

ON THE COEXISTENCE OF SUPERCONDUCTIVITY AND FERROMAGNETISM

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The Cooper problem is studied when oriented spins of internal electrons are present. The interactions between conduction electrons caused both by phonon exchange and spin-wave exchange are taken into account. It is shown that in principle superconductivity and ferromagnetism can coexist in the same spatial regions. It is also shown that in solid solutions of ferromagnetic metals in superconductors the temperature of transition into the superconducting state can increase with increasing concentration of the ferromagnetic component for sufficiently small concentrations. For larger concentrations, increases of concentration should lead to a reduction of the transition temperature.

1. It was recently shown (see [1]) that certain solid solutions of ferromagnetic metals in superconductors are simultaneously superconductors and ferromagnets. It was not clear, however, whether the superconducting and ferromagnetic regions coincide, and the hypothesis was advanced (Matthias and Suhl [1]) that such solutions have a "spongy" structure and consist of ferromagnetic domains separated by superconducting layers.

It is of interest in this connection to study the Cooper problem in the presence of oriented spins of the internal electrons (*d* electrons) of a ferromagnetic component in solid solution. The present paper is devoted to this problem. We consider both the usual interaction between conduction electrons (*s* electrons) caused by phonon exchange, [2] and the specific additional interaction associated with the exchange of spin waves. [3]

Analysis of the Cooper problem shows that superconductivity and ferromagnetism can coexist, and that spatial separation of the superconducting and ferromagnetic regions is not necessary.

The additional interaction between conduction electrons due to the exchange of spin waves leads, as shown in [3], to attraction for a pair of conduction electrons if this pair is in a triplet state with zero spin projection, and to repulsion in a singlet state. It can, therefore, assist or hinder the occurrence of superconductivity, depending upon the spin state in which the electrons pair.

If pairing occurs in a triplet state, a situation is possible in which increasing the concentration of the ferromagnetic component in the solution will increase the temperature of transition into the

superconducting state.¹⁾ Such a variation of transition temperature with concentration was observed in solutions of iron and cobalt in titanium. [4] Unfortunately, however, there is no reliable information on the ordered character of the spins in these solutions.

For sufficiently large concentrations of the ferromagnetic component there occurs, irrespective of the spin state of the paired electrons, a decrease in the effective interaction volume in electron momentum space, owing to which the temperature of transition into the superconducting state starts to fall with increasing concentration.

2. Proceeding to the analysis of the Cooper problem for ferromagnetic superconductors, we dwell at first on the character of the interaction between conduction electrons caused by the exchange of spin waves. The interaction energy *V* between *s* and *d* electrons is proportional to the spin *s* of the *s* electron and to the magnetic moment per unit volume of the ferromagnet *M*:

$$V(r) = \Theta a^3 \mu_0^{-1} sM(r),$$

where Θ is of the order of magnitude of the Curie temperature Θ_C , *a* is the lattice constant, and μ_0 is the Bohr magneton.

¹⁾Considering the atoms of the ferromagnetic component as impurities which cause additional scattering of conduction electrons, Matthias and Suhl [1] concluded that there must be a reduction of the transition temperature with increasing ferromagnetic concentration. However, the interaction between conduction electrons due to spin-wave exchange is of a coherent character, and cannot be interpreted as scattering of the electrons at impurities.

The emission and absorption of spin waves is associated with the components of the vector \mathbf{M} which are perpendicular to the axis of easy magnetization (the z axis). These components have the form

$$M_x = \left(\frac{\mu_0 M_0}{2}\right)^{1/2} \sum_{\mathbf{k}} (a_{\mathbf{k}} + a_{-\mathbf{k}}^+) e^{i\mathbf{k}\cdot\mathbf{r}},$$

$$M_y = -i \left(\frac{\mu_0 M_0}{2}\right)^{1/2} \sum_{\mathbf{k}} (a_{\mathbf{k}} - a_{-\mathbf{k}}^+) e^{i\mathbf{k}\cdot\mathbf{r}},$$

where $a_{\mathbf{k}}^+$ and $a_{\mathbf{k}}$ are the operators for the creation and absorption of a spin wave with momentum $\hbar\mathbf{k}$, and M_0 is the component of magnetic moment along the z axis (the normalization volume is taken as unity). The interaction V causes the energy of an electron to depend on its spin orientation: $\epsilon(\mathbf{p}, \sigma) = \epsilon_0(\mathbf{p}) + 2\sigma\Theta$, where \mathbf{p} is the quasi-momentum of the electrons and $\sigma = \pm 1/2$ is the projection of its spin on the z axis. We put henceforth $\epsilon_0(\mathbf{p}) = p^2/2m$.

The matrix element of the interaction energy between two s electrons due to spin-wave exchange has, in the second approximation of perturbation theory, the following form:^[3]

$$\langle \mathbf{p}_1 \sigma_1, \mathbf{p}_2 \sigma_2 | V | \mathbf{p}'_1 \sigma'_1, \mathbf{p}'_2 \sigma'_2 \rangle$$

$$= \frac{1}{2} \Theta^2 a^3 \left[\frac{(s_1^-)_{\sigma_1 \sigma'_1} (s_2^+)_{\sigma_2 \sigma'_2}}{\epsilon(\mathbf{p}_1, \sigma_1) - \epsilon(\mathbf{p}'_1, \sigma'_1) - \hbar\omega_{\mathbf{k}}} \right.$$

$$\left. + \frac{(s_2^-)_{\sigma_2 \sigma'_2} (s_1^+)_{\sigma_1 \sigma'_1}}{\epsilon(\mathbf{p}_2, \sigma_2) - \epsilon(\mathbf{p}'_2, \sigma'_2) - \hbar\omega_{\mathbf{k}}} \right], \quad (1)$$

where $\mathbf{p}_{1,2}$, $\sigma_{1,2}$ and $\mathbf{p}'_{1,2}$, $\sigma'_{1,2}$ are the momenta and spin projections of the electrons in the initial and final states, $s^\pm = s_x \pm is_y$, $\hbar\mathbf{k} = \mathbf{p}'_1 - \mathbf{p}_1$, and $\hbar\omega_{\mathbf{k}} = \Theta_C(ak)^2$ is the energy of the spin wave.

Since the energy of an s electron depends on the orientation of its spin, various spin orientations correspond to various Fermi-surface radii $p^\pm = \sqrt{2m(\zeta \pm \Theta)}$, where ζ is the chemical potential. The matrix element (1) is non-zero only when $\sigma_{1,2} + \sigma'_{1,2} = 0$. Therefore, if $\mathbf{p}_1 \approx p^\pm$, then $\mathbf{p}_2 \approx p^\mp$; in other words, during emission or absorption of a spin wave, an electron transfers from one Fermi sphere to the other. Whence it is easy to conclude that the minimum value of the momentum of the spin wave is

$$p^+ - p^- = p_0 \Theta / \zeta, \quad p_0 = \sqrt{2m\zeta}.$$

Close to the Fermi boundary [$\epsilon(\mathbf{p}, \sigma) \approx \zeta$] the matrix element (1) has the form

$$\langle \mathbf{p}_1 \sigma_1, \mathbf{p}_2 \sigma_2 | V | \mathbf{p}'_1 \sigma'_1, \mathbf{p}'_2 \sigma'_2 \rangle = -\frac{1}{2} \Theta^2 \frac{a^3}{\hbar\omega_{\mathbf{k}}} Q;$$

$$Q = (s_1^- s_2^+ + s_1^+ s_2^-)_{\sigma_1 \sigma_2, \sigma'_1 \sigma'_2}. \quad (2)$$

When solving the Cooper problem it can be assumed that the interaction between electrons is non-zero and is given by (2) if $\mathbf{p}_{1,2}$ and $\mathbf{p}'_{1,2}$ lie in the ranges $(p^\pm - \Delta p_S, p^\pm + \Delta p_S)$, where $\Delta p_S = p_0 \Theta_C / 2\zeta$.

Noting that $Q = S(S+1) - 3/2 - 2\sigma_1\sigma_2$, where $S = 0$ or 1 is the total spin of the pair, we easily verify that Q is non-zero only in two spin states of the pair: in the singlet state ($S = 0$) and in the triplet state with zero projection of total spin ($S = 1, \sigma_1 + \sigma_2 = 0$). Here $Q = -1$ if $S = 0$, and $Q = +1$ if $S = 1$; in other words, the electrons attract in the triplet state and repel in the singlet state.^{[3]2)}

The effective matrix element of the interaction between electrons due to spin-wave exchange will be denoted $(-1)^S U_S(\mathbf{p}, \mathbf{p}')$, where

$$U_s = \Theta^2 a^3 / 2\hbar\omega_{\mathbf{k}}, \quad 2\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2, \quad 2\mathbf{p}' = \mathbf{p}'_1 - \mathbf{p}'_2,$$

$$\hbar\mathbf{k} = \mathbf{p}' - \mathbf{p}.$$

Adding to this expression the effective matrix element $-U_p(\mathbf{p}, \mathbf{p}')$ of the electron interaction due to phonon exchange [here the momenta $\mathbf{p}_{1,2}$, $\mathbf{p}'_{1,2}$ are assumed to lie in the ranges $(p^\pm - \Delta p_p, p^\pm + \Delta p_p)$, where $\Delta p_p = p_0 \Theta_D / 2\zeta$ and Θ_D is the Debye temperature], we obtain the total matrix element of the interaction

$$U(\mathbf{p}, \mathbf{p}') = -U_p(\mathbf{p}, \mathbf{p}') + (-1)^S U_s(\mathbf{p}, \mathbf{p}').$$

3. We now write the Schrödinger equation for the wave function of the pair $\Psi(\mathbf{p}_1, \mathbf{p}_2) \equiv \Psi(\mathbf{p}, \mathbf{P})$ in the Cooper problem^[5] ($\mathbf{p}_1 = 1/2 \mathbf{P} + \mathbf{p}$, $\mathbf{p}_2 = 1/2 \mathbf{P} - \mathbf{p}$). This function must clearly be symmetric with respect to \mathbf{p} in the singlet state ($\Psi = \Psi_0$) and anti-symmetric in the triplet state ($\Psi = \Psi_1$). The functions Ψ_0 and Ψ_1 satisfy the equations

$$\left(\frac{p^2}{4m} + \frac{p^2}{m} - E\right) \Psi_0(\mathbf{p}, \mathbf{P}) = \frac{1}{2} \int \{U(\mathbf{p}, \mathbf{p}') + U(-\mathbf{p}, \mathbf{p}')\}$$

$$\times \left[n\left(\mathbf{p}' - \frac{1}{2} \mathbf{P}\right) + n\left(\mathbf{p}' + \frac{1}{2} \mathbf{P}\right) - 1 \right] \Psi_0(\mathbf{p}', \mathbf{P}) \frac{d\mathbf{p}'}{(2\pi\hbar)^3};$$

$$\left(\frac{p^2}{4m} + \frac{p^2}{m} - E\right) \Psi_1(\mathbf{p}, \mathbf{P}) = \frac{1}{2} \int \{U(\mathbf{p}, \mathbf{p}') - U(-\mathbf{p}, \mathbf{p}')\}$$

$$\times \left[n\left(\mathbf{p}' - \frac{1}{2} \mathbf{P}\right) + n\left(\mathbf{p}' + \frac{1}{2} \mathbf{P}\right) - 1 \right] \Psi_1(\mathbf{p}', \mathbf{P}) \frac{d\mathbf{p}'}{(2\pi\hbar)^3}, \quad (3)$$

where $n(\mathbf{p})$ is the distribution function of the electrons, and E is the energy of the pair. At absolute zero the expression in square brackets is non-zero

²⁾The erroneous conclusion obtained in [7] regarding the type of interaction between conduction electrons caused by exchange of spin waves is associated with the fact that in [7] the triplet state with $S_z = 0$ and the singlet state are not distinguishable.

only in two ranges of momenta change:

$$n(p_1) + n(p_2) - 1 = \begin{cases} 1, & p_1 < p^\pm, \quad p_2 < p^\mp \\ -1, & p_1 > p^\pm, \quad p_2 > p^\mp \end{cases}.$$

Together with the conditions $|p_1 - p^\pm| < \Delta p$, $|p_2 - p^\pm| < \Delta p$ where $\Delta p = \Delta p_S$ for the potential U_S and $\Delta p = \Delta p_p$ for the potential U_p , these intervals define the region of change of the momenta p_1, p_2 on the right-hand side of (3).

Knowing the intervals of variation of p_1 and p_2 , and noting that the vector $\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$ is geometrically expressed as the median of the triangle constructed with sides p_1, p_2, P , it is simple to determine the range of integration with respect to momentum \mathbf{p} . The modulus of the vector \mathbf{p} varies in the range determined by the inequalities

$$\xi - p_0 \Delta p / m < p^2 / m + P^2 / 4m < \xi + p_0 \Delta p / m,$$

where

$$n(\mathbf{p} \pm \frac{1}{2}\mathbf{P}) + n(\mathbf{p} - \frac{1}{2}\mathbf{P}) - 1$$

$$= \text{sign} \{p^2 / m + P^2 / 4m - 2\xi\}.$$

The angle θ between the vectors \mathbf{p} and \mathbf{P} varies in the ranges (θ_1, θ_2) and $(\pi - \theta_2, \pi - \theta_1)$, where

$$\begin{aligned} \cos \theta_1 &= \min \{(p^+ - p^- + \Delta p) / P, 1\}, \\ \cos \theta_2 &= \max \{(p^+ - p^- - \Delta p) / P, 0\}. \end{aligned} \quad (4)$$

In other words, the end of the vector \mathbf{p} can, in the general case, lie only in two symmetrically disposed zones or segments of the Fermi sphere. The ratio S^0 of the area of this region to the total area of the Fermi sphere is shown graphically as a function of the total momentum of the pair in Fig. 1a for the case $q^- > 0$, and in Fig. 1b for the case $q^- < 0$ ($q^\pm = p^+ - p^- \pm \Delta p$).

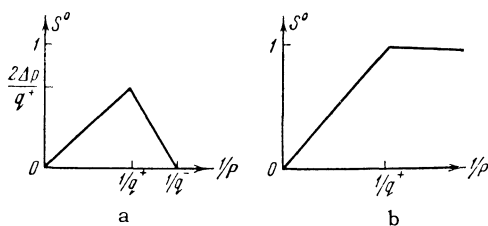


FIG. 1

Taking into account the form of the region of integration on the right-hand side of (3), we must now find the values of the energy E for which these equations have nontrivial solutions, and determine the energy gap $\Delta = E - 2\xi$. Since the region of integration depends essentially on the total momentum of the pair \mathbf{P} , quantity Δ also depends on \mathbf{P} . In fact, in the superconducting state, the electrons

will be paired with total momentum and spin state ($S = 0$ or $S = 1$) such as to yield the largest gap.

4. Expanding the potential $U(\mathbf{p}, \mathbf{p}')|_{\mathbf{p}=\mathbf{p}'=\mathbf{p}_0}$ in a series of Legendre polynomials, and retaining³⁾ only the first two terms $U^0, U^1 \cos \vartheta$ (ϑ is the angle between \mathbf{p} and \mathbf{p}'), we rewrite equations (3) in the form

$$\begin{aligned} & \left(\frac{P^2}{4m} + \frac{p^2}{m} - E \right) \Psi_0(\mathbf{p}, \mathbf{P}) \\ & - \int [U_p^0(\theta, \theta') - U_s^0(\theta, \theta')] \Psi_0(\mathbf{p}', \mathbf{P}) \eta d\tau' = 0, \\ & \left(\frac{P^2}{4m} + \frac{p^2}{m} - E \right) \Psi_1(\mathbf{p}, \mathbf{P}) \\ & - \int [U_p^1(\theta, \theta') + U_s^1(\theta, \theta')] \cos \vartheta \Psi_1(\mathbf{p}', \mathbf{P}) \eta d\tau' = 0, \end{aligned} \quad (5)$$

where $U^l(\theta, \theta') = U^l$, if both angles θ, θ' belong to the region (4) [in the contrary case $U^l(\theta, \theta') = 0$];

$$\eta d\tau = (2\pi\hbar)^{-3} d\mathbf{p} \text{ sign} \{p^2 / m + P^2 / 4m - 2\xi\}.$$

We search for a solution of (5) in the form

$$\Psi_l(\mathbf{p}, \mathbf{P}) = \begin{cases} \sum_{m=-l}^l \psi^{lm}(p, P) e^{im\varphi} P_l^m(\cos \theta) \\ \sum_{m=-l}^l \chi^{lm}(p, P) e^{im\varphi} P_l^m(\cos \theta) \end{cases}, \quad l = 0, 1,$$

where the upper expression defines the function Ψ_l in the range of θ in which both the potentials U_S and U_p act, and the lower expression defines it in the range of θ in which only the potential with the wider range of action is non-zero (in the angular range where both potentials vanish, the wave function is, of course, zero).

Completing the integration over the angle variables, we reduce (5) to

$$\begin{aligned} & \left(\frac{P^2}{4m} + \frac{p^2}{m} - E \right) \Psi^{lm}(p, P) = -U_p^l \int R_p^{lm}(p', P) \eta d\tau' \\ & + (-1)^l U_s^l \int R_s^{lm}(p, P) \eta d\tau', \end{aligned}$$

$$\begin{aligned} & \left(\frac{P^2}{4m} + \frac{p^2}{m} - E \right) \chi^{lm}(p, P) \\ & = \begin{cases} -U_p^l \int R_p^{lm}(p', P) \eta d\tau', & \Delta p_p > \Delta p_s \\ (-1)^l U_s^l \int R_s^{lm}(p', P) \eta d\tau', & \Delta p_p < \Delta p_s \end{cases}; \end{aligned}$$

³⁾An account of higher harmonics of the potential gives nothing qualitatively new; it does, however, greatly complicate the problem, since, owing to the absence of spherical symmetry in the region of integration, (3) do not resolve into independent equations for each spherical harmonic.

$$R^{lm}(p, P) = \frac{(l-|m|)!}{(l+|m|)!} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta \times \int \frac{d\varphi}{(2\pi)^2} e^{-im\varphi} [P_l^m(\cos \theta)]^2 \Psi_l(p, P), \quad (6)$$

where the quantities θ_1 and θ_2 in the expression for $R_S(\theta_{1,2} = \theta_{1,2}^S)$ are determined by formulae (4) with $\Delta p = \Delta p_S$ and in the expression for $R_p(\theta_{1,2} = \theta_{1,2}^p)$ by the same formulae with $\Delta p = \Delta p_p$.

The condition for the existence of nontrivial solutions of (6) is the vanishing of the determinant

$$\det \begin{pmatrix} 1 + (-1)^{l+1} U_s^l S_s^{lm} J & (-1)^{l+1} U_s^l S_q^{lm} J \\ U_p^l S_q^{lm} J & 1 + U_p^l S_p^{lm} J \end{pmatrix} = 0, \quad (7)$$

where S_q is the smaller of the quantities S_p and S_s ,

$$S_r^{lm} = \frac{(l-|m|)!}{(l+|m|)!} \int_{\theta_1}^{\theta_2} [P_l^m(\cos \theta)]^2 \sin \theta \, d\theta, \quad r = s, p, \\ J = \int \left(\frac{P^2}{4m} + \frac{p^2}{m} - E \right)^{-1} \eta \, d\tau = \left(\frac{d\tau}{d\varepsilon} \right)_\zeta \ln \frac{E - 2\xi}{2\Theta_D} \quad (8)$$

(we assume for simplicity that $\ln \Theta_D \sim \ln \Theta_C$, although the quantities Θ_D and Θ_C themselves can differ significantly).

Selecting from the two solutions of (7) that, to which corresponds the larger value of Δ , we obtain finally for the gap which occurs in the energy spectrum when electrons pair in the state (l, m) :

$$\Delta^{lm}(\Theta) = 2\Theta_D \exp \{ -1/G^{lm} \}, \quad (9)$$

$$G^0 = \left(\frac{d\tau}{d\varepsilon} \right)_\zeta \frac{2U_s^0 U_p^0 [S_s^0 S_p^0 - (S_q^0)^2]}{\sqrt{(U_p^0 S_p^0 + U_s^0 S_s^0)^2 - 4U_p^0 U_s^0 (S_q^0)^2 - (U_p^0 S_p^0 - U_s^0 S_s^0)}}, \\ G^{lm} = \left(\frac{d\tau}{d\varepsilon} \right)_\zeta \frac{2U_s^l U_p^l [S_s^{lm} S_p^{lm} - (S_q^{lm})^2]}{\sqrt{(U_p^l S_p^{lm} + U_s^l S_s^{lm})^2 - 4U_p^l U_s^l (S_q^{lm})^2 + 4U_s^l U_p^l (S_q^{lm})^2}}. \quad (10)$$

As in the usual case, the temperature of transition into the superconducting state T_0 differs from the quantity Δ only by a multiplier ~ 0.57 .

If $\Theta = 0$, then $S_p^{lm} = (2l+1)^{-1}$, and, in agreement with a result of Gor'kov and Galitskii,^[6] we have

$$\Delta^{lm} \equiv \Delta^l(0) = 2\Theta_D \exp \{ - (d\varepsilon/d\tau)_\zeta (2l+1)/U_p^l \}.$$

Using this relation, the potential U_p can be expressed in terms of the value of the gap in a substance with disordered atomic spins, but with the same lattice characteristics

$$U_p^l = \left(\frac{d\varepsilon}{d\tau} \right)_\zeta \frac{2l+1}{\ln [2\Theta_D/\Delta^l(0)]}.$$

For the quantity U_s^l , using the explicit form of

the potential U_s and taking into account that $\hbar\kappa \geq p^+ - p^-$ and $\xi \gg \Theta$, we obtain

$$U_s^0 = \frac{1}{3} U_s^1 = (\Theta^2 a \hbar^2 / 2 \Theta_C p_0^2) \ln (\xi / \Theta).$$

Equations (10) become much simpler in three limiting cases: $S_s \ll S_p$, $S_s \gg S_p$ and $|S_s - S_p| \ll S_p$, and take the form

$$G^{lm} = (d\tau/d\varepsilon)_\zeta [U_p^l S_p^{lm} + (-1)^{l+1} U_s^l S_s^{lm}] \\ = (2l+1) \left[S_p^{lm} \left(\ln \frac{2\Theta_D}{\Delta^l(0)} \right)^{-1} + (-1)^{l+1} S_s^{lm} \lambda \frac{\Theta}{\xi} \ln \frac{\xi}{\Theta} \right], \quad (11)$$

where $\lambda = (\Theta/\Theta_C)(3\nu/\pi)^{1/3}/8\pi$ is a constant of the order of unity, and ν/a^3 is the density of conduction electrons. To these conditions correspond, of course, the inequalities $\Theta_C \ll \Theta_D$, $\Theta_C \gg \Theta_D$, and $|\Theta_C - \Theta_D| \ll \Theta_D$. Since formula (11) is valid in opposite limiting cases, it can be used as an interpolation instead of the more complex formula (10).

5. We shall clarify the variation of gap with Curie temperature. To do this it will be assumed that the principal mechanism of interaction between conduction electrons is an exchange of phonons, so that the second component in (11) is small in comparison with the first.

The quantities Δ defined by formulae (11) and (10) contain, along with the explicit dependence on the Curie temperature, an implicit dependence on Θ . In fact, the quantities S^{lm} appearing in the expression for the gap depend, according to (8) and (4), both directly on the Curie temperature (through $p^+ - p^-$ and Δp_S) and also on the total momentum of the pair P , which is determined from the requirement that the gap (for given values of the other quantities) should be the greatest possible, and, therefore, is itself a function of the Curie temperature.

Thus, using (4) and (8), we must find the maximum of each of the expressions (11) [or (10)] with respect to P , and investigate the variation of these maxima with Curie temperature. Here, depending on the relation between the quantities Θ , Θ_C , and Θ_D , there occur, for pairing a singlet and in triplet states, several particular cases, in each of which the solution is trivial. We present only the final results.

If pairing occurs in a triplet state with zero spin projection, then, in the range $\Theta < 1/2 \Theta_D$ (i.e., $p^+ - p^- < \Delta p_p$) the formation of pairs with total momentum $P = p^+ - p^- + \Delta p_S = p_0 / \xi (\Theta + 1/2 \Theta_C)$ is most favorable energetically. Here the range of influence of the potential U_p spreads over the entire Fermi surface ($3S_p^{lm} = 1$), whilst U_s acts in

the region $1 \geq \cos \theta \geq (2\Theta - \Theta_C)/(2\Theta + \Theta_C)$. After calculating the values of S_S^{1m} from (8) and substituting them in (11), we obtain

$$G^{10} = \left[\ln \frac{2\Theta_D}{\Delta^1(0)} \right]^{-1} + \frac{\Theta}{\xi} \lambda_1 \ln \frac{\xi}{\Theta};$$

$$\lambda_1 = \lambda \left\{ 1 - \left(\frac{2\Theta - \Theta_C}{2\Theta + \Theta_C} \right)^3 \right\} \quad (12)$$

(the quantities $G^{1,1} = G^{1,-1}$ are somewhat smaller than half of G^{10}).

If $\Theta > \frac{1}{2}\Theta_D$, then the momentum of the pair which corresponds to the greatest gap is $P = p^+ - p^- + \Delta p_p = p_0/\xi(\Theta + \frac{1}{2}\Theta_D)$, and G^{10} assumes the form

$$G^{10} = \left[\ln \frac{2\Theta_D}{\Delta^1(0)} \right]^{-1} \left[1 - \left(\frac{2\Theta - \Theta_D}{2\Theta + \Theta_D} \right)^3 \right] + \frac{\Theta}{\xi} \lambda_2 \ln \frac{\xi}{\Theta};$$

$$\lambda_2 = \lambda \left\{ 1 - \left(\frac{2\Theta - 2\Theta_c + \tilde{\Theta}}{2\Theta + \Theta_D} \right)^3 \right\} \quad (13)$$

and $\tilde{\Theta}$ signifies the greater of the quantities Θ_D or Θ_C .

As follows from (12) and (13), the quantity G^{10} increases with increasing Θ from $[\ln(2\Theta_D/\Delta^1(0))]^{-1}$ when $\Theta = 0$ to

$$G_{max}^{10} = \left[\ln \frac{2\Theta_D}{\Delta^1(0)} \right]^{-1} + \frac{\Theta_D}{2\xi} \bar{\lambda}_2 \ln \frac{\xi}{\Theta_D}$$

when $\Theta = \Theta_0$, where

$$\Theta_0 = \frac{1}{2} \Theta_D \left\{ 1 + O \left(\frac{\Theta_D}{\xi} \ln \frac{\xi}{\Theta_D} \right) \right\}, \quad \bar{\lambda}_2 = \lambda_2|_{\Theta=\frac{1}{2}\Theta_D} \sim 1.$$

On further increasing the Curie temperature, G^{10} starts to decrease. The value of the gap Δ^{10} behaves identically, according to (9), where the ratio of the maximum value of Δ^{10} to the value of the gap when $\Theta = 0$ is

$$\frac{\Delta_{max}^{10}}{\Delta^1(0)} = \exp \left\{ \frac{\Theta_D}{\xi} \left[\ln \frac{2\Theta_D}{\Delta^1(0)} \right]^2 \frac{\bar{\lambda}_2}{2} \ln \frac{\xi}{\Theta_D} \right\}. \quad (14)$$

6. If pairing occurs in a singlet state and $\Theta < \frac{1}{2}\Theta_D$, the largest gap is attained for pair momenta $P < p_0/\xi(\Theta - \frac{1}{2}\Theta_C)$ when repulsion between electrons associated with spin-wave exchange is completely absent. Here Δ^0 does not depend on the Curie temperature, $\Delta^0(\Theta) = \Delta^0(0)$.

If $\Theta > \frac{1}{2}\Theta_D$, then pairing occurs with $P = p_0/\xi(\Theta + \frac{1}{2}\Theta_D)$. Here the value of the gap

$$\Delta^0(\Theta) = 2\Theta_D \exp \{-1/G^0\},$$

$$G^0 = \left[\ln \frac{2\Theta_D}{\Delta^0(0)} \right]^{-1} \left[1 - \left(\frac{2\Theta - \Theta_D}{2\Theta + \Theta_D} \right)^3 \right] - \frac{\Theta}{\xi} \lambda_2 \ln \frac{\xi}{\Theta}$$

diminishes with increasing Curie temperature.

We note that the formulae of Secs. 5 and 6 were obtained under the assumption that $\Theta > \frac{1}{2}\Theta_C$. If

$\Theta < \frac{1}{2}\Theta_C$ the variation of Δ with Θ has the same character, but with certain quantitative differences. First, the multiplier λ_1 in (12) (which we considered somewhat smaller than λ) must be replaced by λ ; secondly, the gap Δ^0 starts to diminish with increasing Θ , starting from $\Theta = 0$ (and not from $\Theta = \frac{1}{2}\Theta_D$), and for $\Theta < \frac{1}{2}\Theta_D$ takes the form

$$\Delta^0(\Theta) = \Delta^0(0) \exp \left\{ -\frac{\Theta}{\xi} \left[\ln \frac{2\Theta_D}{\Delta^0(0)} \right]^2 \lambda_2 \ln \frac{\xi}{\Theta} \right\}.$$

7. The non-monotonic variation of Δ^{10} with Θ which has been obtained has a simple physical explanation. It is due to the effect of two factors, which arise when there is a transition of spins into the ordered state. On the one hand, during such a transition an additional interaction mechanism between the conduction electrons (exchange of spin waves) occurs and leads to an additional attraction between electrons in the triplet state with $S_z = 0$, where this attraction increases with increasing Θ . On the other hand, commencing at $\Theta = \Theta_0$ ($\Theta_0 \sim \Theta_D$), the basic mechanism of attraction between electrons—phonon exchange—starts to weaken; therefore the gap starts to diminish with further increase in the Curie temperature. In the final analysis this is connected with the second “negative” consequence of the ordering of atomic spins—the dependence of the conduction-electron energy on the spin orientation.

The energy gap which occurs when electrons pair in the singlet state decreases, as we saw, with increasing Curie temperature. Such a variation of Δ^0 with Θ is associated with the fact that both the factors mentioned (not only the dependence of electron energy on its spin orientation, but also the interaction due to spin-wave exchange, which leads in the singlet state to repulsion) diminish the effective attraction between electrons for $S = 0$.

Finally, for pairing of electrons in a triplet state with spin projection ± 1 , the gap does not depend on the temperature and is equal to $\Delta^1(0)$, since the dependence of electron energy on its spin orientation is of no account in these states, and there is no interaction due to spin-wave exchange.

8. If it is taken into account that the Curie temperature in solid solutions increases with increasing concentration of the ferromagnetic component x , while the Debye temperature is almost independent of concentration, depending on the relation between the symmetric and antisymmetric parts of the phonon potential U_p^0 and U_p^1 , the following three possibilities can arise:

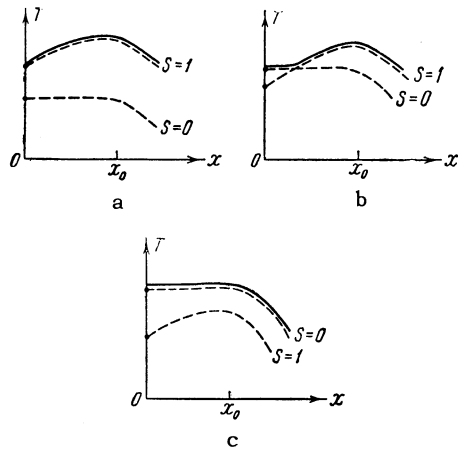


FIG. 2

1. In the pure superconductor pairing occurs in a state⁴⁾ with $S = 1$ ($U_p^0 < \frac{1}{3} U_p^1$). Then the variation of transition temperature T_0 with x has a non-monotonic character (see Fig. 2a; the broken lines show schematically the variation of $0.57 \Delta^{10}$ with x , the continuous line, the variation of T_0 with x).

The highest transition temperature, which is defined by (14), is attained at the concentration x_0 for which $\Theta(x_0) = \Theta_0$.

2. In the pure superconductor, pairing occurs in the state with $S = 0$; however, the state with $S = 1$ lies only a little above the ground state ($0 < U_p^0 - \frac{1}{3} U_p^1 \ll U_p^0$) so that even a small concentration of the ferromagnetic component makes pairing in

⁴⁾Incidentally, such a case is unlikely, since, in the absence of any singular causes the state with $S = 0$ should lie below the state with $S = 1$ (i.e., $U^0 > \frac{1}{3} U^1$).

a triplet state preferable. The variation of T_0 with x is shown schematically for this case in Fig. 2b.

3. In the pure superconductor, the state with $S = 1$ lies significantly above the ground state corresponding to $S = 0$. In this case the temperature of transition into the superconducting state diminishes with increasing x (see Fig. 2c).

In all cases the decrease of transition temperature with increasing concentration proceeds as long as the gap does not become equal to $\Delta^1(0)$. Starting at this stage, pairing will occur in a triplet state with spin projection ± 1 , and the transition temperature will cease to depend on x (if the variation of the Debye temperature and the conducting electron density on concentration is ignored).

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