SPIN WAVES AND PARAMAGNETIC RELAXATION IN A FERMI LIQUID

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In the presence of a magnetic field in a Fermi liquid, there exists a branch of the spectrum corresponding to spin waves. The resonance absorption process leads to the excitation of spin waves. The damping of a spin wave with zero wave vector is connected with interactions leading to the nonconservation of spin and determines the transverse relaxation time. The longitudinal relaxation time is also found.

1. INTRODUCTION

THE investigation of the temperature dependence of the paramagnetic relaxation times in liquid He^3 has apparently been carried out only for $T > 1°K$ [1]. Under these conditions the influence of degeneracy is already small and the results can be qualitatively interpreted in terms of the classical theory [2].

It can now be considered established that for $T < 0.1°K$ liquid He^3 is a Fermi liquid, and hence the theory of paramagnetic relaxation, based on the classical description of the motion of the particles, is inapplicable here. In the present work the microscopic theory of a Fermi liquid is used for the calculation of the paramagnetic relaxation.

2. If the system is situated in a constant magnetic field H_z and, in addition to that, a weak variable field $H_+ = H_x + iH_y$, then in the linear approximation with respect to H_+

$$M_+(t) \equiv M_x(t) + iM_y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\chi_t(\omega)H_+(\omega).$$

It follows from the equations of Bloch [3] that the "transverse" susceptibility χ_t is

$$\chi_t(\omega) = \chi\frac{\omega_0}{\omega_0 - \omega - i/T_2}, \tag{1}$$

where χ is the static susceptibility, $\omega_0 = \beta H_z$, and β is the gyromagnetic ratio of a free particle (a system of units with $\hbar = 1$ is used).

In a weak variable field $H_z(t)$

$$M_z(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\chi_l(\omega)H_z(\omega)e^{-i\omega t},$$

$$\chi_l(\omega) = \chi\frac{i}{T_1}\frac{1}{\omega + i/T_1}. \tag{2}$$

The formulae (1) and (2), as will be shown, follow from the microscopic theory (the first for $\omega \to \omega_0$, the second for $\omega \to 0$). For the transverse (T_2) and longitudinal (T_1) relaxation times one obtains the expressions

$$T_2 = \alpha\mu T^{-2}[1 + (\omega_0/2\pi T)^2]^{-1}, \tag{3}$$

$$T_1 = \alpha\mu/T^2 \tag{4}$$

(μ is the chemical potential, T the temperature in energy units, α a constant of order of magnitude $(\mu a^3/\beta^2)^2 \sim 10^{14}$, and a the interatomic distance). Overhauser [4] obtained an expression analogous to (4) considering the paramagnetic resonance in metals within the framework of the free electron model.

The times determined by (3) and (4) are very large: $T_1 \sim 10^6 - 10^7$ sec for $T \sim 0.01°K$. Therefore in this temperature region the observed relaxation times will be determined by the experimental circumstances (interactions with the container walls etc.). Nevertheless the clarification of the paramagnetic resonance and relaxation mechanism in a Fermi liquid is of interest.

We consider liquid He^3 specifically, but the proposed approach is in principle also applicable to the investigation of paramagnetic resonance in metals. Paramagnetic relaxation is caused by very weak magnetic interactions among the particles, which are accompanied by much stronger interactions of nonmagnetic nature. The latter alone cannot lead to relaxation by themselves but exert an essential influence on the mechanism of the process.

As is well known the Fermi excitations play a fundamental role in the kinetics of a Fermi liquid. In the presence of a magnetic field the energy spectrum of the excitations has the form [5,6]

$$\varepsilon = v(p - p_0) - \gamma(\sigma\mathbf{H}),$$

where γ is different from the gyromagnetic ratio of a free particle β:

$$\gamma = \beta (1 + Z/4)^{-1}. \qquad (5)$$

The constant Z represents the zeroth spherical harmonic of the exchange part of the dimensionless correlation function. In a Fermi liquid (in contrast to a Fermi gas) $Z \sim 1$. Therefore excitation spin flip leads to an energy change $\gamma H \neq \omega_0$. Besides, the excitations have a very small lifetime compared to T_1 and T_2, which leads to a strong diffusion of the absorption band near the frequency γH. The resonance character of the frequency dependence of the magnetic susceptibility χ_t is connected with spin waves, which, as will be shown, can arise in a Fermi liquid in the presence of a magnetic field. The spin waves have the dispersion law $\omega = \omega_0 + bk^2$, where $b \sim v^2/\omega_0$, and the damping for $k \rightarrow 0$ goes to zero if one does not take magnetic interactions into account.

2. TRANSVERSE RELAXATION TIME. SPIN WAVES

1. For the transverse susceptibility χ we can obtain the expression (see, for example, [7])

$$\chi_t(\omega) = \frac{i}{2}\int\limits_{-\infty}^{\infty} dt e^{i\omega t}\langle [M_+(t), M_-]\rangle \theta(t),$$

$$\mathbf{M}(t) = \exp[i\mathcal{H} - M_z H)t]\mathbf{M}\exp[-i(\mathcal{H} - M_z H)t]. \qquad (6)$$

If we do not take the magnetic interactions into account, then $[\mathcal{H}, \mathbf{M}] = 0$, $M_+(t) = e^{-i\omega_0 t}M_+$ and thus

$$\chi_t(\omega) = \chi\frac{\omega_0}{\omega_0 - \omega - i\delta}; \qquad \delta = +0. \qquad (7)$$

To calculate T_2 it is necessary to include in (6) the magnetic interactions leading to nonconservation of spin. The basic interaction of this type in He^3 is the magnetic dipole interaction between the nuclei

$$V(\mathbf{r}) = \beta^2 r^{-3}[\sigma\sigma' - 3(\sigma\mathbf{r})(\sigma'\mathbf{r})/r^2].$$

In practice the explicit form of $V(\mathbf{r})$ cannot be used, since in a Fermi liquid there exist strong nonmagnetic interactions leading to an essential "renormalization" of V. If one expands χ_t in powers of V, then in each order there occur terms containing the factor $(\omega - \omega_0 + i\delta)^{-1}$ to higher powers than in the lower order. For this reason, despite the fact that the interaction V is very weak, for $\omega \rightarrow \omega_0$ it is necessary to select and sum the main terms in all orders of perturbation theory. This series has in general a complicated structure. We shall show, however, that it reduces to a geometrical progression in the case of a Fermi liquid.

2. In the second quantization representation

$$\chi_t(\omega) = \frac{i}{2}\beta^2\int\limits_{-\infty}^{\infty} dt e^{i\omega t}\int\frac{d^3p d^3p'}{(2\pi)^6}\langle[(a^+_{2,\mathbf{p}}a_{1,\mathbf{p}'})_t, a^+_{1,\mathbf{p}}a_{2,\mathbf{p}'}]\rangle\theta(t), \qquad (8)$$

where $a_{\alpha,\mathbf{p}}$ is the annihilation operator of a particle with the spin quantum numbers $\alpha = 1, 2$; the average is taken over the grand canonical ensemble.

As is well known, $\chi(\omega)$ is analytic in the upper half-plane of the variable ω. The application of the temperature diagram technique [8] permits one to find this function at the points $\omega_m = 2\pi m Ti$ (m integer). $\chi(\omega)$ is then determined by means of analytic continuation from the point set ω_m $(m > 0)$ to the real axis. There exists the possibility of a direct graphical description of $\chi(\omega)$, as this was shown earlier for quantities of similar structure [9].

$\chi(\omega)$ (without the factor $\beta^2/2$) is represented by the set of all diagrams in Fig. 1. In this figure a pair of lines corresponds to

$$g(P, \omega) = G^R(P + \omega)G^A(P);$$

$$P \rightarrow (\varepsilon, \mathbf{p}), \quad P + \omega \rightarrow (\varepsilon + \omega, \mathbf{p});$$

the quantity $\mathcal{T}(P, P', \omega)$, connected with the vertex part [9,10], is denoted by the circle, and each of the shaded corners represents the sum of all diagrams not containing intermediate intersections of the type $G^R G^A$ [we denote this quantity by $Q(P, \omega)$].

With the magnetic interaction V accounted for, the single particle Green's function G(P) is a spin matrix. However, we have to take V into account only in $\mathcal{T}$. In this case, in accordance with (8),

$$g(P, \omega) \equiv g_{21}(P, \omega) = G_2^R(P + \omega)G_1^A(P),$$

and for $\chi_t(\omega)$ one can write down the expression

$$\chi_t(\omega) = -\frac{1}{2}\beta^2\Big\{\frac{1}{2i(2\pi)^4}\int d^4P Q^2(P, \omega)\Big(\mathrm{th}\frac{\varepsilon+\omega}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\Big)g_{21}$$
$$\times (P, \omega) + \Big(\frac{1}{2i(2\pi)^4}\Big)^2\int d^4P d^4P' Q$$
$$\times (P, \omega)\Big(\mathrm{th}\frac{\varepsilon+\omega}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\Big)g_{21}(P, \omega)$$
$$\times \mathcal{T}_{21,21}(P, P', \omega)g_{21}(P', \omega)Q(P', \omega)\Big\}. \qquad (9)^*$$

The factor $\tanh[(\epsilon + \omega)/2T] - \tanh(\epsilon/2T)$ is connected with the analytic continuation procedure [9]. The remaining quantities correspond directly to the elements of the diagrams in Fig. 1.

*th = tanh.

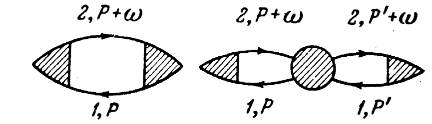

FIG. 1

Clearly small values $\epsilon \sim (T, \omega) \ll \mu$ are essential in the integrals. The quantity $\mathscr{T}(P, P', \omega)$ has the property that for $\epsilon \sim T, \omega$ it vanishes exponentially as a function of ϵ' outside the interval of values $\epsilon' \sim T, \omega$. For small ϵ and $\xi = v(p - p_0)$ the quantity $g_{21}(P, \omega)$ can be written as:

$$g_{21}(P, \omega) \approx \frac{a^2}{[\varepsilon + \omega - \xi - {}^1\!/_2\,\gamma H - ia\,\mathrm{Im}\,\Sigma^R(\varepsilon + \omega)]\,[\varepsilon - \xi + {}^1\!/_2\,\gamma H + ia\,\mathrm{Im}\,\Sigma^R(\varepsilon)]}.$$

Hence it follows that the integrals in (9) converge in the interval of values $\epsilon, \epsilon', \xi, \xi' \sim T, \omega$. The quantity Q, which varies essentially only in the large interval of $\epsilon, \xi \sim \mu$, can be removed from under the integral for zero values of these arguments.

For small ξ and ξ' functions $\mathscr{T}(P, P', \omega)$ depends only on the angle between **p** and **p'**. Since the remaining quantities do not depend on the angles, the angle integration reduces to the substitution for $\mathscr{T}$ of its zeroth spherical harmonic, for which we retain the previous notation. The ξ dependence is thus contained only in g_{21}, where $\mathrm{Im}\,\Sigma$ does not depend on ξ.

After integration over ξ we obtain

$$\chi_t(\omega) = -\beta^2\frac{a^2p_0^2}{8\pi^2 v}Q^2\left\{\int_{-\infty}^{\infty} d\varepsilon\left(\mathrm{th}\,\frac{\varepsilon+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon}{2T}\right)\frac{1}{\Omega(\varepsilon,\omega)}\right.$$
$$+\frac{1}{4}\iint d\varepsilon\,d\varepsilon'\left(\mathrm{th}\,\frac{\varepsilon+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon}{2T}\right)\frac{1}{\Omega(\varepsilon,\omega)}$$
$$\left.\times\ \tilde{\mathscr{T}}(\varepsilon,\varepsilon',\omega)\frac{1}{\Omega(\varepsilon',\omega)}\right\};$$

$$\Omega(\varepsilon,\omega) = \omega - \gamma H - ia\,\mathrm{Im}\,[\Sigma^R(\varepsilon+\omega) + \Sigma^R(\varepsilon)],$$

$$\tilde{\mathscr{T}} = (a^2p_0^2/\pi^2 v)\mathscr{T}_{21,21}. \qquad (10)$$

3. The quantity $\tilde{\mathscr{T}}$ satisfies the equation (neglecting V)

$$\tilde{\mathscr{T}}(\varepsilon,\varepsilon',\omega) = \tilde{\mathscr{T}}^{(0)}(\varepsilon,\varepsilon',\omega)$$
$$+\frac{1}{4}\int_{-\infty}^{\infty} d\varepsilon''\,\tilde{\mathscr{T}}^{(0)}(\varepsilon,\varepsilon'',\omega)\frac{1}{\Omega(\varepsilon'',\omega)}\tilde{\mathscr{T}}(\varepsilon'',\varepsilon',\omega). \qquad (11)$$

In this equation the integration over ξ and the angles has been carried out already. An equation of this type has been obtained and investigated before [9,10]. It has a clear cut graphical structure, since the quantity $\tilde{\mathscr{T}}^{(0)}$ represents the set of all diagrams not having intermediate intersections G^RG^A.

As was shown [10],

$$\tilde{\mathscr{T}}^{(0)}(\varepsilon,\varepsilon',\omega) = \frac{1}{2}C\left(\mathrm{th}\,\frac{\varepsilon'+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon'}{2T}\right) + i\tilde{\mathscr{T}}'(\varepsilon,\varepsilon',\omega), \qquad (12)$$

where $\tilde{\mathscr{T}}' = \mathrm{Im}\,\tilde{\mathscr{T}}^{(0)}$, and the constant C is the zeroth harmonic of the quantity $C(\mathbf{p}, \mathbf{p}')$, which is connected in the following fashion with the k-limit of the forward scattering amplitude for $T = 0$ [11]:

$$(a^2p_0^2/\pi^2 v)\,\Gamma^k(\mathbf{p}\sigma;\,\mathbf{p}'\sigma') = B(\mathbf{p},\mathbf{p}') + (\sigma\sigma')\,C(\mathbf{p},\mathbf{p}').$$

We introduce the notation $\varphi(\epsilon, \epsilon', \omega) = \tilde{\mathscr{T}}(\epsilon, \epsilon', \omega)/\Omega(\epsilon, \omega)$. Then we obtain from (11), taking (12) into account, the equation for φ

$$(\omega - \gamma H)\,\varphi(\varepsilon,\varepsilon',\omega) = \frac{1}{2}C\left(\mathrm{th}\,\frac{\varepsilon'+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon'}{2T}\right) + i\tilde{\mathscr{T}}'(\varepsilon,\varepsilon',\omega)$$
$$+\frac{1}{8}C\int d\varepsilon''\left(\mathrm{th}\,\frac{\varepsilon''+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon''}{2T}\right)\varphi(\varepsilon'',\varepsilon',\omega)$$
$$+\frac{i}{4}\int d\varepsilon''\,\tilde{\mathscr{T}}'(\varepsilon,\varepsilon'',\omega)\,\varphi(\varepsilon'',\varepsilon',\omega)$$
$$+\ ia\,\mathrm{Im}\,[\Sigma^R(\varepsilon+\omega) + \Sigma^R(\varepsilon)]\,\varphi(\varepsilon,\varepsilon',\omega). \qquad (13)$$

If both sides of this equation are multiplied by $\tanh[(\epsilon+\omega)/2T] - \tanh(\epsilon/2T)$ and integrated over ϵ, then the last two terms on the right side of (13) vanish, since

$$\frac{1}{4}\int d\varepsilon\left(\mathrm{th}\,\frac{\varepsilon+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon}{2T}\right)\tilde{\mathscr{T}}'(\varepsilon,\varepsilon',\omega)$$
$$= -a\,\mathrm{Im}\,[\Sigma^R(\varepsilon'+\omega) + \Sigma^R(\varepsilon')]\left(\mathrm{th}\,\frac{\varepsilon'+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon'}{2T}\right). \qquad (14)$$

This relation implies the vanishing of the zeroth spherical harmonic of the collision integral. This as well as the analogous relation

$$\frac{1}{4}\int d\varepsilon'\,\tilde{\mathscr{T}}'(\varepsilon,\varepsilon',\omega) = -a\,\mathrm{Im}\,[\Sigma^R(\varepsilon+\omega) + \Sigma^R(\varepsilon)] \qquad (14a)$$

can easily be verified if one makes use of the explicit expressions for $\tilde{\mathscr{T}}'$ and $\mathrm{Im}\,\Sigma$, which have been obtained earlier [10].

Thus we find that

$$\int d\varepsilon\left(\mathrm{th}\,\frac{\varepsilon+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon}{2T}\right)\varphi(\varepsilon,\varepsilon',\omega) = \frac{4}{1-C/4}\left(\mathrm{th}\,\frac{\varepsilon'+\omega}{2T} - \mathrm{th}\,\frac{\varepsilon'}{2T}\right)$$
$$\times\ \frac{1}{\omega-\omega_0+i\delta}\left(\frac{C}{4}\,\omega - ia\,\mathrm{Im}\,[\Sigma^R(\varepsilon'+\omega) + \Sigma^R(\varepsilon')]\right). \qquad (15)$$

Here we took into account that $1 - C/4 = (1+Z/4)^{-1}$, where Z is related to Γ^{ω}: $(a^2p_0^2/\pi^2 v)\Gamma^{\omega} = F + (\sigma\cdot\sigma')Z$ [11], and we used formula (5).

Substituting (15) into the right hand side of (13) we see that for $\omega \to \omega_0$ the first two terms of the right hand side can be neglected. Near a resonance one can then look for a solution $\varphi(\epsilon, \epsilon', \omega)$ in a form not containing any dependence on the first argument ϵ. The last two terms in (13) then cancel with the aid of relation (14a). As a result we obtain for $\tilde{\mathcal{T}}(\epsilon, \epsilon', \omega)$ an expression which is valid for sufficiently small values of $\omega - \omega_0$:

$$\tilde{\mathcal{T}}(\varepsilon, \varepsilon', \omega) = \frac{2}{(1-C/4)\omega_0}\left(\operatorname{th}\frac{\varepsilon'+\omega_0}{2T} - \operatorname{th}\frac{\varepsilon'}{2T}\right)\frac{\Omega(\varepsilon, \omega_0)\,\Omega(\varepsilon', \omega_0)}{\omega - \omega_0 + i\delta}$$

$$(C\omega_0/4 = \omega_0 - \gamma H). \qquad (16)$$

The first term in (10) (not containing $\mathcal{T}$) has no resonance dependence on ω. Therefore near a resonance

$$\chi_t(\omega) = \beta^2 \frac{a^2p_0^2}{4\pi^2 v}\frac{Q^2}{1-C/4}\frac{\omega_0}{\omega_0 - \omega - i\delta}. \qquad (17)$$

A comparison with the exact formula (7) shows that

$$\beta^2 \frac{a^2p_0^2}{4\pi^2 v}\frac{Q^2}{1-C/4} = \chi \qquad (18)$$

(one can show independently that $aQ = 1 - C/4$ and establish the identity of the formula obtained for χ with the well known expression of the theory of Fermi liquids. [5,6])

We considered the contribution to $\chi_t(\omega)$ of dia-containing at least one cross section of the type $G^R G^A$ (Fig. 1). Diagrams not having such cross sections contain ω along with variables whose range of variation in an integration is of the order of the quantity μ. Therefore such diagrams can be omitted from consideration in an investigation of the frequency dependence of the magnetic susceptibility.

4. Returning to formula (16) we see that $\mathcal{T}$ has a pole at $\omega = \omega_0$. This indicates that in the presence of a magnetic field in a Fermi liquid there exists a Bose branch of the spectrum corresponding to spin waves [12]. We found only the limiting frequency of a spin wave for zero wave vector k.

An investigation of the equation for $\mathcal{T}$ for $k \neq 0$, which we shall not carry out here, shows that for $vk \ll \omega_0$ the spectrum of spin waves is of the form $\omega = \omega_0 + bk^2$, where $b \sim v^2/\omega_0$. The damping of the spin waves goes to zero for $k = 0$ (not taking magnetic interactions into account).

It is interesting to observe that besides this branch of the spectrum, corresponding to a pole of the zeroth spherical harmonic of $\mathcal{T}$, there also exists a branch with $l = 1$, whose limiting frequency is different from ω_0. However, this branch practically does not appear in absorption since its contribution to χ_t is proportional to $(vk/\omega_0)^2 \ll 1$.

5. The presence of a pole in $\mathcal{T}$ for $\omega = \omega_0$ allows one to easily isolate the main terms in an expansion of χ_t in powers of the magnetic interaction V. The problem of the calculation of T_2 reduces to the determination of the damping connected with V in the pole of $\mathcal{T}$.

For the vertex part with inclusion of $V(\mathcal{F})$ one can write the equation

$$\mathcal{F}(\varepsilon, \varepsilon', \omega) = \mathcal{T}(\varepsilon, \varepsilon', \omega) + \frac{1}{2i(2\pi)^4}\int d^4P''\,\mathcal{T}(\varepsilon, \varepsilon'', \omega)\,K_1(P'', \omega)\,\mathcal{F}(\varepsilon'', \varepsilon', \omega)$$
$$+ \left(\frac{1}{2i(2\pi)^4}\right)^2\int d^4P''\,d^4P'''\,\mathcal{T}(\varepsilon, \varepsilon'', \omega)\,K_2(P'', P''', \omega) \times \mathcal{F}(\varepsilon''', \varepsilon', \omega), \qquad (19)$$

where $\mathcal{T}$ is defined by formula (16) $[\tilde{\mathcal{T}} = (a^2p_0^2/\pi^2 v)\mathcal{T}]$, and the quantities $K_1(P, \omega)$ and $K_2(P, P', \omega)$ containing V represent the sum of the diagrams shown in Fig. 2. In this figure the shaded circle is the correction to the self-energy $(\Sigma^{(V)})$; the rectangle is the correction to $\mathcal{T}^{(0)}(\mathcal{T}^{(V)})$.

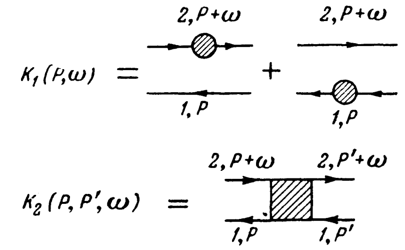

FIG. 2

We substitute into (19) the expressions corresponding to the diagrams:

$$K_1(P, \omega) = g_{21}(P, \omega)\,[G_2^R(P+\omega)\,\Sigma_R^{(V)}(P+\omega) + G_1^A(P)\,\Sigma_A^{(V)}(P)],$$

$$K_2(P, P', \omega) = g_{21}(P, \omega)\,\mathcal{T}^{(V)}(P, P', \omega)\,g_{21}(P', \omega)$$

and carry out the integration over $\xi = v(p - p_0)$ and $\xi' = v(p' - p_0)$. Then, introducing the function $f(\omega)$ by the formula

$$\mathcal{F}(\varepsilon, \varepsilon', \omega) = \frac{2}{(1-C/4)\omega_0}\left(\operatorname{th}\frac{\varepsilon'+\omega_0}{2T} - \operatorname{th}\frac{\varepsilon'}{2T}\right) \times \Omega(\varepsilon, \omega_0)\,\Omega(\varepsilon', \omega_0)\,f(\omega),$$

we obtain

$$f(\omega) = 1/[\omega - \omega_0 - \Pi(\omega_0)]. \qquad (20)$$

The real part of $\Pi(\omega_0)$ determines a small correction to the resonance frequency and will not be considered further. The imaginary part is equal to

$$\mathrm{Im}\,\Pi(\omega_0) = \frac{1}{1-C/4}\frac{1}{2\omega_0}\left\{\int d\varepsilon\left(\mathrm{th}\frac{\varepsilon+\omega_0}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\right)\right.$$
$$\times a\,\mathrm{Im}[\Sigma_R^{(V)}(\varepsilon+\omega_0) + \Sigma_R^{(V)}(\varepsilon)]$$
$$\left. + \frac{1}{4}\iint d\varepsilon\, d\varepsilon'\left(\mathrm{th}\frac{\varepsilon+\omega_0}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\right)\mathrm{Im}\,\mathcal{T}^{(V)}(\varepsilon, \varepsilon', \omega)\right\}. \qquad (21)$$

In order to find the explicit form of the functions $\mathrm{Im}\,\Sigma^{(V)}$ and $\mathrm{Im}\,\mathcal{T}^{(V)}$ one can employ a method analogous to the one which was applied earlier [10] in an investigation of the collision integral. The diagrams of Figs. 3 and 4, where the circle represents the correction to the vertex part linear in V, give the basic contribution to these quantities. The frequency dependence has to be considered only for the isolated lines. The calculations, which do not differ from the ones carried out in [10], lead to the following expression for $\mathrm{Im}\,\Pi(\omega_0)$:

$$\mathrm{Im}\,\Pi(\omega_0) \equiv -1/T_2 = -B\varphi(\omega_0)/(1-C/4); \qquad (22)$$

$$\varphi(\omega_0) = \frac{\pi}{32 p_0 v}\frac{\mathrm{sh}(\omega_0/2T)}{\omega_0}$$
$$\times \iiint \frac{d\varepsilon_1 d\varepsilon_2 d\varepsilon_3}{\mathrm{ch}(\varepsilon_1/2T)\,\mathrm{ch}(\varepsilon_2/2T)\,\mathrm{ch}[(\varepsilon_3+\omega_0)/2T]\,\mathrm{ch}[(\varepsilon_2-\varepsilon_1-\varepsilon_3)/2T]}$$
$$= \frac{\pi^3}{6}\frac{T^2}{p_0 v}\left[1 + \left(\frac{\omega_0}{2\pi T}\right)^2\right], \qquad (23)^*$$

$$B = \int \frac{dO_{p_1} dO_{p_2}}{(4\pi)^2}\delta(|\mathbf{e}+\mathbf{e}_1-\mathbf{e}_2|-1)\left\{\frac{1}{2}\tilde{\Gamma}^{(V)}_{\alpha\beta,\gamma\delta}\tilde{\Gamma}^{(V)}_{\delta\gamma,\beta\alpha}\right.$$
$$\left. - 2\tilde{\Gamma}^{(V)}_{1\beta,1\gamma}\tilde{\Gamma}^{(V)}_{\gamma 2,\beta 2} + \tilde{\Gamma}^{(V)}_{12,\gamma\delta}\tilde{\Gamma}^{(V)}_{\delta\gamma,12}\right\}$$
$$(\tilde{\Gamma}^{(V)}\tilde{\Gamma}^{(V)} = \tilde{\Gamma}^{(V)}(\mathbf{p}, \mathbf{p}_1; \mathbf{p}_2, \mathbf{p}+\mathbf{p}_1-\mathbf{p}_2)\tilde{\Gamma}^{(V)}$$
$$(\mathbf{p}+\mathbf{p}_1-\mathbf{p}_2, \mathbf{p}_2; \mathbf{p}_1\,\mathbf{p})). \qquad (24)$$

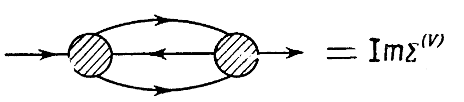

FIG. 3

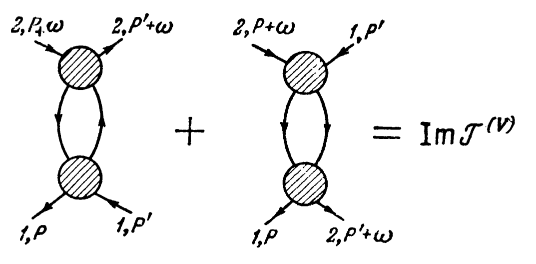

FIG. 4

*sh = sinh, ch = cosh.

The formulae (22)—(24) lead to expression (3). We remark that exchange forces give no contribution to B. Diagrams differing from the diagrams of Figs. 3 and 4 in that one of the vertices does not contain V likewise give no contribution to B. Thus $B \sim V^2$. In connection with this the question could arise whether along with the diagrams of Fig. 2 one has to consider also diagrams, some of which are shown in Fig. 5, where each shaded element is $\sim V$. Calculations similar to the ones which led to (21) and (22) show that the summed contribution of such diagrams equals zero.

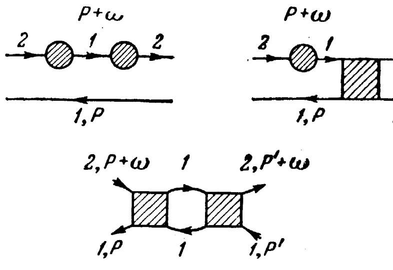

FIG. 5

3. LONGITUDINAL RELAXATION TIME

The "longitudinal" magnetic susceptibility is defined by the formula

$$\chi_l(\omega) = i\int_{-\infty}^{\infty} dt e^{i\omega t}\langle [M_z(t), M_z]\rangle\,\theta(t). \qquad (25)$$

The diagrams entering into $\chi_l(\omega)$ can be split into two groups: diagrams not containing a single cross section $G^R G^A$ and therefore not depending on ω, and diagrams having such cross sections. For the contribution of the second group of diagrams (χ_l') one can obtain a formula analogous to formula (10) for χ_t:

$$\chi_l'(\omega) = -\beta^2\frac{a^2 p_0^2}{4\pi^2 v}Q^2\left\{\frac{1}{2}\int d\varepsilon\left(\mathrm{th}\frac{\varepsilon+\omega}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\right)\frac{1}{\Omega_0(\varepsilon,\omega)}\right.$$
$$+\frac{1}{4}\iint d\varepsilon\, d\varepsilon'\left(\mathrm{th}\frac{\varepsilon+\omega}{2T} - \mathrm{th}\frac{\varepsilon}{2T}\right)$$
$$\left.\times\frac{1}{\Omega_0(\varepsilon,\omega)}\sigma^z_{\alpha\alpha}\mathcal{T}_{\alpha\beta,\beta\alpha}(\varepsilon,\varepsilon',\omega)\sigma^z_{\beta\beta}\frac{1}{\Omega_0(\varepsilon',\omega)}\right\},$$

$$\Omega_0(\varepsilon,\omega) = \omega - ia\,\mathrm{Im}\,[\Sigma^R\ \varepsilon+\omega) + \Sigma^R(\varepsilon)]. \qquad (26)$$

We show now that neglecting V we have $\mathcal{T} \sim (\omega + i\delta)^{-1}$ as $\omega \to 0$. Writing $\mathcal{T}$ in the form

$$\mathcal{T}_{\alpha\beta,\beta\alpha} = \mathcal{T}_1 + \sigma^z_{\alpha\alpha}\sigma^z_{\beta\beta}\mathcal{T}_2,$$

we obtain that $\sigma^z_{\alpha\alpha}\mathcal{T}_{\alpha\beta,\beta\alpha}\sigma^z_{\beta\beta} = \frac{1}{4}\mathcal{T}_2$.

For $\mathcal{T}_2$ one can write an equation which differs from (11) by the replacement of $\Omega(\epsilon,\omega)$ by $\Omega_0(\epsilon,\omega)$. This is connected with the fact that in contrast to χ_t in χ_l both lines in the cross section $G^R G^A$ have unique spin indices. Introducing the quantity $\varphi_0(\epsilon, \epsilon', \omega) = \mathcal{T}_2(\epsilon, \epsilon', \omega)/\Omega_0(\epsilon, \omega)$ we obtain for it the equation

$$\omega\varphi_0(\varepsilon, \varepsilon', \omega) = i\widetilde{\mathcal{T}}'(\varepsilon, \varepsilon') + \frac{\omega}{16T} C \int d\varepsilon'' \operatorname{ch}^{-2} \frac{\varepsilon''}{2T} \varphi_0(\varepsilon'', \varepsilon', \omega)$$
$$+ \frac{i}{4} \int d\varepsilon'' \widetilde{\mathcal{T}}(\varepsilon, \varepsilon') \varphi_0(\varepsilon'', \varepsilon', \omega)$$
$$+ 2ia \operatorname{Im} \Sigma^R(\varepsilon) \varphi_0(\varepsilon, \varepsilon', \omega), \qquad (27)$$

where the constant C is the same as in formula (12).

We are interested in values of ω comparable to T_1^{-1}, i.e., $\omega \ll \operatorname{Im} \Sigma \sim T^2/\mu$ and the more so $\omega \ll T$. This is taken into account in (27). As in the derivation of formula (15), one can obtain that

$$\frac{\omega}{2T} \int d\varepsilon \operatorname{ch}^{-2} \frac{\varepsilon}{2T} \varphi_0(\varepsilon, \varepsilon', \omega) = \frac{2}{T} \frac{1}{1 - C/4} \operatorname{ch}^{-2} \frac{\varepsilon'}{2T} \Omega_0(\varepsilon'),$$
$$\Omega_0(\varepsilon) \equiv \Omega_0(\varepsilon, 0) = -2ia \operatorname{Im} \Sigma^R(\varepsilon). \qquad (28)$$

Substituting this expression into (27) we see that the solution can be sought in the form $\varphi_0(\epsilon, \epsilon', \omega) = \varphi_1(\epsilon')/\omega$ not containing a dependence on the first argument ϵ, since such a substitution reduces the last two terms of the right hand part of (27) to zero due to (14a). $\varphi_1(\epsilon')$ is easily determined from (28). As a result we obtain the following expression for $\widetilde{\mathcal{T}}_2$:

$$\widetilde{\mathcal{T}}_2(\varepsilon, \varepsilon', \omega) = \frac{1}{1 - C/4} \frac{1}{T} \operatorname{ch}^{-2} \frac{\varepsilon'}{2T} \frac{\Omega_0(\varepsilon)\Omega_0(\varepsilon')}{\omega + i\delta}. \qquad (29)$$

Thus $\mathcal{T}_2 \sim \omega^{-1}$ as $\omega \to 0$. We note that (29) is obtained from (16) by means of the limit $\omega_0 \to 0$. Therefore all following calculations connected with the consideration of V shall differ from those carried out in the preceding section only in that ω_0 has to be taken equal to zero. Therefore the expression for the vertex part including the damping will differ from (29) by the replacement of $(\omega + i\delta)^{-1}$ by $(\omega + i/T_1)^{-1}$, where T_1 is defined by formula (4), which is obtained from (3) for $\omega_0 = 0$. Substituting the expression for $\mathcal{T}$ into (26) we find that $\chi_l' = \chi\omega(\omega + i/T_1)^{-1}$. Since in agreement with (25) the total susceptibility $\chi_l(\omega)$ without account of $V(T_1 \to \infty)$ equals zero, the contribution of the diagrams not containing cross sections $G^R G^A$ equals χ. (This can also be verified directly.) As a result we arrive at the formula (2) for $\chi_l(\omega)$.

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