ON THE THEORY OF SOUND ABSORPTION IN FERROMAGNETIC SUBSTANCES AT LOW TEMPERATURES

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The effect of paramagnetic impurity atoms on the absorption of sound in a ferromagnetic dielectric is considered, and the absorption coefficient of such a substance is calculated.

THE absorption of sound in a ferromagnetic dielectric has been considered in a number of works.[1-3] It was shown in them that both the processes associated with the exchange interaction of spin waves with each other (scattering of spin waves by spin waves) and processes connected with the relativistic interaction (splitting of a spin wave into three) are important for sound absorption in an ideal ferromagnetic. If the ferromagnetic contains paramagnetic atoms, another mechanism is possible, connected with the exchange scattering of a spin wave by the paramagnetic impurity. It will be seen below that such a mechanism is of importance at certain temperatures and concentrations of the paramagnetic impurity atoms.

The calculation of the absorption of low-frequency sound, as is well known, amounts to a determination of the departure from its equilibrium value of the distribution function of the elementary excitations, which departure is caused by the sonic field and the influence of the thermal conductivity of the sample. However, following Akhiezer [1] (see also [3,4]), we shall not take the thermal conductivity into account in the calculation.

We shall write the kinetic equation for the change with time of the number of spin waves in the form [2-4]

$$n_{\mathbf{k}} = L_{\mathbf{k}}^{(\xi)}\{n\} + L_{\mathbf{k}}^{(e)}\{n\} + L_{\mathbf{k}}^{(r)}\{n\}, \qquad (1)$$

where $L_{\mathbf{k}}^{(\xi)}\{n\}$, $L_{\mathbf{k}}^{(e)}\{n\}$, and $L_{\mathbf{k}}^{(r)}\{n\}$ are collision integrals associated respectively with the processes of exchange scattering of spin waves on paramagnetic impurity atoms, exchange scattering of spin waves on spin waves, and splitting of one spin wave into three by the relativistic interaction.

The average relaxation time corresponding to the first process is easily calculated if use is made of the expression for $L_{\mathbf{k}}^{(\xi)}\{n\}$ from [4]:

$$L_{\mathbf{k}}^{(\xi)}\{n\} = \frac{2\pi}{\hbar}\sum_{\mathbf{k}_1\mathbf{k}_2} |\Phi_{\mathbf{k}_1\mathbf{k}_2}|^2 [(n_{\mathbf{k}_1}+1)\, n_{\mathbf{k}_2} - n_{\mathbf{k}_1}(n_{\mathbf{k}_2}+1)]\, \delta(\varepsilon_{\mathbf{k}_1} - \varepsilon_{\mathbf{k}_2}),$$

$$\Phi_{\mathbf{k}_1\mathbf{k}_2} = \frac{J_{12}\sigma}{2N}\sum_{m\subset p} \exp[-i(\mathbf{k}_1 - \mathbf{k}_2)\,\mathbf{R}_n], \qquad (2)$$

where the summation over m is carried out over those sites p of the crystal lattice where the paramagnetic impurity atoms are located, N is the total number of atoms, J_{12} is the exchange integral, and σ is the spin of the paramagnetic impurity atom. The mean relaxation time of the two remaining processes has been calculated by other authors.[2,5]

Thus, we have for the mean relaxation times of all three processes

$$\frac{1}{\tau^{(\xi)}} \approx \xi\,\frac{(J_{12}\sigma)^2}{\hbar\Theta_C}\left(\frac{T}{\Theta_C}\right)^{1/2}, \qquad \frac{1}{\tau^{(e)}} \approx \frac{\Theta_C}{\hbar}\left(\frac{T}{\Theta_C}\right)^{4},$$

$$\frac{1}{\tau^{(r)}} \approx \frac{(\mu M)^2}{\hbar\Theta_C}\left(\frac{T}{\Theta_C}\right)^{1/2}, \qquad (3)$$

where ξ is the concentration of paramagnetic impurities in the ferromagnetic substance, Θ_C is the Curie temperature, μ is the Bohr magneton, and M is the saturation magnetic moment.

A comparison of these relaxation times with one another shows that

$$\tau^{(\xi)} \ll \tau^{(e)} \text{ for } T \ll \Theta_C(\xi^{1/2}J_{12}\sigma/\Theta_C)^{4/7}$$

and

$$\tau^{(\xi)} \ll \tau^{(r)} \text{ for } \xi \gg (\mu M/J_{12}\sigma)^2.$$

Consequently, in the temperature and concentration regions for which these conditions are fulfilled, the processes of exchange scattering of the spin waves on the paramagnetic impurity atoms are important in sound absorption.

In order to find the sound absorption coefficient we calculate the time rate of change of the entropy of the system due to the presence of a sonic field:

$$\dot{S} = \sum_{\mathbf{k}} \dot{n}_{\mathbf{k}} \ln \frac{n_{\mathbf{k}}+1}{n_{\mathbf{k}}} \approx \sum_{\mathbf{k}} \frac{\dot{n}_{\mathbf{k}} \delta n_{\mathbf{k}}}{n_{\mathbf{k}}(n+1)} . \quad (4)$$

The quantity $\dot{n}_{\mathbf{k}}$ is determined from the kinetic equation (1), which in view of the inequalities above now has the form

$$(\dot{n}_{\mathbf{k}})_s = (\dot{n}_{\mathbf{k}})_c \equiv L_{\mathbf{k}}^{(\xi)} \{n\}. \quad (5)$$

Here $\dot{n}_{\mathbf{k}}$ represents the change in the average number of spin waves with impulse **k** caused by the sonic field.

We shall consider the sonic field as classical and quasi-static and write the energy of a spin wave as $\epsilon_{\mathbf{k}} + \delta\epsilon_{\mathbf{k}}$, where $\delta\epsilon_{\mathbf{k}}$ is a quantity proportional to the deformation tensor of the body. This can be determined by making use of the Hamiltonian

$$\mathcal{H} = \sum_{m \subset p} \sum_{l=1}^{N} J_{12} (\mathbf{R}_{lm}) s_l \sigma_m,$$

which describes the exchange interaction of the spins of the ferromagnetic atoms **s** with the spins of the paramagnetic atoms σ.

Expanding the exchange integral in the displacement of the atom from its equilibrium position and replacing the operator s_l by the creation and annihilation operator for spin waves by means of the Holstein and Primakov transformation, we easily obtain the increment in spin-wave energy that we seek:

$$\delta\varepsilon_{\mathbf{k}} = 2J_{12}\sigma\xi u_{ll} \sum_{\mathbf{k}} a_{\mathbf{k}}^{+} a_{\mathbf{k}},$$

where u_{ii} is the trace of the deformation matrix. With the aid of the last formula we can write

$$(\dot{n}_{\mathbf{k}})_s = 2 (\partial n / \partial \varepsilon_{\mathbf{k}}) J_{12}\sigma\xi \dot{u}_{ll} . \quad (6)$$

A solution to Eq. (5) will be sought in the form

$$n_{\mathbf{k}} = n_{\mathbf{k}}^0 + \delta n_{\mathbf{k}},$$

where $n_{\mathbf{k}}^0$ is a Bose distribution function and $\delta n_{\mathbf{k}}$ is a small addition to it due to the sonic field. Equation (6) is rewritten in the form

$$(\dot{n}_{\mathbf{k}})_s = (\dot{n}_{\mathbf{k}})_c$$

$$= \frac{1}{32\pi^2} \xi \frac{(J_{12}\sigma)^2}{\hbar\Theta_C} ak\delta n_{\mathbf{k}} \int \left(\frac{\delta n_{\mathbf{k}'}}{\delta n_{\mathbf{k}}} - 1 \right) \delta(k - k')\, dk'do, \quad (7)$$

whence we have for $\delta n_{\mathbf{k}}$

$$\delta n_{\mathbf{k}} = \frac{16\pi}{A(\mathbf{k}/k)} \frac{\hbar\Theta_C}{J_{12}\sigma} \frac{1}{ak} \frac{\partial n_{\mathbf{k}}}{\partial \varepsilon_k} \dot{u}_{ll}, \quad (8)$$

where

$$A\left(\frac{\mathbf{k}}{k}\right) = \frac{1}{4\pi} \int \left(\frac{\delta n_{\mathbf{k}'}}{\delta n_{\mathbf{k}}} - 1 \right) \delta(k - k')\, \delta k'do'$$

depends on the polar and azimuthal angles of the wave vector **k**.

Substituting Eqs. (6) and (8) into Eq. (4) after replacing summation by integration, the dissipation function is easily obtained:

$$T\dot{S} = -Va^{-3}\xi\hbar\, (\dot{u}_{ll})^2 I,$$

$$I = \frac{1}{\pi^2} \int \frac{do}{A(\mathbf{x}/x)} \int \frac{x dx\, (\partial n / \partial x)^2}{n(n+1)} .$$

Proceeding in the usual way, we obtain an expression for the sound absorption coefficient for a ferromagnetic dielectric:

$$k = B\xi\, (\omega/c)^2 \hbar / mc,$$

where c is the speed of sound, $m = \rho a^3$, and B is a numerical coefficient.

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[5] Akhiezer, Bar'yakhtar, and Kaganov, UFN 71, 533 (1960), Soviet Phys. Uspekhi 3, 567 (1960).

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