# MAGNETO-ACOUSTIC OSCILLATIONS AND THE FERMI SURFACE IN ALUMINUM

N. A. BEZUGLYĬ, A. A. GALKIN, and A. I. PUSHKIN

Physico-technical Institute of Low Temperatures, Academy of Sciences, Ukrainian S.S.R.

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The anisotropies of magneto-acoustic oscillations in specimens of aluminum at a temperature of 4.2°K, ultrasonic frequencies of 183 and 223 Mc, and in magnetic fields up to 2500 Oe were investigated for directions of the wave vector of the sound wave q along the crystallographic directions [110], [100], [111]. The results of the experiments are in agreement with the Fermi surface for the second zone, constructed for aluminum by Harrison on the basis of a model of almost free electrons. They reproduce its size and shape and indicate the absence of any sharp intersections on the surface of the second zone.

## INTRODUCTION

$I$T has been established by experimental investigations of ultrasonic absorption in pure metals at low temperatures that the dependence of the ultrasonic absorption coefficient on the magnetic field has an oscillatory character. A qualitative explanation of this phenomenon, called geometric resonance in the literature, was first given by Pippard.[1] In this work, and also in later more rigorous studies carried out by Gurevich[2] under the assumption of an arbitrary law of dispersion for the electrons, and by a series of other authors,[3,4] it was shown that the oscillations of the absorption coefficient are brought about by the fact that the extremal diameter of the electron orbit in a direction perpendicular to q and H (q is the wave vector of the sound, q ⊥ H), becomes equal to an integral multiple of the wavelength of the sound λ if the magnetic field is varied periodically.

Experimental studies of the anisotropy of the oscillations of the ultrasonic absorption coefficient are of considerable interest for the reason that these investigations make it possible to determine the extremal diameters of the Fermi surface for electrons in the metal, and—for certain cases—to determine its shape completely.[2] In this case, it must be kept in mind that the phenomenon of geometric resonance is observed in the case in which the condition $ql \gg qr \gtrsim 1$ is satisfied ($l$ is the mean free path of the electrons in the metal and r is the radius of the electron orbit). Thus the number of oscillations, and hence the reliability of data on the extremal dimensions of the Fermi surface, is established by the quantity $ql$. This means that a study of the anisotropy of the oscillations of the ultrasonic absorption coefficient requires that the measurements be carried out on single-crystal specimens of metals of high purity, using ultrasound of as high a frequency as possible.

The present work is devoted to a study of magneto-acoustic effects in aluminum. Up to the present time, magneto-acoustic effects have been studied in some detail in tin,[5-8] copper,[9-11] lead,[6] indium,[12] zinc,[13] gold and silver.[14,15] So far as aluminum is concerned, two researches are known to date that are devoted to the experimental study of magneto-acoustic effects in this metal. Morse and Bohm,[16] did not observe oscillations of the sound absorption coefficient. The first to observe oscillations of the ultrasonic absorption coefficient in aluminum was Roberts,[17] who studied the dependence of the absorption coefficient upon the magnetic field for longitudinal waves of frequency from 10 to 100 Mc. The inadequacy of these investigations, which, as Roberts has shown, are in qualitative agreement with the model of the Fermi surface proposed by Harrison,[18,19] is that the limiting momentum was determined with low accuracy, inasmuch as not more than two insufficiently delineated oscillations were observed by the author. Therefore, it is of interest to carry out the investigation on specimens of aluminum of higher purity, at the same time raising the frequency of the ultrasound.

## TECHNIQUE OF MEASUREMENT AND CHARACTERISTICS OF THE SPECIMENS

For the investigation of the dependence of the absorption coefficient of the ultrasonic wave on the magnetic field, a pulse technique was em-

ployed that was described previously. [20] The measurements were carried out with the use of longitudinal sound at T = 4.2°K in a magnetic field up to 2500 Oe for two frequencies—183 and 223 Mc —on specimens of aluminum for which $R_{293°K}/R_{4.2°K}$ = 14,000—20,000.

In connection with the fact that the absorption coefficient of sound in aluminum at helium temperatures and at the frequencies used is very large, the specimens of aluminum in the experiments described were employed in the form of discs of diameter ~ 10 mm and thickness ~ 2 mm. The use of comparatively thin specimens required a delay element with the help of which the last sound pulse was separated from the initial sound pulse. As a delay element, bars of crystalline quartz were used with a square cross section of edge 8 mm and length 10 mm, which together with the aluminum specimen, were located between the transmitting and receiving resonant crystals. The absorption of the ultrasound in the quartz in the region of helium temperatures is very small, [21] and therefore the presence of a delay in the form of a quartz rod has practically no effect on the value of the output signal.

Samples of single-crystal aluminum in the form of discs were cut by an electro-erosion method from bars of pure aluminum which had passed through 16—20 zone meltings in such a way that the normals to the plane of the discs approximately coincided with the crystallographic directions [100], [110], [111]. Grinding of the surface of the disc to a plane parallel state with sufficient coincidence of the normals to its surface with the crystallographic directions mentioned was obtained by hand by use of abrasive powders with proper x-ray control of the orientation.[1]

The following samples were used in the experimental investigations: sample No. 1 with a departure of the normal to the surface of the disc from the crystallographic direction [110] of not more than 1°, sample No. 2 with departure of the normal to the surface of the disc from the crystallographic direction [100] not greater than 2°, and sample No. 3 with departure of the normal to the surface of the disc from the crystallographic direction [111] not greater than 2°.

The wave vector of the sound q coincided with the direction of the normal to the plane of the specimen. The velocities of propagation of longitudinal sound waves for the different orientations were taken from the work of Roberts, [17] the numerical

values of which were: for the direction [110] — 6.88 × 10⁵ cm/sec, for the [100] direction—6.75 × 10⁵ cm/sec and for the [111] direction—6.92 × 10⁵ cm/sec.

Recording of the dependence of the absorption coefficient of longitudinal ultrasonic waves in the aluminum specimen on the magnetic field was performed with the aid of a two-coordinate recorder, [20] which registered the intensity of the signal passing through the specimen for various values of the magnetic field. The rate of advance of the recorder chart was changed in proportion to the Hall potential difference arising on the germanium detector which was placed between the poles of the electromagnet close to the pin of the metallic cryostat with the sample.

## EXPERIMENTAL RESULTS

The investigation of the anisotropy of the oscillations of the absorption coefficient $\alpha$ were carried out under conditions in which the wave vector q was directed along one of the principal axes of the crystal— [110], [100], [111] —while the magnetic field vector H ⊥ q was rotated relative to q. For complete certainty in the reproducibility of the results, the recording of the dependence $\alpha(H)$ [for a given direction of propagation of the sound and given direction of the magnetic field vector H] was carried out twice. In all cases the resulting oscillatory curves were found to be identical.

1. q along [110]; $\nu$ = 183 Mc. A characteristic feature of the results obtained with this orientation is the presence (for a number of directions of the magnetic field vector H) of a large number of oscillations (up to 15), bearing witness to the long mean free path of the carriers. A second fact, which points to the large value of $ql$ (for certain directions, $ql$ > 200) is the form of the dependence of $\alpha(H)$: the value of the absorption coefficient at saturation is significantly larger than its value in the absence of the magnetic field.

A large number of clearly marked oscillations with excellent periodicity in the inverse field was observed for the orientation q[110], H[$\bar{1}$10], i.e., for such a direction of the magnetic field H relative to the wave vector q for which the oscillatory behavior of the absorption coefficient was not detected in the experiments of Roberts. [17] The form of the dependence $\alpha(H)$ (here and in the following $\alpha$ is expressed in arbitrary units) for this direction is shown in Fig. 1.

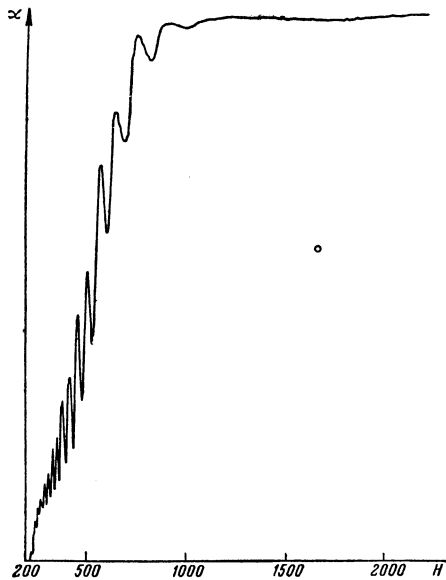With change in the direction of the magnetic field vector H, the period of oscillation in the inverse field first decreased and then increased.

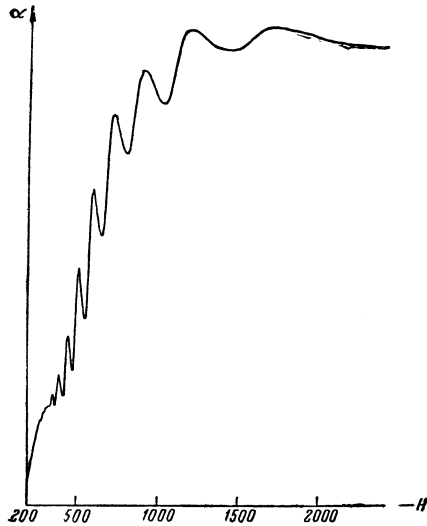FIG. 1. Record of the dependence of the absorption coefficient on the magnetic field for q along [110] and H along [$\bar{1}$10]; $\nu$ = 183 Mc.



FIG. 2. Record of the dependence of the absorption coefficient on the magnetic field for q along [110] and the direction of H making an angle of 35° with [$\bar{1}$10]; $\nu$ = 183 Mc.

The variation of $\alpha(H)$ is shown in Fig. 2 for the case in which the magnetic field vector H had made an angle of 35° with the direction [$\bar{1}$10]. A simple comparison of the oscillatory curves of Figs. 1 and 2 shows that the period of oscillation in the second case is larger than in the first.

In the directions of the magnetic field making angles 0—25° with the direction [00$\bar{1}$], long-period oscillations are observed in addition to the short-period oscillations. Owing to superposition of oscillations of various periods, the interpretation of the curve showing the dependence of $\alpha(H)$ is very

difficult. In this connection, it was possible to estimate the periods in the inverse field for the directions mentioned only for the long wave oscillations, and that only with an accuracy to within about 10 per cent. The long-period oscillations appear also in the case in which the magnetic field vector made angles of 15—25° with the direction [$\bar{1}$10]. However, under these conditions, it was possible to determine the value $\Delta(1/H)$ only for the short-period oscillations.

A marked oscillatory behavior of the absorption coefficient was also observed for the orientation q [110], H [00$\bar{1}$], for which oscillations were likewise not noted in the experiments of Roberts.[17] The distinguishing feature of this orientation was the absence of short-period oscillations. For this direction of the field, long-period oscillations were observed in pure form with periods $\Delta(1/H)$ = (6.4 ± 0.3) × $10^{-4}$ Oe$^{-1}$. The form of the dependence $\alpha(H)$ in this case is shown in Fig. 3.
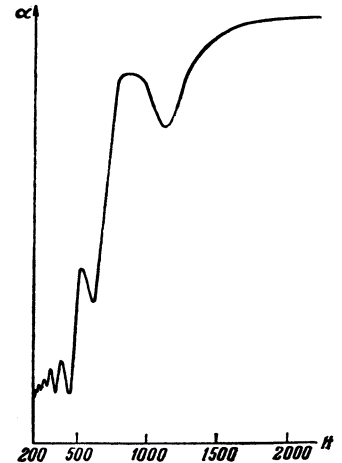


FIG. 3. Record of the dependence of the absorption coefficient on the magnetic field for q along [110] and H along [00$\bar{1}$]; $\nu$ = 183 Mc.

Let us compare the resultant experimental data with the model of the Fermi surface for aluminum proposed by Harrison.[18-19] Aluminum belongs to metals with face-centered cubic lattices, for which the first Brillouin zone has the form of the figure shown by the dashed lines in Fig. 4a. According to Harrison the first Brillouin zone, which is completely filled with electrons, is surrounded by packets of holes of the second zone, the boundaries of the surface of which are shown in Fig. 4a by the solid lines. The third zone (see Fig. 4b) has the form of interconnecting necks with different cross sections.

As is well known,[2,4] the period of oscillations $\Delta(1/H)$ is connected with the extremal distances k to the Fermi surface (k is perpendicular to the vectors q and H) by the following simple relation:
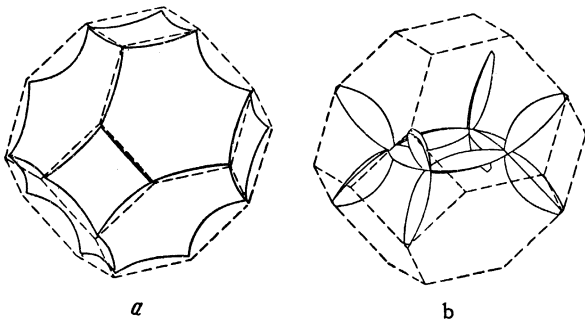
$$\hbar k = \lambda e/2c\Delta (1/H), \tag{1}$$

FIG. 4. The Fermi surface in aluminum according to Harrison: a — region of holes in the second zone; b — region of electrons in the third zone.

where $\lambda$ is the wavelength of the sound, e is the electron charge. If now, using Eq. (1), we express in units of $k_0$ ($k_0 = 2\pi/a_0$, where $a_0 = 4.04 \times 10^{-8}$ cm) the values of k found from the periods of oscillation in the inverse field for each of the directions of H, then they will represent the distance in wave-number space from the center of the Brillouin zone to the Fermi surface of the corresponding zone.

In Fig. 5 are shown the projections of the Brillouin zone on the plane perpendicular to the [110] axis, on which, in corresponding scale, is drawn the shape of the central cross section of the Fermi surface for the second zone plane, also perpendicular to the direction [110].

Data obtained from an experiment on the basis of Eq. (1) are also plotted in the same drawing. Attention is called to the circumstance that oscillations with short periods which correspond to a large value of K in general reproduce the shape of the central cross sectional area of the Fermi surface for the given direction of q very well, testifying in this fashion to the usefulness of the Harrison model. Additional support for the use of this model is found from the sign of the phase,
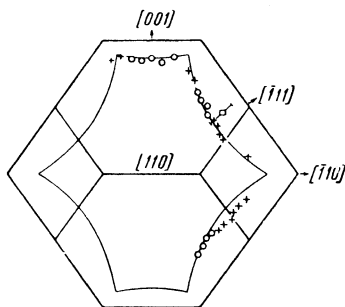


FIG. 5. Projection of the Brillouin zone on a plane perpendicular to the [110] axis, and the shape of the central cross section of the Fermi surface for the second zone. o, + — data of our measurements (o — accuracy to 2-3 per cent, + — accuracy to 5-10 per cent), □ — results of Roberts.[17]

which is obtained by extrapolation of the number of oscillations of the reciprocal of the magnetic field to H → ∞. In particular, for the directions H which form angles close to 45° with the [$\bar{1}$10] direction, the value of the constant phase is seen to be equal to $\pi/4$, in accord with theory;[2] in this case, the positive sign of the phase shows that the diameter of the Fermi surface in a direction close to the [111] direction (see Fig. 5), is seen to be minimal. So far as the long-period oscillations are concerned, for which the angular dependence of the projection of the extremal diameter on the [110] plane is shown in Fig. 6, the problem of their origin requires further analysis. Evidently these oscillations correspond to non-central cross sections close to the boundary of the zone in the [110] direction.
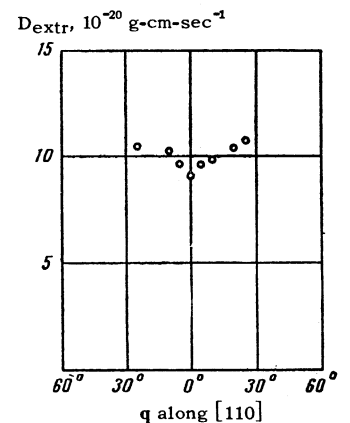


FIG. 6. Angular dependence of the projection of the extremal diameter on the [110] plane, obtained from measurements of $\Delta(1/H)$ of the long period oscillations.

2. q along [100]: $\nu = 183$ Mc. The reliable oscillatory behavior of the absorption coefficient has been found practically for all directions of the magnetic field H that are perpendicular to the wave vector q. However, in contrast with the foregoing case, when the sound was propagated along the [110] direction, the number of observed oscillations was not large (4—5 oscillations). The second distinguishing feature of the oscillations observed for the given direction of the wave vector q is their comparatively long period. Actually, whereas the period of oscillations which reproduce the central cross section of the Fermi surface of the second zone at a frequency of $\nu = 183$ Mc, for the case in which the wave vector of the sound q was directed along the [110] direction, changed upon variation of H from 0—180° in the limits $2.03 \times 10^{-4} - 3.04 \times 10^{-4}$ Oe$^{-1}$, in the case described here the period of oscillation changed within the range $6.1 \times 10^{-4} - 7.35 \times 10^{-4}$ Oe$^{-1}$, i.e., it was approximately 2.5 —3 times as large. This means that these oscillations correspond to non-central orbits.

Along with this, it should be noted that short-period oscillations were also observed for orientations close to the directions q [100], H [001] and q [$\bar{1}$00], H [0$\bar{1}$0], in addition to the long-period oscillations. Of course, because of the superposition of the oscillations of different periods, the accuracy of determination of $\Delta(1/H)$ for the short-period oscillations was not large ($\sim$ 5—10 per cent).

As an illustration of the oscillatory dependence of the absorption coefficient for q directed along the [100] axis, the curve of $\alpha(H)$ is plotted in Fig. 7 for H making an angle of 50° with the [001] direction.

The central cross section of the Brillouin zone is shown in Fig. 8 in a plane perpendicular to the [100] axis, on which (in proper scale) are plotted the central cross sections of the surface of the second zone, and also two other cross sections, which are remote from the central cross section along the perpendicular at differences of 0.75 $k_0$ and 0.90 $k_0$ ($k_0$ = $2\pi/a_0$). As is seen from this drawing, the short-period oscillations observed in the small range of angles reproduce very well the dimensions of the central cross section in the [010] and [001] directions, while the long-period oscillations basically reproduce the shape of the cross section for k = 0.9 $k_0$, which indicates the absence of sharp intersections in accord with the more general theory of Harrison. [19]

The magneto-acoustic effects in aluminum in a direction of sound along the [100] axis were studied at two frequencies—183 and 223 Mc, while the data obtained at these two frequencies were in excellent agreement with one another.
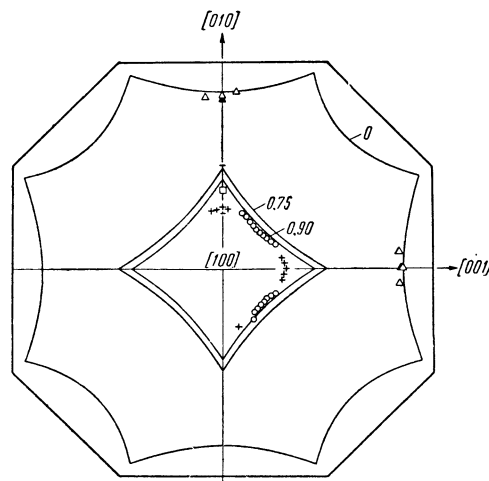


FIG. 8. Central cross section of the Brillouin zone on a plane perpendicular to the [100] axis, on which are plotted curves of the cross sections of the Fermi surface for the second zone. The levels are shown in units of $k_0$. o, + — data obtained by us from measurements of the long-period oscillations (o — accuracy to about 5 per cent, + — accuracy to about 10 per cent), $\triangle$, $\blacktriangle$ — data obtained from measurements of the short-period oscillations ($\triangle$ — frequency 223 Mc, $\blacktriangle$ — frequency 183 Mc), □ — results of Roberts. [17]

3. q along [111]; $\nu$ = 223 Mc. As in the previous case, reliable behavior of the absorption coefficient was observed in practice for all directions of the vector H in the angle range 0—180°. Since the number of oscillations was not large even in the given case, the accuracy in the determination of k [see Eq. (1)] amounted to $\sim$ 10 per cent. Measurements of the wave vector q in the given direction were carried out at frequencies of 183 and 223 Mc. Taking it into account that the measurement data at these frequencies were in excellent agreement and also considering the fact that the oscillations of the coefficient of absorption for $\nu$ = 223 Mc are more clearly pronounced, the comparison with the shape of the surface suggested by Harrison, will be based on the results obtained at 223 Mc.

The form of the Brillouin zone along the [111] direction is shown in Fig. 9. In this drawing, curves are drawn that reproduce the central cross section of the Fermi surface for the second zone in the (111) plane, and also for non-central cross sections which are removed from the central cross section by distances of 0.6 $k_0$ and 0.7 $k_0$. The data shown in this drawing, computed from measurements of the periods of oscillation on the basis of Eq. (1), are in good agreement, within the limits of experimental error, with the shape of the cross section corresponding to the level 0.6 $k_0$.



FIG. 7. Record of the dependence of the coefficient of absorption on the magnetic field for q along [100] and H making an angle of 50° with [001]; $\nu$ = 183 Mc.
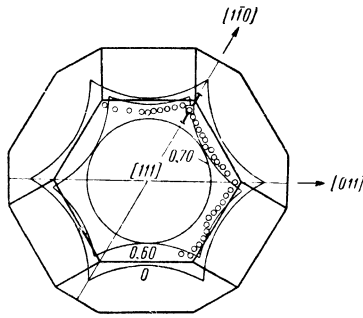
FIG. 9. Projections of the Brillouin zone on the plane perpendicular to [111], on which are plotted curves of the cross sections of the Fermi surface for the second zone. The levels are shown in units of $k_0$; □ — results of Roberts.[17]

## DISCUSSION OF THE RESULTS

The comparison given above of the experimental data obtained in the present work with the shape of the Fermi surface for aluminum suggested by Harrison on the basis of a model of almost free electrons shows that both the dimensions of the second zone, and also its shape, are well represented by the data of magneto-acoustic measurements. As a rule, the divergences are found within the limits of error of the experiment after elimination of the sharp edges of the Fermi surface: the results of the present investigation bear witness to the possibility of the use of the more complete calculations of Harrison,[19] which, without making any changes in the dimensions of the Fermi surface of the second zone, lead to the rounding off of its sharp edges. Unfortunately, it was not possible in the present investigation to study with great care precisely these regions of the Fermi surface: for these directions of the wave vector q and the magnetic field vector H which should give evidence on the dimensions of the Fermi surface in the directions of the sharp intersections, the oscillations of the absorption coefficient are weak. This was evidently brought about by the low density of electron states in the given directions.

In the studies that have been described it was possible to explain more or less in detail the character of the anisotropy of the short period oscillations which represent the central cross sections of the Fermi surface of the second zone for directions of the wave vector q along the [110] and [100] axes, and also the character of the anisotropy of the oscillations with periods of 2.5—3 times as large which represent the non-central cross sections of the Fermi surface of the second zone. In addition to these, for certain directions of q and H, the appearance was noted of oscillations with a still longer period, to which reference was made earlier in our brief note[22]. To estimate this period with sufficient reliability and at the same time to explain the character of the anisotropy of the period has not yet proved possible, although the discovery of such oscillations undoubtedly is

of great interest. The study of the anisotropy of the period of these very long-period oscillations (which is of great difficulty but which is not a hopeless problem) perhaps will shed light on the structure of the third zone.

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