SOVIET PHYSICS JETP VOLUME 17, NUMBER 1 JULY, 1963

LARGE IONIZATION BURSTS AND THE SPECTRUM OF NUCLEAR-ACTIVE PARTICLES AT MOUNTAIN ALTITUDES

Kh. P. BABAYAN, N. G. BOYADZHYAN, N. L. GRIGOROV, Ch. A. TRET'YAKOVA, and V. Ya. SHESTOPEROV

Institute of Nuclear Physics, Moscow State University

Submitted to JETP editor June 27, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 22-34 (January, 1963)

Large ionization bursts were studied with an experimental assembly covering an area of 10 m^2 and containing 192 ionization chambers. An analysis of the data shows that a considerable fraction of the large ionization bursts is induced by the simultaneous incidence of several nuclear-active particles. Simultaneous incidence is responsible for the fact that for bursts registered in an area of 10 m^2 the power exponent of the integral spectrum is $\gamma = 1.38 \pm 0.03$, whereas for bursts induced by individual particles the exponent is 1.92 ± 0.05.

THE absorption of nuclear-active particles in the atmosphere is known to depend essentially on such fundamental interaction properties as the inelasticity parameter, the degree of energy concentration on secondary particles, and the spectrum of primary cosmic radiation.[1] Therefore the study of the energy spectra of nuclear-active particles at different depths in the atmosphere furnishes extremely valuable information regarding the average properties of interactions between high-energy particles and light nuclei.

Numerous investigations of the energy spectra of nuclear-active particles at sea level, at mountain altitudes, and in the stratosphere have been reported in recent years. In most of these investigations pulse ionization chambers were used to register ionization bursts. In practically all cases it was assumed tacitly that the size of the registered pulse is proportional to the energy of the incident particle; therefore the energy spectrum of nuclear-active particles was identified with the ionization burst spectrum. This assumption would be justified (if the inelasticity parameter is independent of energy) except for the fact that there exist certain systematic factors that prevent a unique relationship between the measured burst spectrum and the nuclear-active particle spectrum at any given height.

A considerable amount of experimental information has by now been accumulated regarding the spectrum of ionization bursts induced by nuclear-active particles. The data in the literature exhibit noteworthy discrepancies. Table I gives the results of several mountain-height investigations (3–4 km above sea level). The experimental results obtained by different workers yield a power exponent γ of the integral burst spectrum varying from 2.0[2,3] to 1.4–1.5.[6-9]

Interactions in the energy range 10^{12}–10^{13} eV have recently been studied using nuclear emulsions; the results differ considerably from the ionization burst spectra. The nuclear-active particle spectrum in the stratosphere has an exponent $\gtrsim 2$ according to the emulsion measurements,[10]

Table I

Reference	Height above sea level, m	Range of registered bursts (in number of relativistic particles)	γ	Area of setup, m^2
Lapp[2]	3350	$2 \cdot 10^2$—$6 \cdot 10^3$	2.0±0.10	0.1
Stinchcomb[3]	3550	$2 \cdot 10^2$—$2 \cdot 10^3$	1.88±0.09	0.1
Grigorov et al.[4]	3200	$1 \cdot 10^3$—$1 \cdot 10^4$	1.67±0.05	0.6
Farrow[5]	3200	$1 \cdot 10^3$—$1 \cdot 10^4$	1.65±0.08	0.7
Zatsepin et al.[6]	3860	$6 \cdot 10^2$—$6 \cdot 10^4$	1.50±0.16	1.3
Murzina et al.[7]	3860	$2 \cdot 10^3$—$3 \cdot 10^4$	1.53±0.07*	2.2
Denisov et al.[8]	3860	$3 \cdot 10^3$—$3 \cdot 10^4$	1.50±0.10*	1.2
Babayan et al.[9]	3200	$2 \cdot 10^3$—$2 \cdot 10^5$	1.37±0.03	10.0

*For bursts of more than $\sim 3 \times 10^4$ particles the exponent γ increases to ~ 2.

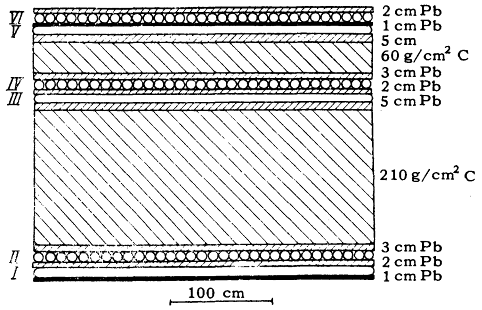

FIG. 1. Schematic diagram of setup. I-VI are rows of ionization chambers.

while the exponent of the burst spectrum at the same heights is 1.5.[11]

Table I shows that the spread of experimental values of γ considerably exceeds the statistical errors of the measurements, thus indicating the existence of systematic errors. Therefore the experimental data hitherto available have not permitted a unique conclusion regarding such a fundamental quantity as the exponent of the nuclear-active energy spectrum. It was the principal purpose of the present investigation to determine the cause of differences between experimental results obtained by different investigators, including those using the same ionization-chamber technique.

APPARATUS

We investigated large ionization bursts at 3200 m above sea level, at the mountain station of the Academy of Sciences of the Armenian SSR, with an experimental setup having an area of 10 m^2 (Fig. 1). The assembly consisted of six rows of ionization chambers; each chamber was 330 cm long and 10 cm in diameter. The rows were placed under different thicknesses of composite lead-graphite absorbers. (A more detailed description of the setup is given in [9]; the ionization chambers and electronics are described in [12,13].) Each chamber was connected to an individual amplifier that measured ionization in the range from ~ 200 relativistic particles passing simultaneously through the diameter of the chamber to ~ 70,000 particles. Summing amplifiers provided for each row enabled the measurement of total ionization in a row from ~ 1500 to a few million particles.

The upper two rows of chambers (V and VI) were placed under lead absorbers 3 and 2 cm thick, respectively and served to register particles of the electron-photon component entering from the air. Arrays III—IV and I—II measured the energy of electron-photon cascades (i.e., of π^0 mesons) generated by nuclear-active particles in the absorbers.

An ionization pulse was registered whenever the ionization in two or more rows of chambers exceeded that due to 2600 relativistic particles in the case of the four lowest rows (I—IV) or 8500 relativistic particles in the case of the uppermost rows.

In the present work we analyze the ionization bursts induced by high-energy nuclear-active particles interacting with the assembly (bursts in rows I—IV). The experimental data were obtained in 730 hours of operation, during which, because of the large area of the assembly, bursts representing the passage of more than 2×10^5 particles were registered. The total energy of π^0 mesons generated in the absorbers reached $> 2 \times 10^{13}$ eV in these cases.

We shall not consider here the energies of the nuclear-active particles inducing bursts, since for this purpose we would need additional information regarding the character of the interactions with nuclei in the absorbers (the inelasticity parameter and its fluctuations). We note only that our experimental results in [9] indicate that the energy of registered electron-photon cascades would have to be multiplied by a factor not exceeding 2 or 3 in order to obtain the energy of the nuclear-active particles inducing bursts.

RESULTS

1. Nature of the ionization bursts. The burst-registration system (where triggering required, as a minimum, the coincidence of pulses in two rows of ionization chambers) practically excluded the registration of bursts due to nuclear disintegration; these bursts are produced by heavily-ionizing short-range particles. Therefore the bursts registered in rows I—IV could have been produced only by cascade showers developing in the absorbers of the assembly. These showers result mainly from interactions between high-energy nuclear-active particles and absorber nuclei. However, they can also be induced by high-energy electrons and photons from the air and by electromagnetic interactions of muons.

Our earlier analysis has shown that at mountain heights the contribution of muons to bursts of more than 2×10^3 particles does not exceed 10% of the total number of bursts, and that this percentage decreases as the size of the bursts increases.[9] The contribution of the electron-photon component impinging on the assembly from the air and passing through the absorbers is also small, because the amount of matter above row IV is 23 t-units (and an even greater amount lies above the other rows). The data obtained by us and by other workers [12,14,15] show that this amount of matter is sufficient to absorb practically the entire electron-photon component entering from the air. However, particles of the electron-photon component having nonvertical paths and passing through the sides of the assembly could in some instances induce registrable bursts in the lower rows (I—IV).

Since high-energy particles have a very steep angular distribution peaked in the vertical direction it can be expected that bursts induced by the electron-photon component impinging on the sides of the assembly from the air would be detected only in the chambers near the ends of each row. In order to estimate this effect we determined the number of bursts of more than 10^3 particles registered by each chamber of a given row, and plotted the dependence of these numbers on the distance between the chamber and the side boundary of the assembly. To enhance the systematic accuracy we connected in each row the chambers located at equal distances from the opposite ends (the 1st and 32nd, the 2nd and 31st chambers etc.) as well as the corresponding chambers of neighboring rows (I and II, III and IV). The results are shown in Fig. 2. A marked increase of the number of bursts registered by chambers near the ends is observed in both distributions; in rows III and IV the effect is observed only in three or four chambers at the ends, whereas in rows I and II a higher number of bursts is observed in five to eight chambers. Figure 2 suggests that about 15% of all bursts registered in rows III and IV are induced by the electron-photon component from the air. The percentage is considerably greater for rows I and II; therefore in determining the spectrum of bursts induced by nuclear-active particles we shall hereafter use only bursts registered in rows III and IV.

We note that some increase of the number of bursts registered by chambers near the ends can theoretically be induced by the nuclear-active particles themselves. This is associated with the fact that nuclear-active particles impinging on the assembly are partly absorbed in the uppermost layer of lead, whereas particles passing through the sides and entering the end chambers are not absorbed in this way. Our calculations show that the increased number of bursts in chambers near the ends as a result of this effect should be considerably smaller than the number observed experimentally.

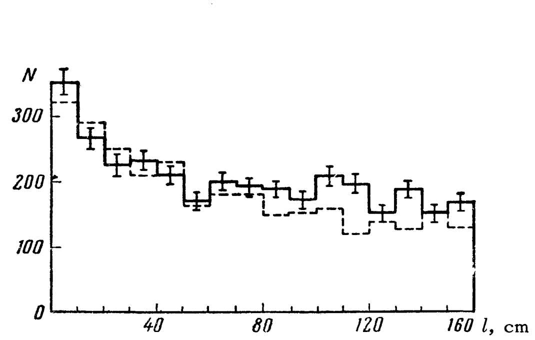

FIG. 2. Number of registered bursts of more than 10^3 relativistic particles vs. distance l of chambers from the sides of the assembly. Continuous line – chambers of rows III and IV; dashed line – chambers of rows I and II. Abscissas – distance (in cm) from the side; ordinates – number of bursts in corresponding chambers.

2. Bursts registered by a setup having a large area (10 m^2). In plotting the spectrum of ionization bursts registered by the entire area of the experimental setup the size of a burst was defined as the combined ionization produced simultaneously in all ionization chambers of a given row. This is equivalent to having each row replaced by a single chamber of 10-m^2 area.

The burst spectra in all four lowest rows are well represented by the power law $N (\geq J) = AJ^{-\gamma}$. The integral spectra of bursts registered by the entire area of rows III and IV are shown in Fig. 3 (curve a), where the abscissas are the sizes of bursts expressed in the numbers of relativistic particles passing simultaneously through a chamber diameter, and the ordinates are the numbers of registered bursts (without corrections for the effect of a transition from lead to the chamber walls).

The exponents of the integral spectra are given in Table II; the average exponent for rows III and IV in the range from 2×10^3 to 2×10^5 particles is 1.38 ± 0.03. We calculated all exponents of spectra and their errors by least squares.

For the spectrum of bursts registered in a 10-m^2 area we obtained a smaller exponent γ than that given by other authors (Table I). This result also differs from that obtained in our earlier work with a setup having an area of 0.6 m^2,[4] where for bursts in the range from 10^3 to 10^4 particles we obtained $\gamma = 1.67 \pm 0.05$.

3. Groups of simultaneously incident nuclear-active particles. As early as 1956 in our work with a setup having an area of 0.6 m^2 [4] we found that in 18% of the bursts of more than 10^3 particles considerable ionization was observed simultaneously in two nonadjacent chambers separated by other chambers in which ionization was small or entirely absent. These bursts were called (laterally) "structured," and comprised only ~ 1% of the bursts of less than 10^3 particles.

An analysis of the experimental data showed that structured bursts are produced when several

Table II

Method of treating experimental data	Row I	Row II	Row III	Row IV	Average for rows III and IV
Bursts in entire area of setup	1.34±0.031	1.32±0.026	1.37±0.030	1.39±0.026	1.38±0.03
Bursts with separate structures taken into account	1.47±0.035	1.52±0.034	1.55±0.034	1.61±0.031	1.58±0.03
Bursts in individual chambers	1.55±0.042	1.59±0.036	1.94±0.052	1.91±0.044	1.92±0.05

Table III

Size of bursts	Percent of structured bursts	Mean number of incident particles (in structural units)	Mean distance between particles ($\bar{l}$, cm)
$1.2\cdot10^3 \leqslant J < 3.6\cdot10^3$	17.6±0.8	2.2	100
$3.6\cdot10^3 \leqslant J < 8.4\cdot10^3$	36.3±2.3	2.9	85
$8.4\cdot10^3 \leqslant J < 2.4\cdot10^4$	63.2±2.4	3.5	80
$2.4\cdot10^4 \leqslant J < 6.0\cdot10^4$	84.2±4.9	4.3	65
$J \geqslant 6.0\cdot10^4$	90.1±7.9	5.5	35

nuclear-active particles of comparable energies impinge simultaneously on the setup. The frequency of these events increases with the size of the bursts, and the mean distance between nuclear-active particles forming the separate "structures" is reduced as their energy increases. All these conclusions were confirmed subsequently when large bursts were registered at sea level in a setup of 10-m^2 area.[12]

The independent registration of ionization in each chamber enabled us to investigate structured bursts that were registered at 3200 m in a setup having a working area of 10 m^2. As in the previous work, we divided all bursts into two categories —single and structured. The first category consisted of cases where the burst in a given row is registered by a small number of chambers, with maximum ionization in one chamber or in two adjacent chambers and with a steep decline of amplitude to the left and right of this maximum. Structured bursts were events where a row contained at least two groups of chambers recording bursts, separated by other chambers that recorded either no bursts or a deep minimum of ionization. Examples of single and structured bursts are given in [4,9,12].

The previously described setup was used to investigate the dependence of the relative number of structured bursts in chambers of rows III and IV on the size of bursts. Table III gives the percentage of structured bursts of different sizes registered in 10 m^2. The systematic accuracy was enhanced by combining data for chambers in row III and IV, which lay under practically the same amount of matter. The data in Table III indicate that the fraction of structured bursts increases monotonically with the size of the bursts. Among all bursts having $\geq 10^4$ particles registered in 10 m^2, ~ 60% are induced by the simultaneous incidence of more than one nuclear-active particle. For bursts having $\geq 6 \times 10^4$ particles this ratio increases to 90%.

Both the probability of simultaneous incident particles and the number of particles in the groups increases with the size of the bursts. Table III gives the mean number of particles impinging on the setup simultaneously (the mean number of separate structures) and inducing structured bursts of different sizes. It is found that an average of two primary particles induce structured bursts of 10^3 particles in 10 m^2, whereas four nuclear-active particles induce bursts of 3×10^4 particles.

With increasing total size of a structured burst, i.e., with increasing total energy of the group of simultaneous nuclear-active particles, we observed, as previously,[4,12] a decreasing distance between the most energetic particles of the group. This is shown by Table III, which gives the distances measured between the chambers registering the most ionization in each structured burst. It was required that the sizes of the bursts in these maxima, i.e., the sizes of bursts induced by separate nuclear-active particles, should differ by a factor less than 2. It must be noted, however, that since $\bar{l}$ is affected by the size of the setup (see [12], for example) absolute values of $\bar{l}$ are not given in the table.

4. Dependence of burst spectrum on area of experimental setup. The foregoing data show that when nuclear-active particles are registered by a setup having a large working area a considerable fraction of the bursts result from the simultaneous incidence of a group of particles. For the purpose of discriminating to some degree the bursts due to individual particles we proceeded as follows. In a considerable number of events the distribution of ionization in chambers involved in structured bursts enabled us to determine the ion-

ization in each of the separate structural units, i.e., the sizes of bursts due to individual nuclear-active particles forming a group. If we could assume that the apparatus enables us to discriminate laterally all incident nuclear-active particles, then by dividing the structured bursts into bursts due to individual particles we would obtain the true burst spectrum, which would be an adequate spectrum of nuclear-active particles.

The integral spectra of bursts in rows III and IV obtained by this procedure are shown in Fig. 3 (curve b), and the values of the exponent γ for all four rows are given in Table II. Since bursts due to individual nuclear-active particles can be discriminated within structured bursts only if their separation exceeds 30—40 cm (and sometimes more) this treatment of the experimental results is equivalent to the registration of bursts by a setup with dimensions of the order of the corresponding geometric dimensions (40×300 cm^2).[1)]

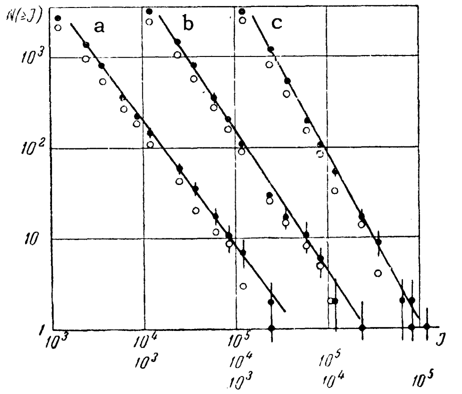

FIG. 3. Spectra of ionization bursts in chambers of rows IV (dots) and III (circles). a – bursts in entire area of setup; b – bursts taking separate structures into account; c – bursts in individual chambers.

Table II shows that the exponent γ in this case is larger than the exponent in the registration of bursts by the entire area of the setup (10 m^2); this difference lies far outside the statistical errors.

It follows from Table III that the higher the energy of nuclear-active particles (the larger the induced bursts), the smaller the average separation of these particles in structured bursts. Thus the measured mean separation of the most energetic particles in structured bursts of $\sim 10^4$ particles is 40—50 cm. The actual distance between these particles is less than the measured distance since the minimum measurable separation in our case is about 20—30 cm. Particles separated by less than 20 cm cannot be discriminated at all. We can therefore not exclude the possibility that some structures in structured bursts and, possibly, some "single" bursts are induced by several particles separated by less than 20—30 cm.

In order to improve the resolving power of the setup and thus to reduce the probability of registering groups of particles we plotted the spectra of bursts registered by individual 10×330-cm chambers. Here it must be taken into account that because of the elongated geometry the effective area of a chamber for registering groups of particles is less than 10% of the geometric area for particles separated by more than 30 cm.

We note that for separations $l \geq 20$ mm the probability of simultaneous registration by a single chamber is 4—5 times smaller than the probability of registration by four chambers connected in parallel.

In order to enhance the statistical accuracy of the spectrum of bursts registered in individual chambers, our data were summed over all chambers in a row. The results of this treatment of rows III and IV are shown in Fig. 3 (curve b) and the values of the exponent γ are given in the third row of Table II. The average value of γ in rows III and IV is 1.92 ± 0.05, which is considerably larger than the exponent for bursts registered over the entire area of the setup.

Similarly, the spectrum plotted and averaged over chambers in rows I and II has the exponent $\gamma = 1.57 \pm 0.03$, which is smaller than for rows III and IV. This difference results from the already indicated fact that a considerable fraction of the bursts in rows I and II is produced by the electron-photon component from the air, and the burst spectrum in individual chambers is characterized by $\gamma = 1.35 \pm 0.03$ (which is like the exponent that we obtained for individual chambers in rows V and VI). Therefore the exponent for I and II will be determined by approximately equal weighting of bursts due to the nuclear-active component ($\gamma = 1.92$) and of bursts due to atmospheric showers ($\gamma = 1.35$).

[1)] The probability of registering two particles whose path separation is $l = 30$-40 cm using apparatus 330 cm long with width $d = 40$ cm is 0.5-0.4. We therefore define the effective area as $S_{\text{eff}} = (0.5\text{-}0.4)S_{\text{geom}}$. We thus define the effective area S_{eff} for registering particle groups as $S_{\text{eff}} = WS_{\text{geom}}$, where W is the probability of registering a group and S_{geom} is the geometric area of the apparatus. If l is the distance between two particles and d is the width of very long registering apparatus, we have

$$W\left(\frac{l}{d}\right) = \frac{2}{\pi}\left[\arcsin\frac{d}{l} + \frac{l}{d}\left\{\sqrt{1-\left(\frac{d}{l}\right)^2} - 1\right\}\right] \text{ for } \frac{l}{d} \geqslant 1,$$

$$W\left(\frac{l}{d}\right) = 1 - \frac{2}{\pi}\frac{l}{d} \text{ for } \frac{l}{d} \leqslant 1.$$

It follows from Table II that the form of the burst spectrum depends greatly on the area of the measuring apparatus and on its power to discriminate laterally the individual particles in groups. In order to test this conclusion on a different setup we reworked our older experimental results obtained with a setup of area 0.6 m^2,[4] which was used to unvestigate bursts in the range from $\sim 10^3$ to 10^4 particles. This setup consisted of two rows of ionization chambers under a lead absorber (and part of the time under a composite lead-graphite absorber). Each row consisted of 22 chambers 90 cm long and 4 cm in diameter. This work was done in 1956—1958 at 3200 m above sea level; the total operating time was about 2000 hours.

The data obtained with this apparatus were used to plot the spectrum of bursts registered by the entire area of the setup as well as the spectrum of bursts registered in individual chambers. These spectra are shown in Fig. 4, where the numbers of bursts registered by both rows are plotted. The figure reveals the considerable difference existing in this case also between the spectra obtained by different treatments of the data. The spectrum of bursts registered in the entire area 0.6 m^2 has the exponent $\gamma = 1.67 \pm 0.05$, while that for individual chambers (each having an area ~ 0.04 m^2 and an effective area ~ 0.004 m^2) is characterized by $\gamma = 1.90 \pm 0.05$.

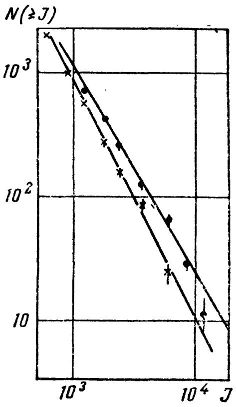

FIG. 4. Spectra of bursts registered by a setup having the area 0.6 m^2. o – bursts in entire area; x – bursts in individual chambers.

5. Investigation of irregularities in the spectrum of large ionization bursts. In connection with some investigations conducted at mountain heights [7,8] it has been stated that a sharp increase of γ is observed for bursts of $\sim 3 \times 10^4$ particles. Other authors did not observe this change of the exponent, because a setup having a large working area is required for the registration of such large bursts. However, it is very important to confirm reliably the change in the exponent, since this could possibly indicate a change in the characteristics of elementary events at high energies.

The setup of 10-m^2 area operated at 3200 m furnished sufficient statistics to show the aforementioned spectral irregularity for bursts of $\sim 3 \times 10^4$ particles. It follows from the experimental data in Fig. 3, independently of the method of treating the data (bursts in the entire area, bursts taking structures into account, and bursts in individual chambers) that our burst spectra exhibit no "bends" in the entire range of bursts from 10^3 to 10^5 particles.

In order to improve the systematic accuracy we summed the data obtained in chambers of rows III and IV. In addition, in order to permit a more valid comparison of our data with the results given in [7,8] we introduced a correction for the lead-to-brass transition effect and expressed the size of bursts as the number of particles passing simultaneously through the mean chord of a chamber, as in [7,8]. Figure 5 shows the spectra obtained in this manner; no bends are observed up to bursts of 2×10^5 particles.

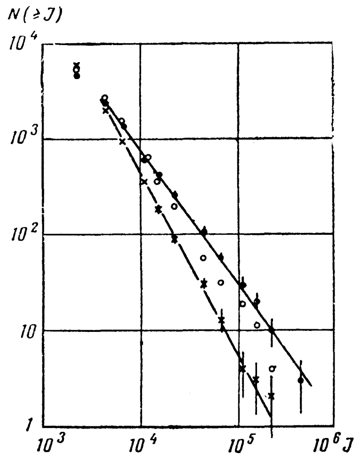

FIG. 5. Spectra of bursts in chambers of rows III and IV. ● – bursts registered in the entire area of the setup; o – bursts taking structures into account; x – bursts in individual chambers.

We also obtained the integral spectrum of large ionization bursts (up to 10^5 particles) at sea level, [12] without observing any irregularities.

DISCUSSION OF RESULTS

The foregoing experimental data confirm our previous result[4,9,12] that several nuclear-active particles often impinge simultaneously on a setup having a large working area. The probability of such events grows with the size of the bursts; the mean number of simultaneously incident particles also increases. While a single nuclear-active particle is usually responsible for a small burst ($\sim 10^3$ particles), several incident particles are usually associated with the registration of large bursts ($\sim 10^4$ particles). Therefore the size of a large burst does not indicate the energy of any single nuclear-active particle. Under these conditions the burst spectrum does not, of course, represent the spectrum of individual nuclear-active particles at a given height.

The results show that when a setup of large dimensions is used a very large difference can be observed between the burst spectrum and the spectrum of individual nuclear-active particles, because of the incident particle groups. Thus the spectra registered in areas of 10 m^2 and 1 m^2 have the exponents $\gamma = 1.38 \pm 0.03$ and 1.58 ± 0.03, respectively, while the spectrum of bursts in individual chambers, where group incidence has its smallest effect, has the exponent 1.92 ± 0.05. It must be mentioned that all these data on the spectra of bursts registered by setups having different areas were obtained by us with the same apparatus and differ only in the treatment of the experimental data. The configuration of the setup, the system used to select events, the operating time, the altitude, and uncontrolled changes, if any, of instrumental parameters were identical in all cases.

It follows from the foregoing experimental results that correct measurements of the energy spectrum of high-energy nuclear-active particles using ionization chambers require apparatus of small dimensions. In this way one reduces the probability that several particles will impinge on chambers simultaneously; the spectrum of measured bursts then approximates the energy spectrum of nuclear-active particles.

Let us consider the case in which cylindrical ionization chambers of radius r are used. We introduce the function $f(t)\,dt$, representing the fraction of the shower particles striking an infinitely long strip of width dt separated by the distance t from the shower axis, i.e., $dn = N_0 f(t)\,dt$ with $\int_{-\infty}^{\infty} f(t)\,dt = 1$, where N_0 is the total number of shower particles. This equation denotes that we assume that the lateral distribution of particles in the electron-photon shower is independent of its intensity (the total number of particles). In unit time under 1 cm^2 of the absorber there can be generated $F(N_0)\,dN$ showers having a total number of particles in the interval N_0, $N_0 + dN_0$, where $F(N_0)\,dN_0 = A\,dN_0/N_0^{\gamma+1}$. We now wish to determine the burst spectrum that will be registered by a chamber of length l with width 2r, assuming l to be long enough so that the effect of the chamber ends can be neglected.

If the shower axis passes at a distance x from the center of the chamber and the shower contains N_0 particles, n particles will enter the chamber:

$$n = N_0 \int_{x-r}^{x+r} f(t)\,dt = N_0 b(x), \quad b(x) = \int_{x-r}^{x+r} f(t)\,dt.$$

The burst spectrum in the chamber (the size of a burst being given by n) for a fixed value of x will be

$$\Phi(n,\ x)\,dn = F\left(N_0 = \frac{n}{b}\right)\frac{dn}{b} = \frac{Ab^{\gamma}(x)\,dn}{n^{\gamma+1}}.$$

Electron-photon showers can be generated by nuclear-active particles passing at different distances from the center of a chamber, i.e., x can assume any value from $-\infty$ to $+\infty$. Therefore the total number of cases when from n to n+dn particles will enter the chamber will be

$$\Psi(n)\,dn = dn \int_{-\infty}^{\infty} \Phi(n,\ x)\,dx = \frac{A\,dn}{n^{\gamma+1}} \int_{-\infty}^{\infty} b^{\gamma}(x)\,dx = Ac\,\frac{dn}{n^{\gamma+1}}.$$

Thus the spectrum of bursts in a narrow chamber will duplicate the spectrum of electron-photon cascades generated by individual nuclear-active particles if the lateral distribution does not depend on N_0.

When cylindrical ionization chambers having diameters $2r \geq 1$ cm are used, the function f(t) will depend mainly on the angular distribution of the electrons emerging from the lead absorber above the chamber, i.e., the function will depend on the "age parameter" of the shower. If the lead layer is sufficiently thick for an electron-photon cascade to develop to its maximum (even for showers having the largest number of particles N_0 in the investigated range), then the great majority of showers will be registered (because of the steep spectrum $F(N_0)\,dN_0$) near the maximum of their development, i.e., with an angular and therefore a lateral distribution of particles that is independent of N_0.

In order to obtain confirmation that for bursts induced by single particles the spectrum measured by a single chamber coincides with the spectrum of bursts in which the total number of cascade particles is determined we proceeded as follows. The spectrum was plotted for bursts registered over the entire area of the setup in the cases of single incident particles (structureless bursts). The spectrum of the same events in individual chambers was then plotted, adding the data for all four lowest rows (I–IV). Figure 6 shows that the two spectra are practically parallel, with $\gamma = 2.3$ (for the entire area) and 2.4 (for individual chambers). This confirms experimentally the hypothesis that the registration of bursts by a single chamber (measuring only part of the ionization induced by a cascade) results in the same spectrum that would be obtained by measuring all of the ionization if bursts were induced only by single particles. We therefore conclude that the data obtained by treating bursts in individual chambers represent

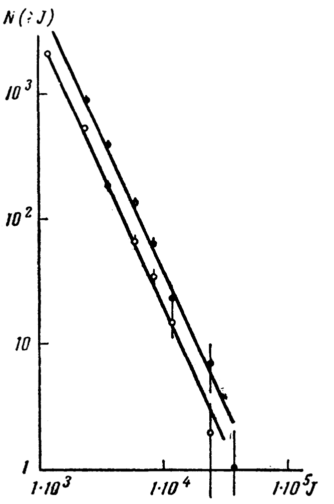

FIG. 6. Spectra of bursts induced in setup by single particles. ● – in entire area of apparatus; ○ – in individual chambers.

most closely the spectrum of bursts induced by single nuclear-active particles and that the spectrum of bursts (in the range 2×10^3—10^5 relativistic particles) has the exponent $\gamma = 1.92 \pm 0.05$. These bursts were registered by a setup having an effective area of only a few hundredths of a square meter (individual chambers of rows III and IV). Our experimental results obtained with an effective area ~ 0.003 m^2 show that in the range of small bursts (5×10^2 to 5×10^3 particles) the exponent has the same value (Fig. 4). In [2,3] spectra were measured in the range from 2×10^2 to 6×10^3 particles with setups of 0.1-m^2 area, yielding $\gamma = 1.9$—2.0.

In order to convert to the spectrum of bursts induced by single nuclear-active particles it must be taken into account that at mountain heights some bursts are induced by muons. As already indicated muons contribute $\sim 10\%$ of bursts having $\geq 2 \times 10^3$ particles; this contribution diminishes as the size of bursts increases. Therefore practically all bursts having $\geq 2 \times 10^3$ particles are induced by nuclear-active particles and our experimental spectrum represents the spectrum of nuclear-active particles. The muon contribution is quite large for bursts having less than 10^3 particles. Thus, when we extrapolate our spectrum of muon-induced bursts[9] to the region of smaller bursts we find that muons contribute ~ 30—40% of bursts having $\geq 2 \times 10^2$ particles. Accordingly, we obtain $\gamma \approx 1.8$ for the spectrum induced by nuclear-active particles in the range from $\sim 2 \times 10^2$ to 2×10^3 particles. We note that the energy spectrum of nuclear-active particles (protons) having energies of a few BeV was measured with a magnetic spectrometer at 3200 m.[16] The results indicate that the exponent of the integral energy spectrum of nuclear-active particles in the given energy range is close to 1.8.

For bursts having $\geq 2 \times 10^3$ particles the exponent of the spectrum due to individual nuclear-active particles is 1.92 ± 0.05. However, it must be taken into account that even when bursts are registered in individual chambers we cannot entirely exclude the possibility that several nuclear-active particles will impinge on a single chamber, especially when large bursts are registered. Therefore the result $\gamma = 1.92 \pm 0.05$ for large bursts is evidently somewhat too small. We can thus assume that for large bursts ($\sim 10^5$ particles) the exponent for individual nuclear-active particles at mountain heights could approach 2.0 or an even larger value. It follows that if the spectrum induced by individual nuclear-active particles in the entire investigated range from 2×10^2 to 10^5 particles at mountain heights is described by a power law, the exponent will vary from 1.8 for bursts of 2×10^2—2×10^3 particles to ~ 2 for bursts of $\sim 10^5$ or more particles.

The foregoing data indicate that at mountain heights the simultaneous incidence of several nuclear-active particles inducing bursts registered by setups having areas ~ 1 m^2 is significant for bursts of more than 10^3 particles. In recent years all investigations[3-9] of large ionization bursts have been conducted with setups having working areas of at least about one square meter. It is interesting to return to Table I, where the last column gives the areas of the setups used in the different investigations. The value of γ is seen to decrease as the area is enlarged; data obtained when setups having approximately equal areas are used exhibit satisfactory agreement. We believe that the spread of γ in the different reports can be associated entirely with the incidence of particle groups. In addition, the data in Table I represent another proof that the burst spectrum depends on the area of the setup and that this dependence becomes stronger as the area increases. Therefore the spectra measured in [3-9] do not represent the spectrum of nuclear-active particles at mountain heights, because group incidence was not taken into account in these investigations.

Our data show that the spectrum of bursts depends on both the size and configuration of the setup. In the lower rows of our arrangement, under a carbon absorber 210 g/cm^2 thick, a considerable fraction of the bursts resulted from the electron-photon component coming from the air. The exponent of bursts in individual chambers of these rows (1.57 ± 0.03) is considerably smaller than the exponent for individual nuclear-active particles (1.92 ± 0.05). A similar effect (the registration of the electron-photon component passing through the sides of the setup) can appear in other arrangements having a height comparable to the lateral dimensions, as in [8]. We must note the following fact. It has now been established that the spectra of extensive air showers with respect to both density and number of particles exhibit a very sharp change of the exponent in the vicinity of $N_S \sim 2 \times 10^6$ particles (at mountain heights). A similar fundamental change can be observed in the spectrum of bursts induced by an incident electron-photon component. It is therefore conceivable that the change of the exponent of the burst spectrum observed in [8] (see Sec. 5) can be accounted for by the changed exponent in the spectrum of the number of particles in extensive air showers.

---

[1] G. T. Zatsepin, JETP **19**, 1104 (1949); N. L. Grigorov, UFN **58**, 599 (1956).

[2] R. E. Lapp, Phys. Rev. **69**, 321 (1946).

[3] T. G. Stinchcomb, Phys. Rev. **78**, 321 (1950).

[4] Grigorov, Shestoperov, Sobinyakov, and Podgurskaya, JETP **33**, 1099 (1957), Soviet Phys. JETP **6**, 848 (1958).

[5] L. A. Farrow, Phys. Rev. **107**, 1687 (1957).

[6] Zatsepin, Krugovykh, Murzina, and Nikolskiĭ, JETP **34**, 298 (1958), Soviet Phys. JETP **7**, 207 (1958).

[7] Murzina, Nikol'skiĭ, and Yakovlev, JETP **35**, 1298 (1958), Soviet Phys. JETP **8**, 906 (1959).

[8] Denisov, Zatsepin, Nikol'skiĭ, Pomanskiĭ, Subbotin, Tukish, and Yakovlev, JETP **40**, 419 (1961), Soviet Phys. JETP **13**, 287 (1961).

[9] Babayan, Babecki, Boyadzhyan, Buja, Grigorov, Loskiewicz, Mamidzhanyan, Massalski, Oles, Tret'yakova, and Shestoperov, Izv. AN SSSR, Ser. Fiz. **26**, 558 (1962).

[10] Duthie, Fisher, Fowler, Kaddoura, Perkins, and Pinkau, Trans. Intern. Conf. on Cosmic Rays, I, Moscow, 1960, p. 28.

[11] Baradzeĭ, Rubtsov, Smorodin, Solov'ev, and Tolkachev, op. cit. [10], p. 152.

[12] Babecki, Buja, Grigorov, Loskiewicz, Massalski, Oles, and Shestoperov, JETP **40**, 1551 (1961), Soviet Phys. JETP **13**, 1089 (1961).

[13] Grigorov, Kondrat'eva, Savel'eva, Sobinyakov, Podgurskaya, and Shestoperov, op. cit. [10], p. 122.

[14] Dmitriev, Kulikov, and Khristiansen, JETP **37**, 893 (1959), Soviet Phys. JETP **10**, 637 (1960).

[15] H. Carmichael, Phys. Rev. **107**, 1401 (1957).

[16] Kocharyan, Saakyan, Aĭvazyan, Kirakosyan, and Aleksanyan, Izv. AN SSSR, Ser. Fiz. **19**, 515 (1955), Columbia Tech. Transl., p. 463.

Translated by I. Emin
5