature.

3. Formation of negative muonium ion. As is well known [14], the neutron-hydrogen atom is capable of capturing a second electron, the binding energy of which is $E_2 = 0.7$ eV. Muonium has the same properties, and in this respect does not differ at all from hydrogen. It is natural to assume that when situated in a substance muonium will retain this ability [3)].

Even within a time $\hbar/E_2 \sim 10^{-15}$ sec both electrons become completely depolarized in the negative ion, since their orbital states are identical and the spins are paired. One can conceive, for example, of the muonium capturing a second electron from a relatively shallow trap and losing it upon collision with a sufficiently deep vacant trap. This mechanism is very intriguing and necessitates further research.

[3)]Such an assumption seems particularly natural with respect to an ionic crystal, since the F-center in it (vacant lattice site from which a negative ion is missing) can likewise capture two electrons.[26]

## 4. ROLE OF DIELECTRIC CONSTANT OF THE MEDIUM

The phenomenon of interest to us, namely the depolarization of the $\mu^+$ meson by the muonium mechanism, continues only so long as the muonium does not enter into chemical reaction with the medium, i.e., so long as its electron does not participate in the chemical bond (see the introduction). However it is precisely for such electrons that the influence of the dielectric properties of the medium on their motion can turn out to be very appreciable.

A splendid illustration is the behavior of certain impurity atoms in semiconductors with large dielectric constants, such as silicon or germanium [25,57]. Namely, the interaction between a singly-charged impurity ion of the "donor" type with an electron obeys the formulas

$$a = m\varepsilon a_0/m^*, \qquad E = m^*\varepsilon^{-2} E_0/m, \qquad (29)$$

where E is the binding energy of the ground state of the electron in the field of the aforementioned ion, a is the corresponding Bohr radius, $\epsilon$ is the dielectric constant, m* is the effective mass of the electron, $a_0 = 0.53 \times 10^{-8}$ cm is the Bohr radius in vacuum, and $E_0 = 13.5$ eV is the ionization potential of the hydrogen atom. It is natural to assume that these relations are valid also for muonium in a dielectric medium [4)].

It is necessary, however, to emphasize in this connection the possible frequency dispersion of the dielectric constant [29]. Formulas (29) contain not the static dielectric constant $\epsilon(0)$, but the dielectric constant $\epsilon(\omega)$ for the frequency $\omega \sim E/\hbar$ of the field produced by the electron on the $\mu^+$ meson. In the case of silicon we can put $\epsilon(0) = 12$ in place of $\epsilon(\omega)$, so that Eq. (29) yields $E = 0.029$ eV [30], and at the corresponding frequencies $1/\omega \sim 2.3 \times 10^{-14}$ sec no appreciable dispersion occurs as yet. An analogous situation takes place also in germanium [31].

The situation is different in the case of silver bromide. The static dielectric constant in AgBr is even somewhat larger than in Si: $\epsilon(0) = 13$ [26]. Substitution in (29) leads to $\hbar/E \sim 10^{-14}$ sec (we assume for estimating purposes m* = m). Yet the dispersion of the dielectric constant in AgBr begins at much lower frequencies, for even when $1/\omega \sim 10^{-13}$ sec the value of $\epsilon(\omega)$ drops to 4.6 [26]. According to (29), the corresponding characteristic time $\hbar/E \sim 10^{-15}$ sec, and for the Bohr radius we have $a \sim 2.4 \times 10^{-8}$ cm. At such small

[4)]A similar hypothesis was first advanced by Feher et al.[28]

distances it is necessary to take into account not only the frequency dispersion of the dielectric constant, but also its spatial dispersion, and also the possible nonlinearity of the connection between the induction **D** and the intensity **E** in fields on the order of atomic [29].

In the limiting case of very small distances these effects lead, in final analysis, to the disappearance of the influence of the electric screening of the Coulomb interaction, and there are grounds for assuming that for silver bromide this limit is attained in practice. Inasmuch as the experimental data (see Sec. 2) confirm our assumption, in our analysis of $\mu^+$-meson depolarization in AgBr we took no account whatever of the dielectric polarizability of this medium [5)].

Another interesting feature of media with large $\epsilon(\omega)$ is the lack of thermodynamic equilibrium of the orbital state of the muonium, in which the $\mu^+$ meson is depolarized. According to (29), the ionization potential E becomes in many practical cases comparable with the temperature T = 0.025 eV (see above for silicon), or even drops somewhat lower (for example, for germanium [27]). As a result, at not too large values of the concentration $N_-$ of the free electrons, the equilibrium probability of filling the ground state of muonium,

$$\bar{n}_0 = 2^{3/2}\pi^{3/2}\hbar^3 m^{*-3/2}T^{-3/2}N_- e^{E/T} \qquad (30)$$

becomes very small compared with unity. On the other hand, the slowing down of the $\mu^+$ meson leads to the state $\bar{n}_0 = 1$ (see the introduction). In the absence of chemical interactions this strong deviation from thermodynamic equilibrium with respect to the parameter $\bar{n}_0$ could be "lifted" only by ionization of the muonium. Thus, along with the $\mu^+$ mesons entering into a chemical reaction, in media with large $\epsilon(\omega)$ there is in principle another possible relaxation mechanism, responsible for the finite mean time $\tau$ of the depolarization process.

The non-equilibrium character of the orbital ground state of muonium in which an electron is situated does not prevent it from exchanging with the free electrons of the medium, which have opposite spin directions (see the preceding section). It is convenient here for us to deal with an estimate of the probability of the "flipping" of the electron spin, expressed in terms of the volume concentration of the free electrons $N_-$:

$$\nu \approx \sqrt{2T/\pi m^*}\,\sigma N_- . \qquad (31)$$

For example, in silicon $m^* = 0.3$ m, $a = 2.1 \times 10^{-7}$ cm; $\sigma \sim 4\pi a^2 = 5.5 \times 10^{-13}$ cm$^2$. As a result we obtain for the average expectation time of the exchange collision, expressed in seconds,

$$\nu^{-1} \sim 2 \cdot 10^5 / N_- , \qquad (32)$$

where $N_-$ is in cm$^{-3}$.

In the experimental work of Feher et al [28], the function $P(N_-)$ was measured at room temperature in the absence of a magnetic field ($x = 0$). The concentration of the free electrons $N_-$ was varied by adding impurities. Let us compare the results of [28] with the theory developed. In the region of p-silicon, with a hole concentration $N_+ > 3 \times 10^{-12}$ ($N_+ N_- = \text{const} = 10^{20}$), it was observed that $P > 1/2$, which corresponds to the case $\nu\tau \ll 1$ (see Sec. 2 and Fig. 1). For five specimens of p silicon with different $N_+$, the experimentally observed P and formula (14) were used to calculate $\omega_0\tau$. We present the results:

| $N_+$ | $3 \cdot 10^{18}$ | $10^{18}$ | $10^{16}$ | $3 \cdot 10^{14}$ | $3 \cdot 10^{12}$ |
|---|---|---|---|---|---|
| $P$ | 0.87 | 0.81 | 0.75 | 0.57 | 0.54 |
| $\omega_0\tau$ | 0.59 | 0.80 | 1.0 | 2.5 | 3.4 |

It must be noted that although relation (1) remains valid also in the case of media with large $\epsilon(\omega)$, the formula $|\psi(0)|^2 = 1/\pi a^3$ no longer holds true for them. The point is that in the immediate vicinity of the Coulomb center one cannot use the macroscopic dielectric constant $\epsilon(\omega)$ which enters in (29). Thus, for semiconductors of the silicon and germanium type the frequency of the hyperfine splitting $\omega_0$ should be more readily regarded as a parameter which must be determined from experiment. We note that it can depend on the concentration of the impurities, inasmuch as an analogous dependence holds true also for the ionization energy $E = \hbar\omega$ [32].

In the intermediate region $3 \times 10^7 < N_- < 10^{12}$ cm$^{-3}$ the polarization of the $\mu^+$ mesons differs little from 1/2, in accordance with the condition $1/\omega_0 \ll \tau \ll 1/\nu$ for the applicability of formula (3). In n-silicon, with $\nu$ increasing in accord with (31) and (32), the condition $\nu\tau \ll 1$ is violated, and immediately past the point $N_- = 10^{12}$ the $\mu^+$-meson polarization decreases rapidly below the value $P = 1/2$. For the interval $3 \times 10^{12} < N_- < 3 \times 10^{13}$ cm$^{-3}$ formulas (16) and (32) yield $\tau \sim 10^{-8}$ sec. The condition $\nu \ll \omega_0$ for the applicability of formula (16) leads to

$$5 \cdot 10^{-9} \gg \omega_0^{-1} \gg 3.6 \cdot 10^{-11} \text{ sec}. \qquad (33)$$

The right half of the inequality denotes that in silicon, owing to the fact that $\epsilon(\omega) \gg 1$, the hyperfine splitting of muonium $\hbar\omega_0$ should be much smaller than in vacuum.

[5)] In analogous fashion we have actually put in Sec. 2 $\epsilon(\omega) = 1$ also in the case of gelatin, for which apparently even the static dielectric constant is close to unity.

We do not consider the experimental data obtained in [28] for specimens with $N_- \geq 10^{14}$, for in this case the form of the $P(N_-)$ curve is influenced by a large number of complicating circumstances. With increasing $N_-$ the condition $\nu \ll \omega_0$ is violated and a transition takes place to the opposite limiting case $\nu \gg \omega_0$; a sharp dependence of the parameter $\tau$ on $N_-$ sets in, and $1/\tau \sim N_-$ at sufficiently large concentrations of $N_-$, when the cessation of depolarization is due to impact ionization of the muonium, and not at all to its entering into chemical reaction with the atoms of the medium (see above).

Finally, let us point out still another effect which depends quadratically on $N_-$: after impact ionization of the muonium, one of the electrons of the medium can return to a Bohr orbit (charge exchange), restoring the depolarization of the $\mu^+$ meson. Apparently to interpret the course of the $P(N_-)$ curve in n-silicon with high concentration of free electrons ($N_- \geq 10^{14}$ cm$^{-3}$) it is necessary to accumulate more experimental data, and also to refine the theory (in particular, to account for charge exchange).

## 5. CONCLUSION

The arguments presented above give grounds for hoping that noticeable progress has been made in understanding the depolarization of $\mu^+$ mesons in condensed media. At the same time, the theory of this question is still insufficiently developed when it comes to several important points, some of which we list here.

1) The theory developed in the present paper starts from the assumption that the depolarization of the $\mu^+$ meson proceeds continuously during a certain time interval, which terminates when the muonium enters into a chemical reaction or is ionized. Let us analyze first the validity of this assumption with respect to the ionization process.

Charge exchange is a phenomenon characteristic of substances of any type when $v \sim v_{at}$, and the cross section of this process is of the same order as the geometrical cross section. Therefore the average time between successive events of muonium formation and its ionization is of the order of $\tau_{c.e} = 1/N\sigma v_{at}$, where N is the number of atoms per unit volume; $\sigma \sim 10^{-16}$ cm$^2$. As a result we obtain $\tau_{c.e} \sim 10^{-15}$ sec. The reduction in the polarization of the $\mu^+$ meson during the time $\tau_{c.e}$ is equal in order of magnitude to $(\omega_0\tau_{c.e})^2$. Thus, the effect becomes noticeable after a time

$$(\omega_0\tau_{c.e})^2\, t/\tau_{c.e} = 1, \qquad t = 1/\omega_0^2\tau_{c.e} \sim 10^{-6} \text{ sec}.$$

Of course, the duration of the Bohr stage $v \sim v_{at}$ is many orders of magnitude less; it is bound to amount to $\sim 10^{-11}$ sec. Consequently, when $v \sim v_{at}$ charge exchange is ineffective, and we are justified in disregarding it. However, in media with large dielectric constant which has a weakly pronounced frequency dispersion, in view of the low binding energy of the muonium electron, in the region $v \ll v_{at}$, the electron sometimes again becomes capable of ionization with subsequent return of one of the electrons to the Bohr orbit.

The theoretical question of the depolarization of $\mu^+$ mesons by charge exchange was considered by Ferrell et al [33], who obtained a formula for the polarization $P(x)$, in which the presence of an external magnetic field $x = H_x H_0$ was taken into account. However, the derivation of this formula raises certain doubts concerning its correctness. In particular, the ionization of muonium was considered statistically (the corresponding probability per unit time was introduced), and the return of the electron was characterized by a fully defined number n, the number of its captures on the orbit. Such an asymmetry in the treatment of these two perfectly equivalent processes does not seem justified at all.

In addition, it must be borne in mind that only in a few n-type semiconductors with high free-electron density can charge exchange be an effective mechanism for the depolarization of the $\mu^+$ mesons (see above). But under these conditions there exists without fail another depolarization mechanism, not connected with the ionization of the muonium and reducing to the "flipping" of the electron spin: we are referring to exchange collisions between the free electrons in muonium (see the preceding section). Apparently only a simultaneous account of both mechanisms can make a theoretical consideration of charge exchange suitable for a direct comparison with experiment.

2) Let us turn now to the question of the validity of the assumption that after the $\mu^+$ meson enters into chemical reaction with the atoms of the medium its depolarization stops. If the nuclei of these atoms are not all even-even, then further depolarization can proceed on account of the interaction of their magnetic moments with the magnetic moment $\boldsymbol{\mu}$ of the $\mu^+$ meson. For the nucleus a the corresponding term in the Hamiltonian has the form

$$\hat{\mathcal{H}}_a = [\boldsymbol{\mu}\boldsymbol{\mu}_a - 3(\boldsymbol{\mu}\mathbf{n}_a)(\boldsymbol{\mu}_a\mathbf{n}_a)]\, r_a^{-3}, \qquad (34)$$

where $\mu_a$ is the magnetic moment of the nucleus, $r_a$ is the distance from the muon, and $\mathbf{n}_a$ a unit vector directed along the line joining the nucleus with the $\mu^+$ meson.

Inasmuch as an expression of this type decreases very rapidly with distance, for a rough estimate of the $\mu^+$ meson depolarization time it is sufficient to take into account its interaction with the nearest neighbors only. It turns out here that a noticeable influence of the effect under consideration, which manifests itself within a time $\tau_\mu = 2.2 \times 10^{-6}$ sec, is not excluded in principle, but, generally speaking, the situation apparently does not favor this. In fact, the magnetic field produced by the nearest nucleus a on the $\mu^+$ meson amounts to

$$H \sim 5\mu_a/r_a^3, \tag{35}$$

where H is in Gausses, $\mu_a$ in nuclear magnetons, and $r_a$ in Angstroms. On the other hand, the Larmor precession frequency $\xi\omega'$ of the $\mu^+$-meson spin in a magnetic field of H Gauss is

$$1/\zeta\omega' = 1.2 \cdot 10^{-5}/H, \tag{36}$$

where the characteristic time $1/\xi\omega'$ is in seconds. Thus, to obtain a noticeable effect a field of $\sim$10 G is necessary. But in a crystal the distance between sites is usually not less than 3 Å, which according to (35) corresponds to fields $\lesssim$1 G.

Of course, the $\mu^+$ meson can be part of a compact chemical formation such as a molecule in which the distances between nuclei are $\lesssim$1 Å. However, even in this case the mechanism under consideration may turn out to be ineffective, and its influence is determined not only by the magnitude of the magnetic moment $\mu_a$ of the nearest nuclei. The point is that in a solid body the sufficiently compact molecules retain their ability to rotate [16], and when averaged over all possible directions of the vector $\mathbf{n}_a$ the expression (34) vanishes. If such an averaging, over times less than $\hbar/|\hat{\mathcal{H}}_a|$, is capable of actually occurring, this will lead to the absence of depolarization of the $\mu^+$ meson. Nevertheless, the arguments presented above do not exclude the possibility of existence of such substances, in which the $\mu^+$ meson becomes depolarized after entering into chemical reaction. This is all the more so since in a solid the rotational state of the molecule, in which it entered, need not of necessity be characterized by total isotropy.

A characteristic feature of the depolarization mechanism considered here is the appreciable influence of external magnetic fields of intensity still too low to influence the muon mechanism. Indeed, even in a field comparable in magnitude with (35) the depolarization in the chemically-bound state should decrease noticeably, and in fields $\sim$100 – 200 G one can expect practically the entire depolarization to be due only to the muonium stage of the process. In this connection it is interesting to note a recently published paper [34] in which depolarization of $\mu^+$ mesons was apparently observed after their entering into chemical reaction. The medium was one of the plastic scintillators (polystyrol plus 2% p-terphenyl). The (longitudinal) external field H ranged from zero to 150 G. Although the experimental data are not yet sufficiently accurate, it can be stated that in this interval the $\mu^+$-meson polarization P(H) has increased by approximately three times, reaching a value close to 1/2.

The question of the depolarization of $\mu^+$ mesons after they have entered into chemical reaction has not yet been thoroughly investigated theoretically. In particular, in the derivation of the dependence P(H) for this mechanism it would be essential to clarify the difference between the behavior in longitudinal and transverse magnetic fields.

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