CONDUCTIVITY OF PLASMA MEDIA IN THE PRESENCE OF A DRIFT

V. I. PUSTOVOĬT

P. N. Lebedev Physics Institute, Academy of Science, U.S.S.R

Submitted to JETP editor July 14, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 2281–2289 (December, 1961)

The alternating-current conductivity of a plasma is considered and it is demonstrated by the kinetic equation that the conductivity becomes negative in the presence of a drift with a velocity exceeding the phase velocity of a certain wave. The appearance of a negative conductivity is connected with the directed motion of the space charge. The dependence of the conductivity on frequency is investigated and it is shown that the effect occurs only at low frequencies. The conductivity is also derived by taking into account the magnetic field and it is shown that in a weak field for which the electron cyclotron frequency ω_H is smaller than the collision frequency ν the elementary formula (16) is valid. In a strong magnetic field, $\omega_H \gg \nu$, the conductivity may also be negative, provided the drift velocity in the magnetic field exceeds the phase velocity of the waves. It is shown that a consequence of taking into account the velocity dependence of the collision frequency is renormalization of the drift velocity.

SEVERAL recent papers have dealt with the conductivity of plasma media such as a gas-discharge plasma, plasma in a solid, etc. The formulas usually used for the conductivity are derived by introducing the collisions as small imaginary additions to the frequency, or by disregarding the collisions completely. This corresponds to a region of frequencies ω much larger than the collision frequencies ν, $\omega \gg \nu$. In many cases, however, it is necessary to deal precisely with the frequency region satisfying the opposite inequality, $\omega \ll \nu$. For example, in a solid the effective frequency of collisions between the carriers and the lattice is on the order of 10^{12}–10^{14} $\sec^{-1}$, and the condition $\omega \ll \nu$ is satisfied for frequencies down to the infrared region. During a time equal to the period of the oscillations the electron experiences many collisions, and the character of its motion depends usually on the friction due to the collisions. Consequently it is necessary to take correct account of the collisions when solving the problem for low frequencies.

Such an analysis was carried out for the simplest case in [1], where it was found that the conductivity of the medium can be negative for certain waves in the low-frequency region. The conditions for the appearance of negative conductivity are similar to the conditions for Cerenkov radiation: the drift velocity must be larger than the phase velocity of the generated wave. However, in directed motion of the electrons the drift velocity is much smaller than the random thermal velocity of the electrons, and it is not obvious beforehand that the imposition of a small directional velocity on the random thermal motion can lead to the appearance of negative conductivity. To this end we undertook a kinetic analysis of the problem [1], which has confirmed in the region of small frequencies the elementary formula for the conductivity, obtained without account of the electron thermal motion [2].

As was shown in [1,2], the appearance of negative conductivity is connected with the directional motion of space charge, which plays here the role of the Cerenkov "emitter." Whereas in Cerenkov radiation each particle radiates separately (provided only its velocity exceeds the phase velocity of the wave), in this case the wave is radiated by an aggregate of particles—the space charge drifting under the influence of the external field. The formation of the alternating component of the electron density itself—the space charge—is due to the alternating field of the wave, which either amplifies or excites the space charge [2]. We therefore have here, as it were, a system (the space charge plus the wave) with a feedback loop. To the contrary, in the case of Cerenkov radiation, the electron and the wave it produces do not interact (in the linear approximation).

The purpose of the present work is to show that the appearance of negative ac conductivity in the presence of a drift exceeding the phase velocity of the wave is a rather general property of such a wave, and to explain further the influence of the magnetic field and the dependence of the collision frequency and the velocity dependence of the collision frequency on the magnitude of the effect.

1. CONDUCTIVITY OF THE MEDIUM AT LOW FREQUENCIES

We start from the kinetic equation for the electronic distribution function f(**r**, **v**, t)

$$\frac{\partial f}{\partial t} + \mathbf{v}\nabla_{\mathbf{r}} f + \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\nabla_{\mathbf{v}} f + S = 0, \quad (1)^*$$

where **E** and **H** are the specified external electric and magnetic fields, respectively, and S is the collision integral.

Assume that only constant time-independent homogeneous fields $\mathbf{E}_=$ and **H** act on the plasma medium. The solution of Eq. (1) is sought in the form of an expansion in Legendre polynomials in velocity space:

$$f(\mathbf{r}, \mathbf{v}, t) = \sum_{k=0}^{\infty} P_k(\cos\alpha) f_k(\mathbf{r}, v, t). \quad (2)$$

Under certain conditions, which will be discussed later on, it is possible to retain only the first two terms of the expansion (2) for the distribution function f:

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, v, t) + (\mathbf{v}/v)\, \mathbf{f}_1(\mathbf{r}, v, t), \quad (2a)$$

where f_0 and $\mathbf{f}_1$ satisfy the following system of equations (see [3,4])

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\operatorname{div}_{\mathbf{r}} f_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2 \mathbf{E}_= \mathbf{f}_1) + S_0 = 0, \quad (3)$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\nabla_{\mathbf{r}} f_0 + \frac{e\mathbf{E}_=}{m}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{H}\mathbf{f}_1] + S_1 = 0, \quad (3a)$$

where S_0 and S_1 are the Legendre transformations of the collision integral [3].

We are interested in the ac conductivity of the medium in the presence of drift. "Turning on" the weak alternating field changes the distribution function, and small additional terms appear in the symmetrical part f_0 and in the asymmetrical part $\mathbf{f}_1$ of the distribution function, owing to the simultaneous presence of the alternating and constant fields:

$$f = f_0 + \varphi_0 + (\mathbf{v}/v)(\mathbf{f}_1 + \boldsymbol{\varphi}_1) + \ldots, \quad (4)$$

In the approximation linear in the weak alternating field $\mathbf{E}_\sim$, assumed to be parallel to $\mathbf{E}_=$, the equations for φ_0 and φ_1, as can be seen from (3) have the form

$$\frac{\partial \varphi_0}{\partial t} + \frac{v}{3}\operatorname{div}_{\mathbf{r}}\boldsymbol{\varphi}_1 + \frac{e\mathbf{E}_=}{3mv^2}\frac{\partial}{\partial v}(v^2\boldsymbol{\varphi}_1) + S_0 = -\frac{e\mathbf{E}_\sim}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{f}_1), \quad (5)$$

*$[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

$$\frac{\partial \boldsymbol{\varphi}_1}{\partial t} + v\nabla_{\mathbf{r}}\varphi_0 + \frac{e\mathbf{E}_=}{m}\frac{\partial \varphi_0}{\partial v} + \frac{e}{mc}[\mathbf{H}\boldsymbol{\varphi}_1] + S_1 = -\frac{e\mathbf{E}_\sim}{m}\frac{\partial f_0}{\partial v}, \quad (5a)$$

where the collision integrals S_0 and S_1 contain φ_0 and φ_1 in lieu of f_0 and f_1 [1)].

If we confine ourselves to purely elastic scattering, then [3]

$$S_0 = -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta_{\mathrm{el}}\nu(v)\left[\frac{\varkappa T}{m}\frac{\partial\varphi_0}{\partial v} + v\varphi_0\right]\right\},$$
$$S_1 = \nu(v)\,\boldsymbol{\varphi}_1, \qquad \delta_{\mathrm{el}} \equiv 2m/M, \quad (6)$$

where T is the temperature of the heavy particles (the lattice).

In accordance with the definition of conductivity we have

$$\sigma_x \equiv \sigma = \frac{e}{E_\sim}\int_0^\infty v^3\varphi_{1x}(v)\,dv, \quad (7)$$

where **x** is the direction corresponding to the vector $\mathbf{E}_=$ [2)]. The dependence of φ on the coordinates and on the time is represented in the form of a plane wave

$$\varphi(\mathbf{r}, \mathbf{v}, t) = e^{i(\omega t - \mathbf{kr})}\varphi(\mathbf{v}); \quad (8)$$

and we then obtain from (5a)

$$\varphi_{1x} = -\frac{eE_\sim}{m\gamma}\frac{\partial f_0}{\partial v} - \frac{eE_=}{m\gamma}\frac{\partial\varphi_0}{\partial v} + \frac{ivs}{\gamma}\varphi_0, \quad (9)$$

$$\varphi_{1y} = \frac{iv(k_y - i\omega_H s/\gamma)}{\nu + i\omega}\varphi_0 + \frac{e\omega_H}{m\gamma(\nu + i\omega)}\left(E_=\frac{\partial\varphi_0}{\partial v} + E_\sim\frac{\partial f_0}{\partial v}\right), \quad (10)$$

$$\varphi_{1z} = \frac{ik_z v}{\nu + i\omega}\varphi_0;$$

$$\gamma \equiv \nu + i\omega + \omega_H^2/(\nu + i\omega), \qquad s \equiv k_x + k_y\omega_H/(\nu + i\omega), \quad (11)$$

$\omega_H = eH/mc$ is the cyclotron frequency, the magnetic field **H** is directed along the **z** axis, i.e., perpendicular to the electric field $\mathbf{E}_=$.

Substituting in (7) the values of φ_{1x} and assuming for the sake of simplicity, for the time being, that the collision frequency ν does not depend on the velocity, we obtain for the conductivity

$$\sigma = \frac{3e^2E_=}{m\gamma E_\sim}\int_0^\infty v^2\varphi_0\,dv + \frac{ise}{\gamma E_\sim}\int_0^\infty v^4\varphi_0\,dv + \frac{3e^2}{m\gamma}\int_0^\infty v^2 f_0\,dv. \quad (12)$$

We now substitute into Eq. (5) the value (8) and

[1)]This is true only for a collision integral in the form of a linear operator relative to the distribution function. If it is nonlinear, as for example when account is taken of the electron-electron collisions, then S_0 and S_1 in (5) and (3) will be different.

[2)]We are interested in the conductivity along the drift only, and although the additions to the Hall emf and the conductivity "transverse" to the drift are of interest, they are beyond the scope of the present paper.

the values of φ_1 from (9), (10), and (11); then, after first multiplying the equation by v^2dv, we integrate it over all velocities. We obtain from (5)

$$\left[\omega - \frac{eE_=}{m\gamma}\left(k_x - \frac{k_y\omega_H}{\nu + i\omega}\right)\right]\int_0^\infty v^2\varphi_0\,dv - \frac{i}{3}\left[\frac{sk_x}{\gamma} - \frac{sk_y\omega_H}{\gamma(\nu + i\omega)} + \frac{k_y^2 + k_z^2}{\nu + i\omega}\right]\int_0^\infty v^4\varphi_0 dv = \frac{eE_\sim}{m\gamma}\left[\left(k_x - \frac{k_y\omega_H}{\nu + i\omega}\right)\right]\int_0^\infty v^2 f_0 dv. \qquad (13)$$

The expression (13) is obviously the continuity equation for the current.

It is seen from (12) and (13) that the conductivity can be determined rigorously in two limiting cases. First, when we can neglect the second terms of (12) and (13) compared with the first, and second, when the first terms can be neglected compared with the second. Let us determine the ratios of these terms and the conditions under which some can be discarded. It is obvious that, apart from a factor of the order of unity, we have

$$\int_0^\infty v^4\varphi_0\,dv \approx v_t^2\int_0^\infty v^2\varphi_0\,dv,$$

where $v_t = \sqrt{2\kappa T_e/m}$ is the thermal velocity of the electrons. From this follows immediately the condition under which one can discard the second term in (13)

$$\omega\,|\,1 - eE_= \cos\theta/m\gamma v_{ph}| \gg \omega^2 v_t^2 / \sqrt{\omega^2 + \nu^2 v_{ph}^2}, \qquad (14)$$

where $v_{ph} = \omega/k$ is the phase velocity of some wave in the medium and θ is the angle between the wave vector **k** and $E_=$; for simplicity, the magnetic field has been left out. Inasmuch as in our case, as will be shown below, we always have $\omega \ll \nu$, inequality (14) can be written in the form

$$v\tau_p v_t^2/v_{ph}^2 \ll |\,1 - v_= \cos\theta/v_{ph}|. \qquad (14a)$$

For semiconductors (see, for example, [5]), v_{ph} is of the order of c_0, the velocity of sound in the lattice, and the quantity v_t^2/c_0^2 plays the same role as the ratio M/m in the plasma.

It is easy to see that if (14a) is satisfied the second term of (12) can also be discarded and then, using the normalization condition

$$\int_0^\infty v^2 f_0(v)\,dv = n_0 \qquad (15)$$

(n_0 is the number of particles per unit volume) we obtain from (12) the following expression for the conductivity

$$\sigma = \frac{\sigma_0}{1 + \omega_H^2\tau_p^2}\left\{1 - \frac{v_=}{v_{ph}}\frac{\cos\theta}{1 + \omega_H^2\tau_p^2} + \frac{v_=}{v_{ph}}\frac{\omega_H\tau_p}{1 + \omega_H^2\tau_p^2}\sin\theta\right\}^{-1}, \qquad (16)$$

where $v_= = eE_=/m\nu$ is the drift velocity, and $\sigma_0 = e^2 n_0/m\nu$ is the dc conductivity. Formula (16) in the absence of a magnetic field and for **k** ∥ $\mathbf{E}_=$ goes over into the well known [1,2] formula

$$\sigma = \sigma_0 / (1 - v_=/v_{ph}). \qquad (16a)$$

From (16) we see that the influence of the magnetic field becomes appreciable only if the cyclotron frequency ω_H is of the same order as the collision frequency.

In the other limiting case, when the inequality inverse to (14) is satisfied, the first terms in (12) and (13) can be neglected compared with the second, and the conductivity will have the form

$$\sigma = \sigma_0\frac{2k_x^2/\gamma + (k_y^2 + k_z^2)/\nu - \omega_H k_y^2(1 - \omega_H/\nu)/\nu\gamma}{k_x^2/\gamma + (k_y^2 + k_z^2)/\nu - k_y^2\omega_H^2/\gamma\nu^2}, \qquad (17)$$

i.e., the appearance of negative conductivity without a sufficiently strong magnetic field is impossible.

We note that inasmuch we do not need the value of the distribution function in the calculation of the conductivity, and condition (15) is sufficient, the heating of the electron gas by the field $\mathbf{E}_=$ does not influence the value of the conductivity. The heating determines the thermal velocity of the electrons and, thus, at higher temperatures the effect merely shifts towards the lower frequencies [see (14a)].

Let us consider now the case of a strong magnetic field, when the cyclotron frequency of the electrons ω_H is much larger than the collision frequency ν, i.e., $\omega_H \gg \nu$. For a solid with $\nu \sim 10^{12}$ sec^{-1} (this is a relatively good crystal), a very strong field is necessary to satisfy this condition: **H** should be of the order of $10^5 - 10^6$ Oe. However, in a relatively dense and weakly ionized plasma, for which this entire analysis is also valid, the collision frequency is $\nu \sim 10^9 - 10^8$ sec^{-1} and the condition $\omega_H \gg \nu$ can be realized for ordinary fields.

A condition analogous to (14) has in this case the form

$$\omega\tau_p v_t^2/v_{ph}^2 \ll |\,1 + v_=^* \sin\theta/v_{ph}\,|, \qquad (14b)$$

where $v_=^* = eE_=/m\omega_H = cE_=/H$ is the drift in the magnetic field. In this case the conductivity will obviously be equal to

$$\sigma \approx \sigma_0\omega_H^{-2}\tau_p^{-2}\,[1 + v_=^*\sin\theta/v_{ph}]^{-1}. \qquad (18)$$

Thus, in a strong magnetic field, if the wave propagates perpendicular to the electric and magnetic fields, it can become intensified (compare with [6]).

2. LOW AND HIGH FREQUENCIES, REGION OF APPLICABILITY

Let us consider now the question of the applicability of the systems (13) and (5), and of the role of the collisions. As shown in [3], the system (3) is valid for a homogeneous plasma (we leave out the magnetic field for simplicity) if

$$e^2E^2/\varkappa T_e(\omega^2+\nu^2)\approx\delta_{\mathrm{el}}\ll 1, \qquad (19)$$

i.e., practically always. Relation (19), if we disregard special cases when the collision frequency ν depends on the velocity (see [3]) is satisfied for the entire region of frequencies, both low $\omega \ll \nu$ and high $\omega \gg \nu$. However, if the medium is inhomogeneous, i.e., $\partial\varphi/\partial\mathbf{r} \neq 0$, the condition for the applicability of the system (3) is different: the functions φ_2, φ_3, etc. in the expansion (2) can be discarded only in the case when [3,4]

$$|\partial\varphi_0/\partial\mathbf{r}|\gg|\partial\varphi_2/\partial\mathbf{r}|. \qquad (20)$$

Further, following [3], we obtain from the equation for φ_2 that

$$\varphi_2\sim\frac{\bar{v}}{\nu+i\omega}\frac{\partial\varphi_1}{\partial\mathbf{r}},$$

if the field $\mathbf{E}_=$ is weak; on the other hand, in the case of a strong field

$$\varphi_2\sim\frac{eE_=}{m}\frac{\bar{v}}{\nu+i\omega}\varphi_1.$$

We then estimate φ_1 and φ_0 from the system (5), putting $S_0[\varphi_0]\sim\delta_{\mathrm{el}}\nu\varphi_0$. As a result we obtain for the condition (20)

$$(\omega^2+\delta_{\mathrm{el}}^2\nu^2)^{-1/2}\gg(\omega^2+\nu^2)^{-1/2}, \qquad (20a)$$

which denotes in fact

$$\omega\ll\nu,\qquad \delta_{\mathrm{el}}\ll 1. \qquad (21)$$

The conditions (20), as shown by Ginzburg and Gurevich [4], give the limitations on the degree of nonstationarity and inhomogeneity of the plasma. Physically this means that the energy and density of the electron should not change appreciably over a time $(\omega^2+\nu^2)^{-1/2}$, nor should the electron current change over the effective range $l_{\mathrm{eff}}\approx v_t\tau_p$ [4], i.e., $l_{\mathrm{eff}}\ll\lambda=2\pi k^{-1}$.

On the other hand, in the region of high frequencies, $\omega\gg\nu$, the collisions can be neglected, and then the kinetic equation (1) for the addition to the equilibrium distribution function f_{00} will be

$$\partial\varphi/\partial t+v\partial\varphi/\partial r+(e\mathbf{E}_\sim/m)\,\partial f_{00}/\partial v=0, \qquad (22)$$

where $\varphi(\mathbf{r},\mathbf{v},t)$ is a small deviation of the true distribution function from the equilibrium value f_{00}. In the case of small frequencies, when the collisions play the decisive role, we obtain the complete system of equations [see (3a)] both for the function f and for the addition φ to it. However, at high frequencies we cannot obtain a complete system of equations for the determination of f_{00} and φ. Therefore the form of the equilibrium distribution function is usually postulated: if there is no external field, then f_{00} is chosen to be a Maxwellian distribution function, while in the presence of a field usually one chooses a Maxwellian distribution with drift:

$$f_{00}=\mathrm{const}\cdot\exp\left[-(\mathbf{v}-\mathbf{v}_=)^2/2\varkappa T_e\right]. \qquad (23)$$

The expression for the conductivity in the case when $\omega\gg\nu$, as can be readily seen from (22) and (23), will be

$$\sigma\sim i\int dv\,\frac{\partial f_{00}/\partial v}{\omega-\mathrm{kv}}, \qquad (24)$$

i.e., the conductivity reverses sign when $\bar{v}_= > v_{\mathrm{ph}}$, where $\bar{v}_=$ is a certain directional velocity.

In the literature (see [7,9] and the bibliography therein) the investigation of the instability of oscillations of a plasma medium entails the use of expression (24) for the conductivity, which is valid only at high frequencies. Therefore, although the condition for the instability of the oscillations ($\bar{v}_= > v_{\mathrm{ph}}$) is formally the same in both cases (16) and (24), viz., the drift velocity should exceed the phase velocity of the generated wave in the system, nevertheless the regions of application (low and high frequencies) and the very nature of the effects are somewhat different in these cases.

On the basis of the purely qualitative picture of the phenomenon [2], the appearance of negative conductivity at low frequencies is connected with the direction of motion of the space charge. This is also clearly seen from the expression for the conductivity (12). The reversal in the sign of σ is due to the first term in (12), i.e., to a quantity proportional to $v_=q_\sim$, where

$$q_\sim\equiv e\int_0^\infty v^2\varphi_0(v)\,dv. \qquad (25)$$

Expression (25) has the physical meaning of the current due to the directional motion of the space charge, which occurs if the condition $\varphi_0\neq 0$ is fulfilled. At high frequencies, no space charge is produced, and the addition to the symmetrical part of the distribution function φ_0 is much smaller than φ_1.

We note still another circumstance. Kovrizhnykh and Rukhadze [10] considered the question of the instability of longitudinal oscillations of the electron-ion plasma in the presence of electron drift. If we trace the dependence of the growth increment on the electron concentration, then the growth of longitudinal waves (ionic sound) upon change in the concentration is independent of the concentration over a wide range of the latter. As regards low frequencies, when the conductivity is given by (16), an analysis of the dispersion equation for the excitation of sound waves in a piezo-semiconductor [11] shows that the growth tends to zero as $n_0 \to 0$, i.e., it depends essentially on the carrier concentration.

3. ACCOUNT OF THE VELOCITY DEPENDENCE OF THE COLLISION FREQUENCY

The foregoing analysis did not take into account the velocity dependence of the collision frequency, so that we shall now discuss this question. It is impossible to obtain any quantitative deductions without knowing the explicit form of the function $\nu(v)$ and of the distribution function φ, and it is necessary to solve first the kinetic equation for φ. However, the scheme developed above for the determination of the conductivity directly from the kinetic equation itself makes it possible to draw certain qualitative conclusions concerning the influence of the dependence of $\nu(v)$ on the value of the conductivity.

We consider only the frequency region (14a) [3)], which in our opinion, is of greatest interest. For the sake of simplicity we assume that there is no magnetic field. The dependence of the collision frequency on the velocity upon collision with a neutral molecule has, as is known, the form

$$\nu(v) = \pi a^2 N_M v, \tag{26}$$

where a is the "radius" of the molecules, and N_M is the molecule concentration; on the other hand, in the case of collisions with ions (see [3,13]) we have

$$\nu(v) = (2\pi N_i e^4/m^2 v^3) \ln(1 + \rho_m^2 m^2 v^4/e^4), \tag{27}$$

where N_i is the ion concentration, and ρ_m is the maximum impact parameter, which is equal in order of magnitude to the Debye screening radius [12].

We confine ourselves to an examination of these two particular cases of the velocity dependence of the collision frequency, since they already make it possible to trace the main quantitative laws. From the definition of the conductivity (7) and (9) it follows that

$$\sigma = -\frac{e^2}{m}\int_0^\infty v^3 \frac{\partial f_0}{\partial v}\frac{1}{\nu(v)}\,dv - \frac{e^2 E_=}{m E_\sim}\int_0^\infty \frac{v^3}{\nu(v)}\frac{\partial \varphi_0}{\partial v}\,dv. \tag{28}$$

As before, integrating (5) with respect to the velocity, we obtain

$$\int_0^\infty \left(1 - \frac{eE_= \cos\theta}{m v_{ph}\nu(v)}\right) v^2 \varphi_0\,dv + \frac{eE_= \cos\theta}{3 m v_{ph}}\int_0^\infty \frac{\nu'(v)}{\nu^2(v)} v^3 \varphi_0\,dv$$
$$= -\frac{eE_\sim}{3 m v_{ph}}\cos\theta \int_0^\infty \frac{v^3}{\nu(v)}\frac{\partial f_0}{\partial v}\,dv. \tag{29}$$

We further proceed in the same manner as in the derivation of (16). We readily see that it follows from (28) and (29) that

$$\sigma = \bar\sigma_0 / (1 - \bar v_= \cos\theta / v_{ph}), \tag{30}$$

$$\bar\sigma_0 = -\frac{e^2}{3m}\int_0^\infty \frac{v^3}{\nu(v)}\frac{\partial f_0}{\partial v}\,dv; \tag{31}$$

The drift velocity $\bar v_=$ is determined by the quantity $\bar v_= = E_=/m\nu_{eff}$.

The effective collision frequency ν_{eff} is determined by the type of scattering

$$\nu_{eff} = \alpha_1 \pi a^2 N_M v_t, \qquad \nu_{eff} \approx \alpha_2 2\pi N_i e^4/m^2 v_t^3 \tag{32}$$

for the scattering by molecules and ions, respectively.

The numerical coefficients α_1 and α_2 are determined from the relations

$$\alpha_1 \equiv -v_t \int_0^\infty v^2 \frac{\partial \varphi_0}{\partial v}\,dv \Big/ \int_0^\infty v^2 \varphi_0\,dv, \tag{33}$$

$$\alpha_2 \equiv -v_t^{-3}\int_0^\infty v^6 \frac{\partial \varphi_0}{\partial v}\,dv \Big/ \int_0^\infty v^2 \varphi_0\,dv, \tag{34}$$

where by virtue of the symmetry of the function φ_0 the quantities α_1 and α_2 are always positive. This pertains also to the conductivity $\bar\sigma_0$, inasmuch as f_0 is also a symmetrical function in velocity space. We note that from (33) we can draw certain conclusions concerning the temperature dependence of the drift (mobility). In addition, the frequency region where this effect takes place is in itself dependent on the temperature and on the type of scattering. For example, in scattering by neutral molecules the inequality (14) assumes the form

$$\omega m^{-1/2}\sqrt{2\varkappa T_e} / \alpha_1 \pi a^2 N_M v_{ph}^2 \ll |1 - v_= \cos\theta / v_{ph}|, \tag{35}$$

i.e., it shifts towards the lower frequencies with

[3)] It is obvious that condition (14a) must now be taken in the sense that the value of ν is taken at the point $v = \bar v$, where $\bar v$ is a certain velocity at which the function $\varphi_0(v)$ has a maximum ($\bar v \approx v_t$ in order of magnitude).

increasing temperature in proportion to the square root of T_e; in the case of (27) (scattering by a charged ion) we have

$$\omega m^{-1/2}(2\varkappa T_e)^{3/2}/2\pi\alpha_2 N_i v_{ph}^2 \ll |1 - v_{=}\cos\theta / v_{ph}|, \quad (35a)$$

i.e., the shift is proportional to $T^{5/2}$ [4)].

We see therefore that when we take into account the dependence $\nu(v)$, the main formula for the conductivity [compare (16) with (30)] remains the same as before, but the drift and conductivity are redefined.

Let us make a few general remarks. We have shown under rather general assumptions that formula (16) holds for the conductivity. In its derivation we chose for the sake of simplicity the collision integral for the symmetrical part φ_0 in the Davydov form [see [5], formula (6)], and used the condition

$$\int_0^\infty v^2 S_0[\varphi_0]\,dv = 0. \quad (36)$$

However, even other forms of the collision integral, more general than (6), can also satisfy condition (36). Thus, the results will not change if we use a different form for S_0 (it must always be remembered here that φ_0 is not the distribution function itself, but only an addition to it, see footnote [1)]).

We note also that an account of the electron-electron collisions, in an analysis of problems connected with a dense weakly-ionized plasma and a plasma in a solid body, is immaterial. This is connected with the fact that the relative concentration of the electrons $\mu = n_0/N_i$ or n_0/N_M is exceedingly small. For semiconductors, for example, we always have $\mu < 10^{-4}$.

In conclusion we note that the buildup of sound oscillations without a magnetic field was observed experimentally by Hutson et al [13]. In the case of a strong magnetic field ($H \approx 2 \times 10^4$ Oe) at helium temperature ($T \sim 2°K$), Esaki [13] also observed in bismuth crystals low frequency current oscillations ($\omega \approx 1$ Mc), and the conditions for their appearance were the same as in our experiment, namely $v_{\pm}^{*} > c_0$—the velocity of longitudinal waves in bismuth.

I take this opportunity to express sincere gratitude to Professor V. L. Ginzburg, S. V. Bogdanov, M. E. Gertsenshteĭn, A. V. Gurevich, and L. V. Keldysh for valuable useful remarks and hints.

[4)]The estimates (35) are tentative, since α_1 and α_2 can also be temperature-dependent.

[1] M. E. Gertsenshteĭn and V. I. Pustovoĭt, JETP, **43**, 536 (1962). Soviet Phys. JETP **16**, 383 (1963).

[2] M. E. Gertsenshteĭn and V. I. Pustovoĭt, PTÉ, No. 7, 1009 (1962).

[3] V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plazma), IFML, 1960, Sec. 38.

[4] V. L. Ginzburg and A. V. Gurevich, UFN **70**, 2 (1960). Soviet Phys. Uspekhi **3**, 1 (1960).

[5] B. I. Davydov, JETP, **7**, 1069 (1937).

[6] R. F. Kazarinov and V. G. Skobov, JETP, **42**, 910 (1962). Soviet Phys. JETP **15**, 628 (1962).

[7] D. Bohm and E. P. Gross, Phys. Rev. **75**, 1864 (1949).

[8] V. P. Silin and A. A. Rukhadze, Elektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasmas and Plasma-like Media), Atomizdat, 1961, Sec. 25.

[9] D. Pines and J. R. Schriffer, Phys. Rev. **124**, 1387 (1961).

[10] L. M. Kovrizhnykh and A. A. Rukhadze, JETP, **38**, 850 (1960), Soviet Phys. JETP **11**, 615 (1960).

[11] Hutson, McFee, and White, Phys. Rev. Lett. **7**, 237 (1961).

[12] A. V. Gurevich, JETP, **30**, 1112 (1956). Soviet Phys. JETP **3**, 895 (1956).

[13] Leo Esaki, Phys. Rev. Lett. **8**, 4 (1962).

Translated by J. G. Adashko
390