EQUATIONS OF MOTION FOR MAGNETIZATION IN DEFORMED ANISOTROPIC MEDIA

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Submitted to JETP editor April 29, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 43, 2128-2135 (December, 1962)

Two kinds of equations of motion for the magnetization in deformable anisotropic media are established by means of the method of the thermodynamics of nonequilibrium processes. Under certain assumptions and approximations the more general equations (16) and (23) obtained transform into the equations of Landau and Lifshitz, Bloch, Gilbert, and others.

MANY works, of which a survey is given in [1], for example, are devoted to the derivation of various kinds of equations of motion for the magnetization in magnetic media. In deriving these equations, however, certain model representations are used and some restrictions or approximations are imposed, for which the criterion is not always even clear. We establish here certain more general equations for the rate of change of the magnetization in deformable anisotropic media.

With account of (1), (6), (24), and (17) as well as the equations of state (25) or (26), the obtained equations of motion (16) or (23) (which go over into the known equations of Landau and Lifshitz, Bloch, Gilbert, etc. under certain assumptions) can be used to solve such problems as the propagation and absorption of magneto-elastic waves, the resonance line width, resonance conditions near the Curie point, etc., in which the use of existing less general equations can (and indeed does) lead to failure to take into account a number of singularities of the phenomena under consideration. These equations are established by the method of nonequilibrium thermodynamics, which have already been used under less general assumptions [2-4].

1. Let us isolate in the system under investigation a magnetic subsystem characterized by the magnetization $\mathbf{M}$ or the associated density of the mechanical moment $\mathbf{J}$:

$$\mathbf{M} = g\mathbf{J}, \tag{1}$$

where g is a non-symmetric tensor of the second rank in the general case.

We consider processes in which the deviation from equilibrium occurs at constant temperature T of the magnetic subsystem and at constant magnetic field intensity $\mathbf{H}$. Under these assumptions and taking account of the relations presented in [5], we obtain the following expression for the rate of change of the entropy:

$$\dot{S} = -\int \frac{\rho}{T}\frac{d\Phi}{dt}\,d\mathbf{r} + \oint s\dot{u}\,d\Sigma, \tag{2}$$

where $d\Phi/dt$ is the material time derivative of the specific thermodynamic potential, ρ the density, and S and s the total entropy and the entropy density of the system, respectively.

When the media are deformed, volume elements of the medium experience rotations ω along with deformations ϵ_{ij} and displacements $\mathbf{u}$.

From the viewpoint of the thermodynamics of nonequilibrium processes, the change in any vector or tensor quantity with respect to the medium is important. Consequently, let us put

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial \mathbf{I}}\frac{D\mathbf{I}}{Dt} + \frac{\partial\Phi}{\partial(\partial I_i/\partial x_j)}\frac{D}{Dt}\left(\frac{\partial I_i}{\partial x_j}\right) + \frac{\partial\Phi}{\partial\varepsilon_{ij}}\frac{D\varepsilon_{ij}}{Dt}, \tag{3}$$

where the derivative D/Dt defines the rate of some quantity in the coordinate system moving and rotating together with the volume element of the medium. Here, the specific thermodynamic potential should also be written in the coordinate system moving and rotating together with the volume element.

Let us take into account that

$$\Phi = F - \mathbf{mH}. \tag{4}$$

Here $\mathbf{I}$ and $\mathbf{m}$ are the specific values of the magnetic and mechanical moments, and F the specific free energy written in a coordinate system rotating together with the volume element. This means that the gyromagnetic free energy [6] must be taken as one of the components of this energy:

$$F_{\text{gm}} = -\lambda_{ij}^{\text{gm}}\,\omega_i m_j. \tag{5}$$

The form of the remaining components is defined by the kind of magnetic medium. For example, in the case of ferromagnets it is necessary to take account of the following kinds of free energy: 1) the

exchange energy, which depends on the modulus of I and determines mainly the very existence of the spontaneous magnetization; 2) the exchange energy associated with the inhomogeneity of the magnetization; 3) the energy of crystallographic magnetic anisotropy; 4) the magneto-elastic energy; in the case of paramagnets only the components $\frac{1}{2}\alpha_{ij}I_iI_j$, etc., can be taken into account.

Let us introduce the notation

$$\mathbf{H}^* = -\partial\Phi/\partial\mathbf{I}. \tag{6}$$

The quantity H* is associated with the effective field

$$\mathbf{H}^{\text{eff}} = -\partial\Phi/\partial\mathbf{m}, \tag{7}$$

according to (1), by the relation

$$\mathbf{H}^* = \tilde{g}\mathbf{H}^{\text{eff}}, \tag{8}$$

where the sign ~ denotes the transpose. When taking account of the dependences of the thermodynamic potential on the magnetization gradients in (6) and (7), the variational derivatives of the total thermodynamic potential with respect to **I** or **m** should be taken in place of the derivatives of the specific thermodynamic potential with respect to **I** (or **m**).

Taking account of (3) and (6), expressions for the thermodynamic forces and fluxes can be obtained on the basis of (2).

2. Let us first express the thermodynamic fluxes in terms of forces by neglecting the magneto-elastic kinetic phenomena:

$$\rho\frac{DI_i}{Dt} = \int L_{ij}(\mathbf{I}, \mathbf{r}, \mathbf{r}')\frac{1}{T}H_j^*d\mathbf{r}'. \tag{9}$$

The Onsager [2] relations:

$$L_{ij}(\mathbf{I}, \mathbf{r}, \mathbf{r}') = L_{ji}(-\mathbf{I}, \mathbf{r}', \mathbf{r}) \tag{10}$$

must be satisfied for the kinetic coefficients L_{ij}.

Furthermore, let us assumed that the coefficients L_{ij} are even functions of the difference $\mathbf{r}-\mathbf{r}'$ (as can be done for the case of a homogeneous medium in the ground state) and let us expand the tensor L_{ij} into symmetric and antisymmetric components, $L_{ij} = L_{ij}^{s} + L_{ij}^{a.s.}$. It follows from (10) that the symmetric tensor L_{ij}^{s} is an even function and the antisymmetric tensor $L_{i,j}^{s}$ is an odd function of **I**. Hence, $L_{ij}^{a.s.}$ can be represented as $L_{ij}^{a.s.} = -e_{ijk}L_{kl}L_l$, where e_{ijk} is an absolutely anti-symmetric unit tensor of third rank. Then Eq. (9) becomes

$$\rho\frac{DI_i}{Dt} = \int\frac{1}{T}[L^*\mathbf{J}, \mathbf{H}^*]_i d\mathbf{r}' + \int L_{ij}^{s}\frac{1}{T}H_j^*d\mathbf{r}'. \tag{11}*$$

*$[\mathbf{J}, \mathbf{H}] = \mathbf{J} \times \mathbf{H}$.

It follows from (11) that the rate of change of the vector **I** (and, therefore, of the magnetization, too) can be defined at a point **r** by the values of **H*** (and, hence, also of the effective fields H^{eff}) not only at the point **r** but also in neighboring points.

For a homogeneous distribution of **I**, **H***, and T we have in place of (11)

$$\rho\frac{D\mathbf{I}}{Dt} = \frac{1}{T}[\lambda^*\mathbf{J}, \mathbf{H}^*] + \lambda\mathbf{H}^*,$$

$$\lambda_{ij}^* = \int L_{ij}^*d\mathbf{r}', \qquad \lambda_{ij} = \int L_{ij}^{s}T^{-1}d\mathbf{r}'. \tag{12}$$

Equation (12) can also be used for the case of very weak inhomogeneity of **I** and **H*** when the tensor constants L_{ij}^* and L_{ij}^{s} can be represented as:

$$L_{ij}^* = \lambda_{ij}^*\delta(\mathbf{r}-\mathbf{r}'); \quad L_{ij}^{s} = T\lambda_{ij}\delta(\mathbf{r}-\mathbf{r}').$$

The case of weak inhomogeneity is realized when the dimensions characterizing the macroscopic inhomogeneity of **I** and **H*** exceed the microscopic characteristic dimensions (mean free path, etc).

The equation of motion (12) takes account of absorption and consequently in the absence of absorption this equation must transform into the mechanical equation of motion. If it is assumed that the last term in (12) is responsible for the energy dissipation and all the components of the tensor λ_{ij} are set equal to zero, then Eq. (12) becomes

$$\rho\frac{D\mathbf{I}}{Dt} = \frac{1}{T}[\lambda^*\mathbf{J}, \mathbf{H}^*]. \tag{13}$$

In order that this equation might be treated as the mechanics equation for the rate of change of the mechanical moment **I** under the effect of a force couple due to certain effective "fields" **H***, it is necessary to put

$$\lambda_{ij}^* = T\delta_{ij}. \tag{14}$$

The situation here is apparently analogous to that which holds in the derivation of the dissipation function presented in [5]. Namely, Landau and Lifshitz note that the mechanics equations in the Hamilton form, one of which is substantially equation (13), are in formal correspondence with the principle of symmetry of the kinetic coefficients. However, the "kinetic coefficients" in the mechanics equations degenerate into the scalar quantity defined in (14).

Apparently the tensor L_{ij}^* is also defined by an expression of the form (15)

$$L_{ij}^* = T\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \tag{15}$$

If it is now assumed that the tensor L_{ij}^{s} is not exactly a δ-function of $\mathbf{r}-\mathbf{r}'$, but is a certain delta-like function which drops sharply with increasing $\mathbf{r}-\mathbf{r}'$, (15) is taken into account, **H*** is expanded

in a series of $\mathbf{r}-\mathbf{r}'$ up to quadratic terms and the evenness of L^s_{ij} relative to $\mathbf{r}-\mathbf{r}'$ is taken into account, then, for a slightly inhomogeneous distribution of I, H*, and T, the following equation of motion for an unbounded medium can finally be obtained in place of (11):

$$\rho \frac{DI_i}{Dt} = [\mathbf{JH}^*]_i + \lambda_{ij} H_j^* + D_{ijkl} \frac{\partial^2 H_j^*}{\partial x_k \partial x_l}, \quad (16)$$

$$D_{ijkl} = \frac{1}{2}\int (x_k - x_k')(x_l - x_l')\frac{1}{T} L_{ij}^s \, d\mathbf{r}',$$

$$\rho \frac{D\mathbf{I}}{Dt} = \rho \frac{d\mathbf{I}}{dt} - \rho[\dot{\boldsymbol{\omega}}\mathbf{I}] = \frac{\partial \mathbf{J}}{\partial t} + \frac{\partial}{\partial x_k}(\mathbf{J}\dot{u}_k) - [\dot{\boldsymbol{\omega}}\mathbf{J}], \quad (17)$$

and the derivatives $\partial/\partial t$ and d/dt define, respectively, the local velocity and the rate of change of some quantity in a coordinate system moving forward together with the volume element. The last term in (16) defines the diffusion processes.

3. Now, neglecting the magneto-elastic kinetic phenomena, let us express the thermodynamic forces in terms of the fluxes:

$$H_i^* = \int B_{ij}(\mathbf{I}, \mathbf{r}, \mathbf{r}') \frac{\rho}{T} \frac{DI_j}{Dt} d\mathbf{r}'. \quad (18)$$

Taking into account that Onsager relations analogous to (10) hold for the kinetic coefficients B_{ij}, we obtain in place of (18) by expanding the tensor B_{ij} into symmetric B^s_{ij} and anti-symmetric $B^{a.s.}_{ij}$ terms, putting $B^{a.s.}_{ij} = e_{ijk}B^*_k J/J^2$,

$$H_i^* = \int \frac{\rho}{TJ^2}\left[\frac{D\mathbf{I}}{Dt}, B^*\mathbf{J}\right]_i d\mathbf{r}' + \int B^s_{ij} \frac{\rho}{T} \frac{DI_j}{Dt} d\mathbf{r}'. \quad (19)$$

For a homogeneous distribution we have

$$H_i^* = \frac{\rho}{TJ^2}\left[\frac{D\mathbf{I}}{Dt}, \beta^*\mathbf{J}\right]_i + \beta_{ij}\rho \frac{DI_j}{Dt}. \quad (20)$$

From (20) it is possible to obtain the mechanics equation of motion for the perpendicular component of DI/Dt (relative to J)

$$\rho (DI/Dt)_\perp = [\mathbf{JH}^*],$$

in which the right side can be integrated as a couple acting on the mechanical moment if the last component in (20) is set equal to zero and the tensor β^*_{ij} equal to

$$\beta^*_{ij} = T\delta_{ij}. \quad (21)$$

The tensor B^*_{ij} apparently satisfies also the relation

$$B^*_{ij} = T\delta_{ij}\delta(\mathbf{r} - \mathbf{r}'). \quad (22)$$

Just as before, assuming that the tensor B^s_{ij} is a delta-like function of $\mathbf{r}-\mathbf{r}'$ which decreases rapidly with increasing $\mathbf{r}-\mathbf{r}'$ we obtain in place of (19) for a weakly inhomogeneous distribution of I, H*, and T, taking (22) into account,

$$H_i^* = \frac{\rho}{J^2}\left[\frac{D\mathbf{I}}{Dt}, \mathbf{J}\right]_i + \beta_{ij}\rho \frac{DI_j}{Dt} + D'_{ijkl}\rho \frac{\partial^2}{\partial x_k \partial x_l}\left(\frac{DI_j}{Dt}\right), \quad (23)$$

4. Thus, Eqs. (16) and (23), with (1), (6), and (17) taken into account, define the equations of motion for magnetization with a slightly inhomogeneous distribution of I, H*, and T.

The form of the tensor constants in these equations is determined by the symmetry of the medium. Moreover, the possible dependence of these constants on the magnetization should be taken into account. The essential dependence of the tensors λ_{ij} and β_{ij} on the magnetization follows here from the experimental results, for in paramagnets it determines the presence of the longitudinal and lateral relaxation times, and in ferromagnets possessing cubical symmetry it is impossible to explain completely the observed anisotropy of the resonance line width without it.[1] This circumstance can be taken into account by expanding these constants in a series in the magnetization, i.e., by putting for example

$$\lambda_{ij} = \lambda^0_{ij} + \lambda_{ijkl} M_k M_l + \lambda_{ijklmn} M_k M_l M_m M_n + \ldots. \quad (24)$$

From the Onsager relations and from the properties of the expansion (24) it follows that all the tensor components λ^0_{ij}, λ_{ijk}, λ_{ijklmn}, ... with the first pair of subscripts transposed and with subscripts following the first pair transposed are equal.

As will be shown below, Eq. (16) is applicable to the case of weak absorption in the sense indicated below. However, (16) may prove to be more convenient than (23) in the case of weak absorption.

Setting the rate of change of the magnetization in the coordinate system moving and rotating together with the volume element, i.e., DI/Dt, equal to zero in (19), we obtain an equation of state for the equilibrium value of the magnetization

$$\mathbf{H}^* = 0, \quad (25)$$

which corresponds, as is seen from (6), to the condition of minimum thermodynamic potential.

Taking account of (4), we have according to (25) for a homogeneous initial state

$$\mathbf{H} = \partial F / \partial \mathbf{m}. \quad (26)$$

Condition (25) for the equilibrium value of the magnetization also satisfies Eqs. (11), (12), and (16).

5. In order to disclose the correspondence between the equations of motion (16) or (23) and the equations of motion customarily used for the magnetization, and to establish the region of their applicability, let us investigate the case of an isotropic medium.

If it is assumed that the tensor g_{ij} is independent of the magnetization and an expansion such as (24) is valid for the tensors λ_{ij} and β_{ij} then $g_{ij} = g_0\delta_{ij}$, $\lambda^0_{ij} = \lambda^0\delta_{ij}$, and $\beta^0_{ij} = \beta^0\delta_{ij}$, while the tensors λ_{ijkl} and β_{ijkl} have only two independent components of the type λ_{1111} and λ_{1122}, where $\lambda_{1212} = \lambda_{1313} = \ldots = (\lambda_{1111} - \lambda_{1122})/2$.

When the term responsible for the diffusion [1)] is neglected, Eqs. (16) and (23) take the form

$$\rho D\mathbf{m}/Dt = g_0[\mathbf{M}, \mathbf{H}^{\text{eff}}] + \lambda_\perp \mathbf{H}^{\text{eff}}_\perp + (\lambda_\parallel - \lambda_\perp)\mathbf{M}(\mathbf{M}\mathbf{H}^{\text{eff}}/M^2) \quad (27)$$

or

$$\rho(D\mathbf{m}/Dt)_\perp = g_0[\mathbf{M}, \mathbf{H}^{\text{eff}}] + \lambda_\perp \mathbf{H}^{\text{eff}}_\perp, \quad (28)$$

$$\rho(D\mathbf{m}/Dt)_\parallel = \lambda_\parallel \mathbf{H}^{\text{eff}}_\parallel \quad (29)$$

and

$$g_0\mathbf{H}^{\text{eff}} = \frac{\rho}{M^2}\left[\frac{D\mathbf{m}}{Dt}, \mathbf{M}\right] + \beta_\perp\rho\frac{D\mathbf{m}}{Dt} + (\beta_\parallel - \beta_\perp)\rho\mathbf{M}\left(\frac{D\mathbf{M}}{Dt}\mathbf{M}\right)/M^2, \quad (30)$$

or

$$\rho(D\mathbf{m}/Dt)_\perp = [\mathbf{M}, g_0\mathbf{H}^{\text{eff}} - \beta_\perp\rho D\mathbf{m}/Dt] \quad (31)$$

and

$$0 = g_0\mathbf{H}^{\text{eff}}_\parallel - \beta_\parallel\rho(D\mathbf{m}/Dt)_\parallel, \quad (32)$$

where

$$\lambda_\perp = g_0^2(\lambda^0 + \lambda_{1122}M^2 + \ldots);$$

$$\beta_\perp = (\beta^0 + \beta_{1122}M^2 + \ldots)/g_0;$$

$$\lambda_\parallel = g_0^2(\lambda^0 + \lambda_{1111}M^2 + \ldots);$$

$$\beta_\parallel = (\beta^0 + \beta_{1111}M^2 + \ldots)/g_0,$$

the vectors provided with the subscripts $\parallel$ and $\perp$ and are understood to be quantities of the type

$$\mathbf{A}_\parallel = \mathbf{M}(\mathbf{A}\mathbf{M})/M^2; \quad \mathbf{A}_\perp = [\mathbf{M}[\mathbf{A}\mathbf{M}]]/M^2. \quad (33)$$

If it is assumed now that the modulus of the magnetization vector is constant, then (29) and (32) transform into identities and (28) and (31) for the undeformed media go over into the Landau and Lifshitz and Gilbert equations [1,9] [2)].

In deformed media under the same condition and for $\lambda_0^{gm} = g_0^{-1}$ Eq. (28) with (17) taken into account becomes

$$\partial\mathbf{M}/\partial t + \partial(\mathbf{M}\dot{u}_k)/\partial x_k = g_0[\mathbf{M}\mathbf{H}^{\text{eff}}] + \lambda_\perp\mathbf{H}^{\text{eff}}_\perp, \quad (34)$$

where $\mathbf{H}^{\text{eff}} = \mathbf{H}^{\text{eff}'} + \lambda_0^{gm}\dot{\boldsymbol{\omega}}$, and $\mathbf{H}^{\text{eff}'}$ is defined by (7) if the gyromagnetic energy is not taken into account in Φ. This equation differs from the equation used by Akhiezer et al [10] in just the last term, in which the gyromagnetic phenomena have now been taken into account.

The Landau and Lifshitz equation is valid for the weak absorption case—weak in the sense that the second term of (28), which determines the transverse component of the rate of change of the magnetization, must be less than the first. No other restrictions are ordinarily imposed on the Gilbert equation. Consequently, it can be expected that the more general equation (16) is also applicable only for the case of weak absorption in the above-mentioned sense, and Eq. (23) is applicable for any absorption to a certain degree. This latter can easily be illustrated by an example examining small changes in the magnetization.

Let us analyze processes under which the magnetization $\mathbf{M} = \mathbf{M}_0 + \boldsymbol{\mu}$ changes slightly ($\mu \ll M_0$) near its initial, uniform, equilibrium value $\mathbf{M}_0$ determined by (25) or (26). Then, expanding the free energy near its equilibrium value in a series in the magnetization, we obtain instead of (28) and (29) for the case of a non-deformed medium

$$\dot{\boldsymbol{\mu}}_\perp = \frac{1}{\gamma_\perp\tau_L^0}\left[\frac{\mathbf{M}_0}{M_0}, \mathbf{h}^{\text{eff}}\right] + \frac{1}{\gamma_\perp\tau_\perp}\mathbf{h}^{\text{eff}}_\perp, \quad (35)$$

$$\dot{\mu}_\parallel = \mathbf{h}^{\text{eff}}_\parallel/\gamma_\parallel\tau_\parallel, \quad (36)$$

where $\gamma_{||}$ and $\gamma_\perp$ are components of the tensor of the inverse static susceptibility γ_{ij}; $h_i^{\text{eff}} = h_i - \gamma_{ij}\mu_j$; $\tau_{||}$ and $\tau_\perp$ are components of the relaxation-time tensor τ_{ij}:

$$\tau_\perp = 1/\lambda_\perp\gamma_\perp; \quad \tau_\parallel = 1/\lambda_\parallel\gamma_\parallel, \quad (37)$$

and τ_L^0 is the period of Larmor precession

$$\tau_L^0 = 1/g_0\gamma_\perp M_0. \quad (38)$$

For free oscillations of the vector $\mathbf{M}$ (for $M_0 = M_z^0$) we obtain the following solutions of (35) and (36)

$$\mu^+ = \mu_x + i\mu_y = \mu_0^+ \exp(-it/\tau_L - t/\tau_\perp), \quad (39)$$

$$\mu_z = \mu_z^0 \exp(-t/\tau_\parallel), \quad (40)$$

where

$$\tau_L = \tau_L^0, \quad (41)$$

from which it is seen that all the components of the relaxation time tensor τ are defined in terms of the components of the tensor λ in the same manner, and that the precession period τ_L equals τ_L^0 and is independent of the absorption.

Equations (31) and (32) reduce to

$$\mathbf{h}^{\text{eff}}_\perp = \tau_L^0\gamma_\perp[\dot{\boldsymbol{\mu}}, \mathbf{M}_0/M_0] + \tau'_\perp\gamma_\perp\dot{\boldsymbol{\mu}}_\perp, \quad (42)$$

$$\mathbf{h}^{\text{eff}}_\parallel = \tau_\parallel\gamma_\parallel\dot{\boldsymbol{\mu}}_\parallel; \quad (43)$$

[1)] A survey of papers devoted to the account of diffusion is given in [7]; see also [8].

[2)] It is possible to go over from (27) to an equation of the Bloch type [1] by introducing a quasi-static susceptibility and assuming it to be small.

$$\tau'_\perp = \beta_\perp/g_0\gamma_\perp;\ \tau_\parallel = \beta_\parallel/g_0\gamma_\parallel. \quad (44)$$

For the free oscillations of the vector **M**, we again obtain the solutions (39) and (40), where

$$\tau_\perp = \tau_L^0\,[1 + (\tau'_\perp/\tau_L^0)^2];\ \tau_\perp = \tau_L^{02}\,[1 + (\tau'_\perp/\tau_L^0)^2]/\tau'_\perp. \quad (45)$$

It is seen from (44) and (45) that all the components of the relaxation-time tensor τ are expressed in terms of the components of the tensor α in the identical manner (i.e., they are proportional to one another) only for the case $\tau'_\perp/\tau_L^0 = \beta_\perp M_0 \gg 1$, i.e., when the second term in (42) exceeds the first. This corresponds to high damping since in this case the transverse relaxation time $\tau_\perp \approx \tau'_\perp$ is less than the period of precession $\tau_L \approx \tau_\perp(\tau'_\perp/\tau_L^0)$. If $\tau'_\perp/\tau_L^0 \ll 1$, then $\tau_L = \tau_L^0$ (weak absorption), but according to (44) and (45) the longitudinal relaxation time $\tau_\parallel$ is proportional to $\beta_\parallel$, whereas the transverse relaxation time $\tau_\perp$ is inversely proportional to $\beta_\perp$. This circumstance can be still more inconvenient for the case of an anisotropic medium, when the equations for the longitudinal and transverse components of the rate of change of the magnetization do not separate in the general case.

6. By having the equations of magnetization in the form of (16) or (23) at our disposal, definite judgments and conclusions can be made on the approximations and on criteria advanced in certain earlier works. Thus, for example, the equation of motion used in [11] for investigations of the anisotropy of the resonance frequency due to the anisotropy of the magneto-mechanical ratio is obtained from (23) by neglecting terms responsible for diffusion ($D_{ijkl} = 0$) and absorption ($\beta_{ij} = 0$). It is seen, with the isotropic medium as the example, that by assuming $\beta_{ij} = 0$ we allow, according to (44) and (45), the longitudinal relaxation time $\tau_\parallel$ to approach zero and the transverse relaxation time $\tau_\perp$ to approach infinity. In turn, this means that, according to (32) or (43), we obtain the equation of state for the longitudinal component in the change in magnetization and, according to (31) and (42), the mechanical equation of motion without damping for the transverse component. The examined case, when $\tau_\parallel \ll \tau_\perp$ is apparently realized quite often when the crystallographic magnetic anisotropy is sufficiently large [1].

In the interpretation of the formulas obtained in [3] for the angle of rotation of the plane of polarization of elastic waves in magnetically polarized media, the conclusion was drawn that this rotation can be due to gyromagnetic phenomena. However, no analysis was made there of this phenomenon and, in particular, no analysis of the consequences of the fact that the dynamic equations of "state" used therein must be written in a coordinate system bound to the volume element.

This analysis is now carried out easily on the basis of (16) or (23). Let us set $\beta_{ij} = 0$ and $D_{ijkl} = 0$ in (23) and let us isolate from $\mathbf{H}^*$ the component $H^* = H^{*\prime} + \lambda^{gm}_{ik} g_{km}\dot{\omega}_i$, which depends on $\dot{\omega}$ and is determined, according to (4), (5), and (6), by the derivative of the gyromagnetic energy with respect to **I**. Then Eq. (23) reduces to the equation of mechanics for the perpendicular component of the rate of change of the magnetization

$$\rho\,(d\mathbf{I}/dt)_\perp = [\mathbf{J}, \tilde{g}\mathbf{H}^{\text{eff}}] \quad (46)$$

in a non-rotating coordinate system in which $\dot{\omega}$ does not enter, under the condition that the tensor λ^{gm}_{ik} is the inverse of the tensor g_{km}, i.e., it is the tensor of the magneto-mechanical ratio.

The absence of $\dot{\omega}$ in (46) means the absence of a relation between the elastic waves and the precession of a magnetic moment (which can cause rotation of the plane of polarization) due to the gyromagnetic phenomena. However, a difference in the magnitudes of the spectroscopic splitting factor and the magneto-mechanical ratio is observed in experiment, and this difference can apparently cause the rotation of the plane of polarization associated with the gyromagnetic phenomena.

In conclusion, I am grateful to S. V. Vonsovskiĭ and E. A. Turov for interest in the research and for discussions.

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Translated by M. D. Friedman
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