RADIATIVE CAPTURE OF THE μ^- MESON BY A PROTON

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Submitted to JETP editor June 1, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1821-1825 (November, 1962)

Radiative capture of μ^- mesons by protons is considered anew. In distinction to Bernstein's paper [3], the present analysis of the contributions from induced pseudoscalar interaction [1,2] not only takes into account diagrams in which a photon is emitted by a μ^- meson, but also diagrams in which photons are emitted by protons and virtual π^+ mesons. If the pseudoscalar constant is assumed to be $g_P = +8g_A$, and the indicated diagrams are taken into account, the probability for radiative capture of a μ^- meson by a proton is about twice that computed by Bernstein [3]. A change in sign of the pseudoscalar constant changes the radiative capture probability by 2.5 times. A relation is obtained between μ^--meson radiative capture due to induced pseudoscalar interaction and the photoproduction of π^- mesons on neutrons ($n + \gamma \rightarrow \pi^- + p$) and radiative decay of the π^+ meson ($\pi^+ \rightarrow \mu^+ + \nu + \gamma$).

1. INTRODUCTION

THE β-decay and μ^--capture processes are described by matrix elements which in general contain four constants for the coupling between the nucleon and lepton fields [1,2]: g_A—axial constant, g_V—vector constant, g_M—weak-magnetism constant, and g_P—effective pseudoscalar constant. If we start from the hypothesis of universal interaction and conservation of the vector current, then the greatest interest from the point of view of theory and experiment apparently attaches to the effective pseudoscalar constant g_P.

The present paper is devoted to a clarification of the role of the pseudoscalar interaction in the radiative capture of a negative muon by a proton ($\mu^- + p \rightarrow n + \nu + \gamma$). The contribution of the pseudoscalar interaction in this process was taken into account in several investigations [3-5]. In all these, however, only some of the Feynman diagrams responsible for the photon emission were considered. Thus, Bernstein [3] and Primakoff [4] took into account only the diagram of Fig. 2a; Wolfenstein [5], on the other hand, apparently did not take into account the diagram of Fig. 2b. In the present paper, unlike those mentioned earlier [3-5], we consider all the diagrams shown in Figs. 1 and 2.

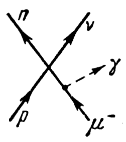
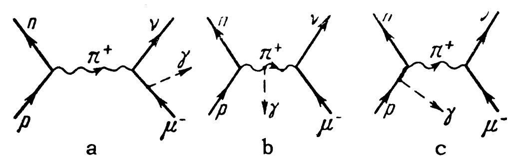


FIG. 1



a b c

FIG. 2

The main results of the present work is that in spite of the statements contained in [3] the contribution of the diagram of Fig. 2 to the total amplitude of the process is comparable with the contribution from the diagram of Fig. 1, corresponding to the V-λA interaction with account of the weak magnetism. The probability of radiative muon capture is therefore strongly dependent on the sign and magnitude of the pseudoscalar constant g_P (this result agrees with Wolfenstein's conclusion [5]).

2. PROBABILITY OF THE $\mu^- + p \rightarrow n + \nu + \gamma$ PROCESS. CIRCULAR γ-QUANTA POLARIZATION

The matrix element M of radiative μ^- capture is best represented in the form of a sum

$$M = M_1 + M_2. \tag{1}$$

Here M_1 is the matrix element corresponding to the Feynman diagram of Fig. 1, with constants g_V, g_A, and g_M determined in [6]:

$$M_1 = (2\pi)^4 ie\sqrt{4\pi}\,\{\bar{u}_\nu\gamma_\lambda\,[m_\mu - i\,(\hat{\mu} - \hat{k})]\,\hat{\varepsilon}u_\mu\}\,\Big\{g_V\,(\bar{u}_n\gamma_\lambda u_p)$$
$$\times - g_A\,(\bar{u}_n\gamma_\lambda\gamma_5 u_p) - \frac{ig_M}{2M}\,(\bar{u}_n\sigma_{\lambda\rho}\,(p - n)_\rho\,u_p)\Big\}. \tag{2}$$

Here $\sigma_{\lambda\rho} = (\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda)/2i$; p, n, and k are the 4-momenta of the proton, neutron, and photon, respectively; M and m_μ are the nucleon and muon masses; $\hat{\epsilon} = \gamma_i\epsilon^i$, ϵ^i is the 4-vector of photon polarization, and $e^2 = \alpha = 1/137$ ($\hbar = c = 1$).

The matrix element corresponding to the Feynman diagrams of Fig. 2, with pseudoscalar interaction, can be represented in the pole approximation in the form

$$M_2 = (2\pi)^4\, ie\sqrt{4\pi}\,\frac{m_\mu F(-m_\pi^2)\sqrt{4\pi}g}{(\pi_2^2 + m_\pi^2)\sqrt{2k}}$$

$$\times\Big[-\frac{i}{2m_\mu k}\{\bar{u}_\nu(1-\gamma_5)\,[m_\mu - i(\hat{\mu}-\hat{k})]\,\hat{\varepsilon}u_\mu\}\{\bar{u}_n\gamma_5 u_p\}$$

$$+\frac{i\varepsilon(\pi_1+\pi_2)}{\pi_1^2+m_\pi^2}\{\bar{u}_\nu(1-\gamma_5)\,u_\mu\}\{\bar{u}_n\gamma_5 u_p\} + \frac{i}{2Mk}\,\frac{\pi_2^2+m_\pi^2}{\pi_1^2+m_\pi^2}$$

$$\times\{\bar{u}_\nu(1-\gamma_5)\,u_\mu\}\{\bar{u}_n\gamma_5\,[M - i(\hat{p}-\hat{k})]\,\hat{\varepsilon}u_p\}\Big]. \quad (3)$$

Here $F(-m_\pi^2)$ is the form factor of the π^+-meson decay ($\pi^+ \to \mu^+ + \nu$), taken on the mass surface, $g^2 = 14.5$, $\pi_1 = \nu - \mu$, and $\pi_2 = p - n$. It is easy to see that the matrix element M_2 represented in this form is clearly gauge-invariant, i.e., it vanishes under the substitution $\epsilon \to k$. After making the usual calculations, we obtain the following expression for the bremsstrahlung spectrum in the radiative capture of a μ^- meson by a proton:

$$N(x)\,dx = [N_1(x) + N_2(x)]\,(1-x)^2\,x\,dx, \quad (4)$$

where

$$N_1(x) = \frac{\alpha\mathscr{E}^4|g_P|^2}{8\pi^3 a_\mu^3 M^2}\Big\{\frac{1}{3}(3-8x+8x^2) + \frac{1}{(1-\lambda x)^2}$$

$$\times\Big[2 + \frac{2\mathscr{E}x}{M} + \frac{8\mathscr{E}^4(1-x)^2[(1-x)^2+x^2]}{3(m_\mu^2+m_\pi^2)^2} - \frac{8\mathscr{E}^2(1-x)^2}{3(m_\mu^2+m_\pi^2)}\Big]$$

$$+\frac{\alpha\mathscr{E}^4}{8\pi^3 a_\mu^3 M m_\mu}\,\frac{1}{1-\lambda x}$$

$$\times\Big[2(g_A g_P^* + g_P g_A^*)\Big(1 + \frac{m_\mu}{2M} - \frac{2}{3}\,\frac{m_\mu\mathscr{E}(1-x)}{m_\mu^2+m_\pi^2}\Big)$$

$$+(g_V g_P^* + g_P g_V^*)\,\frac{m_\mu}{M}(1+\mu_p-\mu_n)\Big]\Big\}, \quad (5)$$

$$N_2(x) = \frac{\alpha\mathscr{E}^4}{2\pi^3 a_\mu^3 m_\mu^2}\Big\{|g_V|^2 + 3|g_A|^2 + \frac{\mathscr{E}}{M}(|g_V|^2 + |g_A|^2)$$

$$-\frac{2}{3}x\,\frac{\mathscr{E}}{M}(|g_V|^2 - |g_A|^2)$$

$$+2\,\frac{\mathscr{E}}{M}(1+\mu_p-\mu_n)(g_V g_A^* + g_A g_V^*)\Big(x - \frac{1}{2}\Big)\Big\}. \quad (6)$$

Here x is the photon energy in units of its maximum energy ($k = \mathscr{E}x$, $\mathscr{E}$ is the maximum photon energy); $\mathscr{E} \cong m_\mu(1 + (E_n - M)/m_\mu) \cong m_\mu$; a_μ is the Bohr radius of the μ^--meson K-orbit in the hydrogen atom, and $\lambda = 2m_\mu m_\pi/(m_\mu^2 + m_\pi^2) = 0.73$.

For the pseudoscalar interaction constant we have

$$g_P = -\frac{g\sqrt{2}m_\mu F(-m_\pi^2)}{m_\mu^2+m_\pi^2}\sqrt{4\pi} = +8g_A, \quad (7)$$

if we put [1]

$$F(-m_\pi^2) = -m_\pi g_V. \quad (8)$$

The expression for the degree of circular polarization of the photons has the form

$$\beta(x) = 1 - \frac{2N_L(x)}{N_1(x)+N_2(x)}, \quad (9)$$

where

$$N_L(x) = \frac{\alpha\mathscr{E}^4|g_P|^2}{8\pi^3 a_\mu^3 M^2}\Big\{\frac{1}{3}(3-8x+8x^2) + \frac{1}{(1-\lambda x)^2}$$

$$\times\Big|\Big[1 + \frac{\mathscr{E}x}{M} + \frac{4\mathscr{E}^4(1-x)^2[(1-x)^2+x^2]}{3(m_\mu^2+m_\pi^2)^2}\Big] - \frac{4\mathscr{E}^2(1-x)^2}{3(m_\mu^2+m_\pi^2)}\Big|\Big\}.$$

3. DISCUSSION OF RESULTS

Figure 3 shows the spectra of the photons emitted in radiative μ^- capture. Curve A represents the spectrum of the photons corresponding to diagram 1 only. Curves B and C correspond to an account of all the diagrams of Figs. 1 and 2, with $g_P = \pm 8g_A$ for curves B and C respectively.

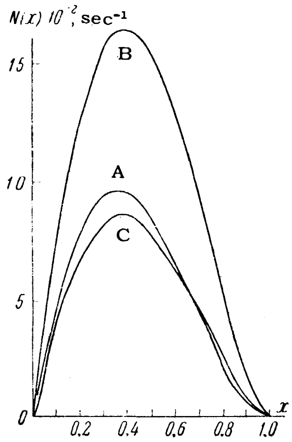


FIG. 3

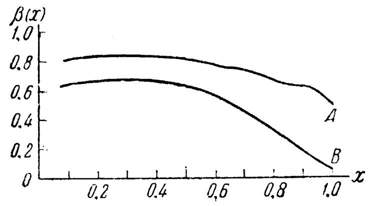


FIG. 4

Figure 4 shows the photon circular-polarization curves for the two signs of the pseudoscalar-interaction constant (curve A corresponds to $g_P = +8g_A$, and for curve B we have $g_P = -8g_A$).

As can be seen from Fig. 4, the presence of an induced pseudoscalar interaction can strongly change the magnitude of the circular polarization near the end of the photon spectrum [$\beta(1)$ changes by a factor of 5 when the sign of g_P is reversed].

It is clear from these figures that an account of diagrams b and c on Fig. 2 nearly doubles the probability of radiative μ^- capture compared with the results of [3], where these diagrams were not taken into account. The probability of radiative μ^- capture without account of diagrams b and c is equal to 5.7×10^{-2} sec^{-1}, whereas the value with account of these diagrams 10.2×10^{-2} sec^{-1}. In addition, it is seen from the curves of Fig. 3 that reversal of the sign of the pseudoscalar-interaction constant (i.e., if we take $g_P = -8g_A$) decreases the probability W of radiative μ^- capture by approximately 2.5 times ($W = 10.2 \times 10^{-2}$ sec^{-1} if $g_P = +8g_A$ and $W = 4.2 \times 10^{-2}$ sec^{-1} if $g_P = -8g_A$). It must also be noted that W depends strongly on the absolute value of the constant g_P [see formulas (5) and (6)]. These conclusions coincide with those of Wolfenstein[5]. We note that the contribution of the diagram 2b, which was not accounted for in [5], to the total probability amounts to $\sim 20\%$.

Thus, measurement of the total probability of the radiative capture of a μ^- meson by a proton leads to certain conclusions regarding the magnitude of g_P, provided we fix its sign, which at present is apparently known ($g_P/g_A > 0$, see [7]). It must be emphasized that an experimental determination of this constant is most important both as a check on the accuracy of the pole approximation used in its calculation[1,2] and as a check on our notions regarding the π-decay mechanism (the sign of g_P/g_A). We note also that the matrix element M_2 can be expressed in the pole approximation in terms of the form factors $F_1, \ldots F_4$ of the radiative decay of the π^+ meson ($\pi^+ \to \mu^+ + \nu + \gamma$) and the form factors $f_1, \ldots, f_4$ of the photoproduction of a π^- meson on a neutron ($n + \gamma \to p + \pi^-$):

$$M_2 = (2\pi)^4 i \frac{1}{\sqrt{2k}} \Big\{ \frac{g(\bar{u}_n \gamma_5 u_p)}{\pi_2^2 + m_\pi^2} [F_1 \{\bar{u}_\nu (1 - \gamma_5) \hat{\varepsilon} u_\mu\} + (\pi_1 \varepsilon) F_2 \{\bar{u}_\nu (1 - \gamma_5) \hat{k} u_\mu\} + (\pi_1 \varepsilon) F_3 \{\bar{u}_\nu (1 - \gamma_5) u_\mu\} + F_4 \{\bar{u}_\nu (1 - \gamma_5) \hat{k}\hat{\varepsilon} u_\mu\}] + \frac{F(-m_\pi^2)}{\pi_1^2 + m_\pi^2} \{\bar{u}_\nu (1 - \gamma_5) u_\mu\} [f_1 \{\bar{u}_n \gamma_5 \hat{\varepsilon} u_p\} + (\pi_2 \varepsilon) f_2 \{\bar{u}_n \gamma_5 \hat{k} u_p\} + (\pi_2 \varepsilon) f_3 \{\bar{u}_n \gamma_5 u_p\} + f_4 \{\bar{u}_n \gamma_5 \hat{k} \hat{\varepsilon} u_p\}]\Big\}. \quad (10)$$

The number of form factors F and f cannot be decreased, from general considerations (relativistic invariance, gauge invariance, weakness of electromagnetic interaction).

In the pole approximation (Fig. 5) the form factors F are functions of the invariance $t_1 = (\nu - \mu)^2$ and $u_1 = (k - \mu)^2$, while the form factors f are functions of the variables $t_2 = (p - n)^2$ and $u^2 = (k - p)^2$.

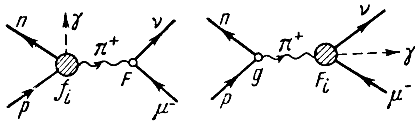


FIG. 5

At the present time the experimental data on the photoproduction of pions and radiative decay of π^+ meson are insufficient for a determination of the form factors F and f.

In perturbation theory [formula (3)] we have

$$F_1 = F_2 = f_1 = f_2 = 0, \quad F_3 = \frac{4\pi i e m_\mu F(-m_\pi^2)}{t_1 + m_\pi^2}, \quad f_3 = 4\pi i e m_\mu,$$

$$F_4 = \frac{4\pi e m_\mu F(-m_\pi^2)}{u_1 + m_\mu^2}, \qquad f_4 = -\frac{4\pi e m_\mu}{u_2 + M^2}(t_2 + m_\pi^2).$$

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Translated by J. G. Adashko