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K_{e4} DECAYS AND THE ISOSCALAR PION-PION RESONANCE AT LOW ENERGY

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The probabilities for K_{e4} decays are computed and it is shown that the assumption of the existence of a pion-pion resonance with I = 0 and l = 0 at an energy of 310 MeV leads to decay rates which exceed the upper limit of the experimental data.

In a previous paper by one of the authors [1] it has been shown that the two-pion effective-mass spectra of the K_{e4} decays

$$K^0 \to e^+ + \nu + \pi^- + \pi^0, \qquad (1)$$

$$K^+ \to e^+ + \nu + \pi^0 + \pi^0, \qquad (2)$$

$$K^+ \to e^+ + \nu + \pi^+ + \pi^- \qquad (3)$$

are determined by the partial amplitudes $F^l(s)$, l = 0 and 1 of the reaction

$$\pi + \pi \to K + \overline{K} \qquad (4)$$

and by the effective coupling constant of the $K_{\mu 2}$ decay. Therefore the experimental data on these decays could yield information on the K-π and π-π interactions.

In the present paper we shall utilize the results obtained earlier [1] in order to compute the decay probabilities for the modes (1)—(3) under the assumption of the existence of a K-π-resonance [2] with spin 1 (the vector meson K*) for two cases: assuming the existence of a pion-pion resonance in the state I = 0, l = 0 at 310 MeV [3] and assuming a pion-pion scattering length in the same state equal to $2.5/m_\pi$ [4]. We shall show that the assumption of the indicated pion-pion resonance leads to values of the K_{e4} decay rates exceeding the upper limit of the experimental data.

In order to remove the kinematic singularities we replace $F^1(s)$ by the quantity

$$f^1(s) = [(s - 4m^2)(s - 4M^2)]^{-1/2} F^1(s), \qquad (5)$$

where m and M are the masses of the pion and kaon, respectively, s is the square of the effective mass of the two pions. For the sake of convenience we denote the partial wave amplitude $F^0(s)$ by $f^0(s)$:

$$f^0(s) = F^0(s).$$

Assuming a Mandelstam representation [5] without subtractions for the amplitudes of the process (4) and for pion-kaon scattering, and taking into account only the contribution of the pion-kaon resonance with a small width in the calculation of the contribution of the left-hand cut, we obtain the following dispersion relation for $f^l(s)$

$$f^l(s) = \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{\operatorname{Im} f^l(s')}{s' - s} ds' + G^l(s); \qquad (6)$$

$$G^l(s) = 8\sqrt{6}\, W\Gamma \left[1 + \frac{2sW^2}{[W^2 - (M+m)^2][W^2 - (M-m)^2]}\right] g^l(s), \qquad (7)$$

$$g^0(s) = \begin{cases} [(s-4m^2)(4M^2-s)]^{-1/2} \operatorname{arc\,tg} \dfrac{[(s-4m^2)(4M^2-s)]^{1/2}}{[W^2 - (M^2+m^2) + s/2]} & \text{for } 4m^2 \leqslant s \leqslant 4M^2 \\ [(s-4m^2)(s-4M^2)]^{-1/2} \dfrac{1}{2} \ln \dfrac{[W^2 - (M^2+m^2) + s/2] + [(s-4m^2)(s-4M^2)]^{1/2}}{[W^2 - (M^2+m^2) + s/2) - [(s-4m^2)(s-4M^2)]^{1/}} & \text{for } s \leqslant 4m^2,\ s \geqslant 4M^2 \end{cases} \qquad (8)^*$$

$$g^1(s) = \frac{2}{\sqrt{6}} [(s-4m^2)(s-4M^2)]^{-1} \left(1 + 2\left[W^2 - (M^2+m^2) + \frac{s}{2}\right] g^0(s)\right), \qquad (9)$$

where W and Γ are the energy and the half-width of the kaon-pion resonance.

In the region $4m^2 \leq s \leq 16m^2$ unitarity implies

$$\operatorname{Im} f^l(s) = f^l(s) e^{-i\delta_l} \sin \delta_l, \qquad (10)$$

where δ_l are the pion-pion scattering phase shifts. In the region $s > 16m^2$ the unitarity condition does not yield a simple relation like Eq. (10). However in the present paper we consider $f^l(s)$ only in the low energy region $4m^2 \leq s \leq M^2 < 16m^2$; there-

*arctg = $\tan^{-1}$.

fore the influence of the high-energy region is inessential.

One can expect Eq. (8) to hold also for $s > 16m^2$ and the $f^l(s)$ as well as the pion pion amplitudes vanish for $s \to \infty$. In this case the solutions of the integral equations are unique. Expressions for such solutions have been obtained by Omnès and Muskhelishvili [6].[1)]

From the above results and Eqs. (14) and (15) in [1] one can compute the rates of the processes (1)—(3). We choose the phase shift, δ_1 in accordance with the existence of a resonance at 750 MeV (the ρ-meson) [7]. For the phase shift δ_0 we envisage two cases: either the existence of a pion-pion resonance in the state I = 0, l = 0 at an energy 310 MeV and half-width 15 MeV or a pion-pion scattering length in the same state equal to $2.5/m_\pi$. As a result we obtain the following values for the decay probabilities for the modes (1)—(3):

$$W_1 = 3\cdot 10^2 \text{ sec}^{-1},\ W_2 = 1.5\cdot 10^4 \text{ sec}^{-1},\ W_3 = 3\cdot 10^4 \text{ sec}^{-1}$$

for the first assumption, and

$$W_1 = 3\cdot 10^2 \text{ sec}^{-1},\ W_2 = 5\cdot 10^2 \text{ sec}^{-1},\ W_3 = 1.1\cdot 10^3 \text{ sec}^{-1}$$

for the second assumption.

The decay mode (3)

$$K^+ \to e^+ + \nu + \pi^+ + \pi^-$$

would look like an anomalous τ-decay. From the fact that among 2000 τ events no event of the type (3) has been observed we obtain the following upper limit of the rate for this mode

$$W_{3\,\text{exp}} \leqslant 2.5\cdot 10^3 \text{ sec}^{-1}.$$

Thus the assumption of the existence of a pion-pion resonance in the I = 0, l = 0 state at an energy of 310 MeV, yields a value for the decay rate for the mode (3) larger by a factor of ten than the upper limit of the experimental data.

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[1)]The results obtained in this manner are valid only in the low energy region and are incorrect at higher energies.

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