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SOME REMARKS ON LOW-ENERGY ππ SCATTERING

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RECENTLY many authors [1-5] solved numerically the equations of low-energy pion scattering. Among the solutions obtained by Serebryakov and Shirkov there are some with resonances in the A_0-wave and in the A_1-wave (waves with $I = S = 0$ and $I = S = 1$, respectively).

The resonance in the p-wave is completely due to the large A_0-wave. In the absence of resonance in the A_0-wave, there are no solutions with p-wave satisfying the threshold condition

$$A_1(\nu)|_{\nu=0} = 0.$$

For the width of the p-resonance there is an upper limit of 50 MeV, connected with saturation of the A_0-wave. In many papers [2-5], the Chew-Mandelstam equations were solved for the ππ-scattering s- and p-waves. To obtain resonance in the p-waves, it is necessary to cut off in these equations the left-half unphysical cut. We note that the left cut is cut off either very far ($\Lambda = 10^8$ [4]), or else the left cut is replaced by a far pole $\nu = -10^3$ [5]. Solutions were obtained with resonance only in the p-wave, due to the "bootstrap" mechanism. We shall show that such a resonance is determined by the contributions of large singularities of the left cut. [5]

Let us cut off the left cut in the Chew-Mandelstam equations

$$A_i(\nu) = \frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im} A_i(\nu')}{\nu'-\nu}d\nu' + \frac{1}{\pi}\int_0^\Lambda \frac{f_i(\nu')}{\nu'+\nu+1}d\nu',$$

$$f_i(\nu) = \frac{1}{\nu+1}\int_0^\nu \left(1-2\frac{\nu'+1}{\nu+1}\right)\Big[\alpha_{i0}\operatorname{Im} A_0(\nu')$$

$$+\alpha_{i2}\operatorname{Im} A_2(\nu') + 3\left(1-2\frac{\nu}{\nu'}\right)\alpha_{i1}\operatorname{Im} A_1\Big]d\nu'.$$

We shall show that the solutions with coinciding resonances in the s- and p-waves satisfy the Chew-Mandelstam equation for a small cutoff parameter.

We consider the δ-approximation

$$\operatorname{Im} A_1(\nu) = \lambda\pi\alpha_1\delta(\nu-\nu_r),$$

$$\operatorname{Im} A_0(\nu) = \lambda\pi\alpha_0\delta(\nu-\nu_r), \qquad \operatorname{Im} A_2(\nu) = 0.$$

Here $\lambda\alpha_i = \Gamma_i$, Γ_i is the total resonance width, and ν_r is the position of the coinciding resonances. Then

$$f_1(\nu) = \frac{\pi\lambda}{\nu+1}\left(1-2\frac{\nu_r+1}{\nu+1}\right)\left[\frac{2}{3}\alpha_0 + 3\left(1-\frac{\nu}{\nu_r}\right)\alpha_1\right].$$

The threshold condition of the p-wave yields

$$2\nu_r\frac{\lambda-\nu_r}{(\Lambda+1)^2}\alpha_0 = 3\alpha_1\left[1-6\ln\frac{\Lambda+1}{\nu_r+1}+3(4\Lambda+2-\nu_r)\right]. \tag{1}$$

We put $\alpha_0 = \varphi(\Lambda)\alpha_1$; the function $\varphi(\Lambda)$ is shown in Fig. 1. At not too large values of Λ, this is the condition for the correlation between the A_0- and A_1-waves, and consequently, confining ourselves to low-energy contributions, we find that resonance in the p-wave is due to resonance of the A_0-wave.

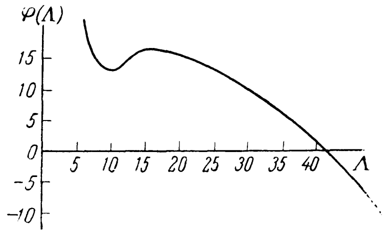

FIG. 1

For $\Lambda > 40$ (energy ~ 2 BeV) the function $\varphi(\Lambda)$ becomes negative and there is no wave correlation. The A_0-wave satisfies the threshold condition independently of the A_1 wave; the resonance is produced by the "bootstrap" mechanism; it is impossible at low cut-off. When $\Lambda = 10$—15, condition (1) leads to an exact connection between α_0 and α_1 in Eqs. (3.8) of [1]:

$$\alpha_0 = 14\alpha_1.$$

Combining the threshold conditions with the normalization of the A_0 wave: $A_0(0) = 5\lambda$,

$$\alpha_0\left[1+\frac{2}{3}\nu_r\frac{\lambda-\nu_r}{(\Lambda+1)(\nu_r+1)}\right]$$

$$+6\alpha_1\left[\frac{\Lambda-\nu_r}{1+\Lambda}\frac{2+\nu_r}{1+\nu_r}-2\ln\frac{\Lambda+1}{\nu_r+1}\right] = 5\nu_r, \tag{2}$$

we obtain the dependence of α_1 on Λ (see Fig. 2). For $\Lambda = 10$—15 we get $\alpha_1 = \Gamma_1/\lambda = 2.14$, Γ/λ

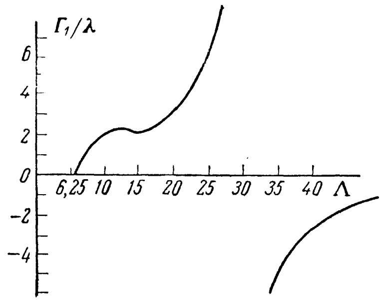

FIG. 2

= 0.34. The width Γ_1 of the p-wave differs then from the value of Γ introduced by Frazer and Fulco by a factor $\nu_r (\Gamma_1 = \nu_r \Gamma)$. This result coincides with the results of Serebryakov and Shirkov. It is seen from the curves that there exists a whole region of low cutoff ($\Lambda_{eff} = 9-15$), in which the Serebryakov-Shirkov solution with resonances in the A_0 and A_1 waves satisfies the Chew-Mandelstam equation. It is shown here that an account of the high-energy contributions (large Λ) changes radically the character of the solution. Apparently this limits the value of the cutoff of the left cut in the Chew-Mandelstam equation, if we wish to obtain a closed-form description of the low-energy scattering. It is interesting to note that the value of the parameter Λ coincides with the value of the cutoff parameter of the Chew-Mandelstam equation, which guarantees convergence of the expansion of the amplitude in Legendre polynomials. However, the question of the possibility of a closed-form description remains open. A probable experimental check of this question involves the question of the maximum of the width of the p-wave resonance.

Let us show how this maximum arises in the case of low cutoff. We consider the saturation of the A_0 wave on a large interval, i.e.,

$$\operatorname{Im} A_0(\nu) = \sqrt{(\nu+1)/\nu}, \quad 0 < \nu < \Lambda,$$

$$\operatorname{Im} A_0(\nu) = 0, \quad \nu > \Lambda; \qquad \operatorname{Im} A_1(\nu) = \pi \Gamma_1 \delta(\nu_r - \nu).$$

Condition (1) goes over when $\Lambda \gg 1$ into

$$\frac{\Gamma_1}{\nu_r} + \frac{3\Gamma_1}{\nu_r}\left[\frac{(4\Lambda + 2 - \nu_r)(\Lambda - \nu_r)}{\Lambda^2} - 2\ln\frac{\Lambda+1}{\nu_r+1}\right] - \frac{1}{3\pi}\left(1 + \frac{1}{\Lambda}\ln 4\Lambda\right) = 0. \tag{3}$$

When $\lambda = 10-15$, we get $\Gamma_{max} = 0.43$, which corresponds to a bipion width of 43 MeV. No such maximum exists for large Λ.

Thus, if we confine ourselves to the low-energy region in the Chew-Mandelstam equation without subtraction, assuming that the contribution of the high-energy region is small, then the corresponding solution coincides with the Serebryakov-Shirkov solution for low-energy scattering.

In conclusion, I am deeply grateful to V. V. Serebryakov and D. V. Shirkov for formulating the problem and for continuous interest in it.

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NONMETALLIC NICKEL AT HIGH COMPRESSIONS

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IT is well known that, beginning from a certain density, dielectrics necessarily become metals on compression (see the final paragraph of this letter). It is natural to expect, therefore, that metals would retain their metallic properties on compression. Up to now, as far as we know, no one has noted the fact that on compression a metal may be transformed into a dielectric within a certain range of densities. Our calculations suggest that this unusual behavior is exhibited by nickel. Calculations similar to those carried out by one of the present authors [1] indicate that, beginning from a density corresponding to the compression $\delta = 6.5$, i.e., from a density of 60 g/cm^3 (obtained at a pressure of 250×10^6 atm), nickel becomes an insulator.

The reason for this lies in the fact that an atom of nickel has 28 electrons, i.e., the number