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DECAY OF ULTRAHIGH ENERGY PARTICLES IN CONDENSED MATTER

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Decay of a fast particle involving a small transfer of momentum from to the atoms of the matter as a result of Coulomb scattering is considered. It is shown that interference of Coulomb scattering of the initial particle and the decay products leads to an appreciable decrease of the decay probability at ultrahigh energies.

1. INTRODUCTION

IN the case when an unstable particle decays not in vacuum but in matter, the Coulomb scattering of the particles participating in the decay process is usually disregarded. The argument usually used in this connection is that a sufficiently large momentum is transferred during the decay, so that distances comparable with interatomic distances cannot be appreciable. Actually, these considerations are not accurate, for during the course of the decay a finite small momentum can be transferred to the external field of the atoms of the medium, and then large distances can become significant.

Since the arrangement of the atoms of the matter is not known, it is meaningful to speak of a decay probability averaged over the atom positions. We can separate here two different cases of Coulomb scattering during the course of decay. The simplest is the case when the Coulomb scatterings of the initial and final particles are independent of each other (the average momentum is transferred to the matter only by the initial particle or only by the final particle). Then the Coulomb scattering does not differ from multiple scattering by a single particle, and since there is only one preferred direction of the initial particle momentum, the momentum averaged over the atom positions has the same direction as initially. It follows therefore that there is no average momentum transfer to the matter, the kinematics of the decay remains the same as in vacuum, and the argument advanced that large distances are insignificant remains valid.

Consequently the Coulomb scattering prior to the instant of decay or after the instant of decay can be neglected.

The situation is different if the Coulomb scattering occurs within the effective region of the decay. In this region we can no longer use the concept of initial and final particle, and consequently we cannot establish precisely which particle transfers the momentum. The momentum transfer is essentially connected in this case with the existence of both the initial and final particles, and we have here not one but at least two preferred momentum directions, of the initial and final particles. Consequently the average momentum transferred to the matter is not zero and the kinematics differs from that in vacuum; thus, if the momentum transferred to the matter is sufficiently small, large distances can become influential.

It follows from the foregoing that the effect under consideration has essentially a quantum character. In fact, the effect due to the transfer of momentum inside the interaction region, i.e., interference between the scattering of the initial and final particles, cannot be obtained by classical calculations.

We develop a general method for analyzing decay processes in matter with allowance for the possible momentum transfer to the medium during the decay. The analysis is confined to the case of sufficiently fast particles, the momenta of which are large compared with the reciprocal of the Thomas-Fermi radius of the atom $\lambda = me^2Z^{1/3}$ ($\hbar = c = 1$), so that we can consider only Coulomb scattering on the atomic nuclei, the presence of the electron shell being really regarded as a screen factor. The effective values of the momentum transferred to the atoms of the medium are small in this case, so that it is possible to neglect the recoil of the atoms, regarding the summary potential of the atoms of the medium as an external field:

$$U(x) = \sum_a U_0(x - x_a) - \left\langle \sum_a U_0(x - x_a) \right\rangle,$$

where $U_0(x)$ is the potential of an individual atom and the symbol $\langle \ldots \rangle$ denotes averaging over the coordinates x_a of the atoms. By virtue of the neutrality of the medium as a whole, $\langle \sum_a U_0(x - x_a) \rangle$ is a constant independent of x.

The problem consists of calculating the probability of decay in an external field U(x), averaged over the locations of the atoms of the medium. The calculations simplify appreciably if the averaging over the coordinates of the atoms is carried out not in the final result, but at an earlier stage, using the diagram technique developed by Abrikosov and Gor'kov [1] to take into account the effect of impurities in superconductivity theory, and by Edwards [2] in the theory of electric conductivity.

2. AVERAGED TRANSITION PROBABILITY

The decay probability in a medium can be represented as a series in powers of a potential U(x) (if the momentum representation is used, the Fourier component V(p) will be involved), so that the averaging of the probability reduces to averaging the products $V(p_1)V(p_2)\ldots V(p_n)$. If it is assumed that the Born approximation holds for scattering by a single atom, then we can take into account for each fixed degree of the potential of the individual $V_0(p)$ only the highest degree in the number n_0, of atoms per unit volume, neglecting the remaining powers of n_0.

In this approximation the average of the product of the potentials is represented by a sum over all possible combinations of pairwise averages, in the form [3]

$$\langle V(p)V(p')\rangle = F(p)\,\delta(p_\nu - p'_\nu);$$

$$F(p) = n_0(2\pi)^3 |V_0(p)|^2 \delta(p_4). \qquad (2.1)$$

The presence of $\delta(p - p')$ in (2.1) allows us to make an analogy between (2.1) and the propagation function of the "quasiparticle." The analogy with quantum field theory can be made even more complete if we formulate a rule for obtaining the averaged transition probability from a known matrix element which, as can be readily verified, assumes the same form as the Wick theorem, except that the convolution of the operators is replaced by paired averaging of the potentials (2.1).

It follows therefore that we can use a graphic method, analogous to the diagrams of the quantum field theory. However, since it is necessary to average the transition probability and not a matrix element, we cannot confine ourselves to ordinary diagrams for matrix elements, and we must consider generalized Feynman diagrams, which show in addition to the diagram for the matrix element M also the mirror reflection of the diagram, corresponding to the Hermitian-conjugate matrix element M^+. An example of such a diagram is shown in Fig. 1, in which the pair averaging (2.1) is shown by a dashed line.

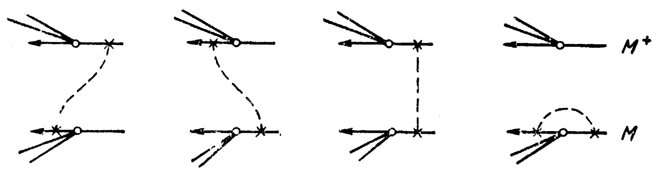

FIG. 1

The use of the graphic method enables us to separate the most essential diagrams in each approximation, and then sum the diagrams. It is possible to disregard, first of all, diagrams of the type of Fig. 1c, which pertain to the scattering by a single particle, and the diagrams (Fig. 1d) which do not describe the transfer of momentum to the medium. The main diagrams are those of type 1a and 1b, which describe the transfer of momentum to the medium within the effective region of interaction. Summation of diagrams of this type is facilitated by the fact that the quantity $n_0\lambda^{-3}$, which is the analog of the interaction constant, is $\sim 10^{-4}$ even for dense media. It is therefore sufficient to use an approximation of the Bethe-Salpeter type.

Let us consider the decay of a charged particle into two neutral particles and one charged particle. If we integrate over the momenta of the neutral particles, then the transition probability will be a function of the momenta p_1 and p_2 of the initial and final charged particles only. Graphically this is represented by joining the lines of neutral particles on Fig. 2. In order to take into account the scattering of the medium, it is necessary to sum diagrams of the type 1a and 1b.

Let us sum first all diagrams of type 1a. Such a sum, obviously, satisfies an integral equation of the type 2a. The similarity between this equation and the Bethe-Salpeter equation becomes obvious

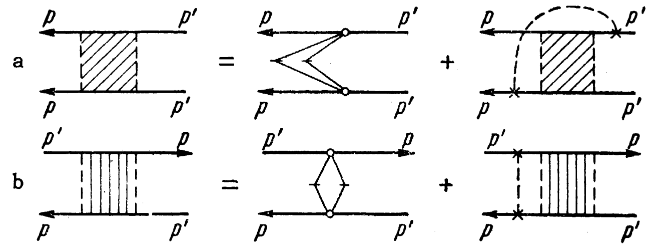

FIG. 2

if the lines of the momenta for the conjugate matrix element are turned in the opposite direction (Fig. 2b).

Let us denote the sum of the diagrams of type 1a by $R_a(p_2, p_1; p_2', p_1')$. Then for $p \gg \lambda$ the equation of Fig. 2a assumes the form

$$R_a(p_2,p_1;\, p_2',p_1') = R_0(p_2,p_1;\, p_2',p_1') + 4E_1E_2\int \frac{d^4l\, F(l)\, R_a(p_2+l,\, p_1,\, p_2',\, p_1'-l)}{[(p_1'-l)^2-m^2-i\delta]\,[(p_2+l)^2-m^2+i\delta]}, \quad (2.2)$$

where $F(l)$ has been defined in (2.1). The effective values of l in the integral (2.2) do not exceed λ, for when $p \gg \lambda$ we can assume that $R_a(p_2+l, p_1; p_2', p_1'-l) \approx R_a(p_2, p_1; p_2', p_1')$, hence the solution (2.2) has the form

$$R_a(p_2,\, p_1;\, p_2',\, p_1') = R_0(p_2,p_1;\, p_2',\, p_1')\,(1 - B(p_2,p_1'))^{-1};$$

$$B(p,\, p') = n_0\,(2\pi)^3\, 4E_1E_2\int d^3l\,|V_0(\mathbf{l})|^2\,(\mathbf{l}^2 - 2\mathbf{p}'\mathbf{l} - i\delta)^{-1} \times (\mathbf{l}^2 + 2\mathbf{p}\mathbf{l} + i\delta)^{-1}. \quad (2.3)$$

Let us sum now the diagrams of the type 1b and all the cross diagrams, in which there are lines of type 1a and 1b. In this case it is convenient to start from the sum of the diagrams R_a, surrounding it with lines of the type 1b, which leads to an interval equation (Fig. 3) in analytic form

$$R(p_2,\, p_1;\, p_2',p_1') = R_a(p_2,p_1;\, p_2',p_1') + 4E_1E_2\int \frac{d^4l\, F(l)\, R(p_2,\, p_1-l;\, p_2'+l,\, p_1')}{[(p_1-l)^2 - m^2 + i\delta]\,[(p_2'+l)^2 - m^2 - i\delta]}, \quad (2.4)$$

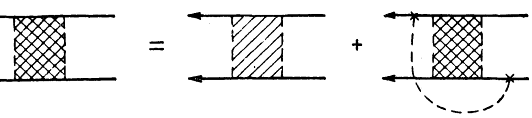

FIG. 3

The solution of (2.4) is found in analogy with the solution of (2.2) and has the form

$$R(p_2,\, p_1;\, p_2',p_1') = R_0(p_2,\, p_1;\, p_2',p_1') \times [1 - B(p_2,\, p_1')]^{-1}\,[1 - B^*(p_2',\, p_1)]^{-1}.$$

The transition probability is connected with the value of $R(p_2, p_1'; p_2', p_1')$ for pairwise coinciding arguments. Therefore the decay probability in the medium $dW(p_2, p_1)$, integrated over the momenta of the neutral particles, is connected with the corresponding decay probability in vacuum $dW_0(p_2, p_1)$ by the relation

$$dW(p_2,\, p_1) = dW_0(p_2,\, p_1)\,|\,1 - B(p_2,\, p_1)\,|^{-2}. \quad (2.5)$$

In calculating the integral (2.3) for very high energies of the initial particle, it is necessary to take into account the final energy width of the particle state with definite momentum in the matter, arising as a result of Coulomb scattering by the atoms of the matter. In other words, we cannot assume that the Green's function of the particle in the medium coincides with its vacuum value, but it is necessary to use the Green's function of the particle in the matter, averaged over the atom locations. An account of this circumstance causes the quantity δ in (2.3) to be finite and in the exponential-screening approximation,

$$U_0(r) = (Ze^2/4\pi r)\, e^{-\lambda r}, \qquad V_0(\mathbf{p}) = Ze^2\,(2\pi)^{-3}\,(\mathbf{p}^2 + \lambda^2)^{-1},$$

it is determined by the formula

$$\delta_1 = \alpha E_1, \qquad \delta_2 = \alpha E_2, \qquad \alpha = (n_0\lambda^{-3})\, Z^2e^4\,(\lambda/2\pi). \quad (2.6)$$

Calculation of the integral (2.3) in the approximation employed yields for the region $\vartheta \ll 1$ of small angles of charged-particle emission, which is of greatest interest at high energies,

$$B = -(\alpha/4\lambda)\,\pi\,(\vartheta^2 + \vartheta_{cr}^2)^{-1/2}\,(1 + \vartheta_{cr}^2\,(\vartheta^2 + \vartheta_{cr}^2)^{-1});$$

$$\vartheta_{cr} = (\alpha/2\lambda)\,[(E_1^2 - E_2^2)/E_1E_2]. \quad (2.7)$$

It follows from (2.7) that Coulomb scattering decreases the decay probability. An appreciable decrease in the decay probability occurs when $B \gtrsim 1$, i.e., when the condition $\vartheta \lesssim (\alpha/\lambda)$ is satisfied.

If this condition is satisfied for the angles $\vartheta \sim M/E$ that are effective during the decay, then the influence of the Coulomb scattering will greatly affect not only the differential but also the total decay probability. It follows therefore that the total probability of any decay in matter decreases appreciably at energies above the critical value

$$E_{cr} \sim M\,(\lambda^3/Z^2e^4n_0), \quad (2.8)$$

which for dense media near the end of the periodic system yields energies on the order of $(10^4 - 10^5)M$.

3. DECAY OF A FAST MUON IN MATTER

Let us illustrate the application of the method with μ-e decay in matter as an example. Assuming the meson to be sufficiently fast, we can use (2.5)—(2.7). As is well known, the differential decay probability integrated over the momenta of the neutral particles of a completely polarized muon in vacuum has the form

$$dW_0 = \frac{G^2 d^3k}{3(2\pi)^4 E\omega}\,\{[3M^2 - 4\,(p_\nu k_\nu)]\,(p_\nu k_\nu) - \omega\,(p - E\cos\vartheta)\,(M^2 - 4\,(p_\nu k_\nu))\}, \quad (3.1)$$

where p_ν and k_ν are the 4-momenta of the meson and decay electron; we have neglected here the ratio of the electron mass m_e to its energy ω. The range of variation of k in (3.1) is limited by the condition

$$Mm_e \leqslant p_\nu k_\nu \leqslant \tfrac{1}{2}(M^2 + m_e^2), \qquad (3.2)$$

in a system traveling together with the meson; this condition thus determines the maximum and minimum energies of the decay electron.

In calculating the total probability, only the upper limit is important, and we can neglect the electron mass in (3.2), too. Integrating (3.1) with account of (3.2), we readily obtain the total decay probability of a moving meson in vacuum

$$W_0 = \tau_0^{-1}(M/E), \qquad \tau_0 = 192\pi^3/G^2M^5, \qquad (3.3)$$

which agrees with the Lorentz transformations for the lifetime of the meson. The total decay probability in matter can be readily estimated for energies which are larger than the critical energy (2.8). In this case the effective values of angle ϑ are small compared with ϑ_{cr}, so that the ratio dW/dW_0 does not depend on the decay angle and is determined by the meson and electron energies:

$$dW \cong dW_0\left(\frac{E^2-\omega^2}{E^2-\omega^2+\pi E\omega}\right)^2, \qquad E \gg E_{cr}, \qquad \vartheta \ll \vartheta_{cr}. \qquad (3.4)$$

Substituting (3.1) in (3.4) and integrating with account of (3.2), we can obtain the total decay probability in matter in the form

$$W \cong 0.3W_0, \quad E \gg E_{cr}. \qquad (3.5)$$

If the meson energies are on the order of critical, then it is necessary to take into account the dependence of dW/dW_0 on the decay angle, which complicates the integration. For energies less than critical the decay probability in matter coincides with the decay probability in vacuum. It follows from the foregoing that at ultrahigh energies the lifetime of the muon relative to decay into an electron and neutrino increases appreciably. This can give rise to a situation wherein the probability of decay into another final state is comparable with or larger than (3.5). In particular, if the characteristic angle of emission of a charged particle in decay exceeds M/E, for example $\sim \sqrt{M/E}$, then the Coulomb scattering hardly influences the probability of such a decay.

It must be noted that the effect considered here is an analog of the Landau-Pomeranchuk effect for bremsstrahlung[4]. In both cases there is a decrease in the probability of the process because the Coulomb scattering inside the effective interaction region disturbs the coherence of the particles participating in the process. The only difference is that the bremsstrahlung for soft quanta has a classical limit, and in this limiting case it is possible to consider the effect classically.

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