INELASTIC INTERACTIONS OF HIGH-ENERGY PARTICLES IN RENORMALIZED STRONG-INTERACTION THEORIES

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The expansion of the Green's functions and differential cross sections for inelastic processes in powers of the reciprocal of the energy $1/s$ is deduced in renormalized theories. In some cases the first terms of the series are the usual peripheral diagrams, whereas in other cases they are somewhat more complicated. The region of applicability of the results obtained is much larger than that for the usual pole theory of peripheral collisions.

MANY recent papers are devoted to a description of strong interactions at high energies (see, for example, [1-3], where a detailed bibliography can be found). The corresponding processes are usually inelastic. They are frequently described by a peripheral model, according to which the most appreciable contribution to the matrix elements is introduced by diagrams with exchange of a minimum number of particles [2,4-6]. The experimental data apparently do not contradict such a model (see, for example, [1-3,7-9] etc.). However, in many cases doubts arise concerning the correctness of this model and the region of its applicability, because the theoretical premises on which the model is usually based are not very convincing, and the experiments so far have not been too accurate (the available exact theoretical results [10,11] are all unphysical and pertain to the case of large 4-squares of the momenta.

In the present paper we present for the analysis of strong interactions at high energies in renormalized theories a method which is sufficiently convincing theoretically. In many cases the results agree with the ordinary assumptions of the peripheral model of inelastic interactions, but there are also cases when the ordinary peripheral model is deformed in a certain manner.

The starting points in the developed construction are the general properties of renormalized theories, formulated for example in the book of Bogolyubov and Shirkov [12]. Unlike in other studies [4-6], the small parameter is $1/s$. Many of the assumptions which we must make here concerning the properties of the perturbation-theory series as a whole have not been demonstrated. These assumptions are quite usual, and are considered to be valid, for example, in electrodynamics. In meson theories their correctness raises grave doubts, but we cannot get along without them.

In the first section we consider the kinematics of the investigated inelastic processes as $s \to \infty$. They are divided into two types: processes in which the momentum transfer l between the fast and slow particles and the momentum p_0 are such that $\lim_{s\to\infty} |p_0 l|\, s^{-1} = u > 0$, and processes in which $\lim_{s\to\infty} |p_0 l|\, s^{-1} = 0$. All further analysis will be made with processes of the first type as an example.

In the second section all the perturbation-theory diagrams for the given processes are subdivided into a finite number of groups of diagrams of given topology. It is shown further that with power-law accuracy the contributions of all the diagrams of a given topology to the Green's functions are the same. To compare the importance of diagrams of a given topology it is therefore sufficient to compare diagrams of the same topology, the high-energy parts of which correspond to the first nonvanishing approximations of perturbation theory.

In Sec. 3 the known method of generalized diagrams [11,13] is developed for this purpose. This method is used in Sec. 4 to compare the importance of diagrams of different topologies. It turns out here that in the limit as $s \to \infty$ the main contribution to the Green's function of the process is made by diagrams of a certain specified topology, corresponding to exchange of one or a small finite number of particles between high-energy and low-energy blocks.

In Sec. 5 we consider the simplest properties of the corresponding expansion of Green's functions in powers of $1/s$ and the region of applicability of the results obtained.

1. FORMULATION OF THE PROBLEM AND KINETICS

A. We investigate inelastic scattering at energies, in which two groups of particles are formed: fast p_i and slow k_j. We have ($\nu \geq 2$)

$$k_0 + p_0 = \sum_{j=1}^{\nu'} k_j + \sum_{i=1}^{\nu} p_i, \qquad 1 \leqslant i, j \leqslant \nu, \nu',$$

$$0 \leqslant \alpha, \beta \leqslant \nu, \nu'; \tag{1.1}$$

$$k_\alpha^2 = \mu_\alpha^2, \qquad p_\beta^2 = m_\beta^2; \qquad \mu_\alpha^2, m_\beta^2 \to m_\gamma^2; \qquad k_\alpha^0, p_\beta^0 > 0. \tag{1.2}$$

This process is described by a Green's function $G(k_\alpha, p_\beta)$. The purpose of the present work is to study the asymptotic behavior of $G(k_\alpha, p_\beta)$ at high energies in renormalized theory, under relations between k_α and p_β which will be considered below.

We introduce the momentum transfer between the fast and slow particles

$$l = k_0 - \Sigma k_j = \Sigma p_i - p_0, \qquad l^2 = t. \tag{1.3}$$

The conservation law (1.1) has for fast particles the form

$$l + p_0 = \sum_{i=1}^{\nu} p_i. \tag{1.3'}$$

We put further

$$s = (k_0 + p_0)^2 = \mu_0^2 + m_0^2 + 2k_0p_0 = m_0^2 + \mu_0^2 + 2\mu_0 E. \tag{1.4}$$

B. Assume now that for all α, β, γ, and δ

$$|k_\alpha p_\beta| \gg |k_\gamma k_\delta|, \; m_\gamma^2; \qquad s \gg |t|. \tag{1.5}$$

In this case in the c.m.s., for example, the particles p_β move forward and k_α move backward. In the laboratory system of the particle k_0 ($k_0^0 = \mu_0$, $\mathbf{k} = 0$), the p_α are fast particles and k_β are slow.

We now let E and all the p_i^0 become infinite simultaneously for fixed k_j, so that

$$\lim_{s\to\infty} \frac{k_\alpha p_\beta}{s} = a_{\alpha\beta} \neq 0 \qquad \left(\sum_{\beta=0}^{\nu} a_{\alpha\beta} = 0\right). \tag{1.6}$$

We are interested in the asymptotic behavior of G with respect to s at fixed k_α [that is, fixed l as well, cf. (1.3)]. Let us determine what limitations are imposed on p_α in this asymptotic limit by virtue of (1.3), (1.5), and (1.6). We shall find it more convenient in what follows to operate in the laboratory system of the k_0 particle.

We denote by b, f and b′, f′ respectively the numbers of fast and slow bosons and fermions

$$b + f = \nu + 1, \qquad b' + f' = \nu' + 1. \tag{1.7}$$

Conditions (1.6) denote that when $s \to \infty$

$$p_i^0 = c_i E + o(E); \qquad \sum_{i=1}^{\nu} c_i = 1, \qquad c_i > 0, \qquad c_0 = 1. \tag{1.8}$$

Further, $p_\alpha^{02} = \mathbf{p}_\alpha^2 + m_\alpha^2$, that is as $E \to \infty$ we have $|\mathbf{p}_\alpha| = p_\alpha^0 - m_\alpha^2/2p_\alpha^0$. Therefore, accurate to the constants in the quantities p_i and p_j, we can assume that

$$|\mathbf{p}_\alpha| = p_\alpha^0. \tag{1.9}$$

The conservation laws (1.3) are now written in the form

$$p_0^0 = E = \sum_{i=1}^{\nu} p_i^0 - l^0; \tag{1.10a}$$

$$|\mathbf{p}_0| = E = \sum_{i=1}^{\nu} p_i^0 x_i - |\mathbf{l}|\, x_0; \qquad |x_\alpha|,\ |a_{\alpha\gamma}| \leqslant 1; \tag{1.10b}$$

$$\sum_{i=1}^{\nu} p_i^0 a_{i\gamma} - |\mathbf{l}|\, a_{0\gamma} = 0; \qquad \gamma = 1, 2; \qquad x_\alpha^2 + a_{\alpha 1}^2 + a_{\alpha 2}^2 = 1 \tag{1.10}$$

(here x_α and $a_{\alpha\gamma}$ are the direction cosines of the vectors in a coordinate frame in which one of the axes is parallel to $\mathbf{p}_0$). Here

$$p_\alpha p_\beta = p_\alpha^0 p_\beta^0 (1 - x_\alpha x_\beta - a_{\alpha 1} a_{\beta 1} - a_{\alpha 2} a_{\beta 2}),$$

$$p_\alpha l = p_\alpha^0 [l_0 - |\mathbf{l}| (x_0 x_\alpha + a_{01} a_{\alpha 1} + a_{02} a_{\alpha 2})] = c_\alpha s u_\alpha + o(s). \tag{1.11}$$

Relations (1.10a), (1.10b) and (1.8) for l = const and $E \to \infty$ are compatible with one another only if asymptotically

$$1 - x_i = \frac{\mu_0 d_i}{p_i^0} + o\left(\frac{1}{E}\right) = \frac{\mu_0 b_i}{E} + o\left(\frac{1}{E}\right), \qquad d_i \geqslant 0. \tag{1.12}$$

It then follows directly from (1.10a), (1.10b), and (1.8) that

$$\sum_{i=1}^{\nu} d_i = \frac{1}{\mu_0} (l^0 - |\mathbf{l}|\, x_0) = u \geqslant 0, \tag{1.12a}$$

that is, (1.6) is satisfied asymptotically only if

$$p_0 l = su + o(s), \qquad u \geqslant 0 \quad (l^0 \geqslant 0). \tag{1.13a}$$

When $u = 0$ we should also have

$$l^2 = t < 0. \tag{1.13b}$$

From (1.10) and (1.12) we get as $E \to \infty$

$$a_{i1}^2 + a_{i2}^2 = \frac{2b_i\mu_0}{E} + o\left(\frac{1}{s}\right) = \frac{2d_i\mu_0}{p_i^0} + o\left(\frac{1}{s}\right). \tag{1.14}$$

In this limit

$$p_0 p_i = c_i d_i s + o(s),$$

$$p_i p_j = c_i c_j (d_i + d_j - 2\sqrt{d_i d_j}\cos\theta_{ij})\, s + o(s), \quad (1.15)$$

$$\cos\theta_{ij} = (a_{i1}a_{j1} + a_{i2}a_{j2}) / \sqrt{(1 - x_i^2)^2 (1 - x_j^2)};$$

$$\sum_{j=1}^{\nu} p_i p_j = c_i s\,(d_i + u_i) + o(s). \quad (1.15a)$$

Thus, when $\alpha \neq \beta$ and $s \to \infty$, generally speaking, $p_\alpha p_\beta \sim p_\alpha k_\gamma \sim s$, and in addition to (1.5)

$$|p_\alpha l|,\ |p_\alpha p_\beta| \sim s \gg |k_\gamma k_\delta|,\ m_\gamma^2. \quad (1.16)$$

However, if

$$p_0 l = o(s), \quad (1.17)$$

all the d_i vanish and all

$$p_\alpha p_\beta = o(s). \quad (1.18)$$

Only in this latter case can the transverse momenta of all the particles p_i ($p_i^0 \sqrt{1 - x_i^2}$) be of the order of a constant as $s \to \infty$. With this, the subdivision of particles into groups p_α and k_β is ambiguous, and the positions of the groups can be interchanged.

The experimental data on interactions at high energies [3] show that such a subdivision is possible at least not too rarely. It is precisely the presence of such angular anisotropies that has given rise to the so-called peripheral models in multiple-production theories.

2. REPRESENTATION OF G BY MEANS OF DIAGRAMS AND THEIR CLASSIFICATION

A. Let us set in correspondence with G the series of the ordinary renormalized perturbation theory

$$G = \Sigma G_\sigma. \quad (2.1)$$

Here G_σ is one of the Feynman diagrams of the investigated process. The representation (2.1) involves certain difficulties. These were already discussed in [11] and will not be analyzed here.

Let us present a certain classification of the diagrams G_σ. The investigated process can not be represented by a sum of two independent processes. Therefore no singularities of the type of resonance denominators can occur in G_σ, and the G_σ diagrams are all connected. We assume, in addition, that whatever divergences G_σ may contain have already been eliminated in the usual manner.

Let us now assume that the diagram G_σ can be broken up into strongly connected parts, which are linked in succession with the remaining parts only through one vertex each. Then $G_\sigma = G_{\sigma 1} \times G_{\sigma 2} \times \ldots \times G_{\sigma k}$. We call the parts $G_{\sigma 1}$ subdiagrams.

Let further some subdiagrams of G_σ not contain fast particles p_α, and let the momenta l_β, transferred at vertices connecting these subdiagrams with that remaining part of G_σ which contains fast particles, be small (that is, $|p_\alpha l_\alpha| \sim s \gg |l_\delta k_\gamma|$ for $s \to \infty$). We then denote by b_σ and f_σ respectively ($b_\sigma + f_\sigma = n_\sigma$) the numbers of Bose and Fermi lines joining the aggregate of such subdiagrams of G_σ with the remaining part of G_σ, where each of the subdiagrams contains fast particles p_α. If G_σ is a strongly connected diagram, then $n_\sigma = \nu' + 1$. It is easy to see that in general

$$f_\sigma \leqslant f'; \qquad n_\sigma \leqslant \nu' + 1; \qquad f_\sigma + f = 2k,$$
$$k = 0, 1, 2, \ldots . \quad (2.2)$$

We say that diagrams G_σ and G_τ, in which $b_\sigma = b_\tau$ and $f_\sigma = f_\tau$ are topologically equivalent.

Let us consider now the "high-energy" part B_σ of the diagram G_σ. The external lines for this diagram are p_α and l_β (l_β are the lines of the set (b_σ, f_σ), $1 \le \beta \le n_\sigma$). We have

$$G_\sigma = B_\sigma \times A_\sigma. \quad (2.3)$$

Here A_σ is the aggregate of all subdiagrams of G_σ which do not depend on s as $s \to \infty$. The asymptotic properties of G_σ (and accordingly of the differential cross section T) with respect to s are determined by the properties of B_σ, which we shall investigate later on.

B. We note first that

$$B_\sigma = B_\sigma (s_\rho, a_\lambda, m_\theta^2; g_i), \quad (2.4)$$

where $s_\rho = p_\alpha p_\beta$, $p_\alpha l_\beta$; $a_\lambda = l_\alpha l_\beta$, l_α^2; m_β^2, m_θ^2 are the masses of the virtual particles on the internal lines of the diagram B_σ; g_i are the coupling constants of the theory. We also put $y_\rho = s_\rho/s$, $x_\lambda = a_\lambda/s$, $x_\theta = m_\theta^2/s$. As $s \to \infty$ we have $x_\lambda x_\theta \to 0$. From dimensionality considerations we see that

$$B_\sigma = B_\sigma (s;\ x_\lambda,\ x_\theta;\ y_\rho;\ g_i) = \prod g_i^{c_{i\sigma}} s^{N_\sigma} \Psi_\sigma (x_\lambda, x_\theta, y_\rho). \quad (2.5)$$

Here Ψ_σ is generally speaking the sum of products of γ and τ matrices, taken with different weights.

C. If we choose $l_\alpha^1 = c_\alpha l_1^1$, $l_1^2 = 0$ (c_α—numbers), then all $l_\alpha l_\beta = l_\alpha^2 = 0$. If in addition we choose $p_\alpha^2 = m_\alpha^2 = 0$, we obtain the diagram corresponding to the contribution of B_σ (s_ρ, 0, m_θ^2; g_i). For the function Ψ_σ this means that $x_\lambda = 0$. In the vicinity of this point, as everywhere else, Ψ_σ is a continuous function for any fixed s. Therefore for any $\epsilon > 0$

$$\lim_{x_\lambda \to 0} x_\lambda^\epsilon \Psi_\sigma (x_\lambda, x_\theta; y_\rho) = 0. \quad (2.6a)$$

Analogously, as $m_\theta^2 \to 0$ there can arise in B_σ

singularities of the infrared catastrophe type. Yennie, Frautschi, Suura, et al [14] have shown that these singularities factor out in polynomial fashion via the (logarithmic) singularities of lower orders of perturbation theory (although the analysis in [14] is presented only with quantum electrodynamics as an example, it essentially does not depend on the form of the renormalized Lagrangian). Thus, as $x_\theta \to 0$ there can arise in Ψ_σ singularities only of the type $\ln^k x_\theta$, that is, for any $\epsilon > 0$ and fixed s we have

$$\lim_{x_\theta \to 0} x_\theta^\varepsilon \Psi_\sigma (x_\lambda, x_\theta; y_\rho) = 0. \tag{2.6b}$$

The transitions to the limit in (2.6a) and (2.6b) are independent. We therefore can expect (2.6a) and (2.6b) to remain in force also when all the a_λ and m_θ^2 tend to zero simultaneously. However, (2.6a) and (2.6b) have been written out for x_λ and x_θ. They therefore signify also that for any $\epsilon > 0$

$$\lim_{s\to\infty} s^{-\varepsilon} \Psi_\sigma (a_\lambda/s, m_\theta^2/s; y_\rho) = 0. \tag{2.7}$$

The obtained limiting relations show that, with power-law accuracy, the dependence of $B_\sigma(s)$ is the same as in the topologically equivalent lowest-order perturbation-theory diagram.

3. GENERALIZED $L_{\sigma\tau}$ AND $T_{\sigma\tau}$ DIAGRAMS

A. We now form in accordance with the usual rules the matrix element of the investigated process

$$F = \sum_\sigma F_\sigma,$$

$$F_\sigma = \bar{u}(k_0)\,\bar{u}(p_0)\,G_\sigma u(p_1)\ldots u(p_\nu)\,u(k_1)\ldots u(k_{\nu'}). \tag{3.1}$$

We sum the quantity $|F|^2$ over all permissible states of the outgoing fast particles p_i (including summation over the polarization states) and average the result over all polarization states of the particle p_0 and of the slow particles k_α.

The obtained quantity T is proportional to a certain differential cross section $d\sigma/d\Omega$. Here, according to (3.1),

$$T = \sum_{\sigma\tau} T_{\sigma\tau}, \tag{3.2}$$

$$T_{\sigma\tau} \sim \sum \int F_\sigma^* F_\tau d\mathbf{p}_1 \ldots d\mathbf{p}_\nu \delta (p_0 + l - \Sigma p_i) = \int F_\sigma^* F_\tau dq. \tag{3.3}$$

We further sum T also over all the momenta of the particle p_0, that is, we form a quantity of the type $\int dp_0 d\sigma/d\Omega$:

$$L = \sum_{\sigma\tau} L_{\sigma\tau}, \qquad L_{\sigma\tau} = \int T_{\sigma\tau} d\mathbf{p}_0; \tag{3.4}$$

$$L_{\sigma\tau} \sim \sum \int F_\sigma^* F_\tau d\mathbf{p}_0 d\mathbf{p}_1 \ldots d\mathbf{p}_\nu \delta \Big(p_0 + l - \sum_{i=1}^{\nu} p_i\Big) = \int F_\sigma^* F_\tau dQ. \tag{3.5}$$

The summation in (3.3) and (3.5) extends over all the values of the spin and other polarization variables k_β and p_α, which are simultaneously admissible by the conservation laws for fixed k_β (that is, as well as l).

In many investigations [13] it was shown that the quantities $L_{\sigma\tau}$ and $T_{\sigma\tau}$ can be set in correspondence with certain generalized Feynman diagrams. Unlike the analogous ordinary $M_{\sigma\tau}$ diagrams used in [11] (see Fig. 1), the blocks $\bar{G}_\sigma$ and G_τ in $L_{\sigma\tau}$ ($T_{\sigma\tau}$) are interconnected by the crossed lines p_α (p_i). One of the crossed lines in $L_{\sigma\tau}$ is directed opposite to all others. These crossed lines in L and T correspond to functions $D^{(+)}(p_\alpha)$ [1)], and not $D^C(p_\alpha)$. This means that

$$T_{\sigma\tau} \sim \sum \int \bar{G}_\sigma \frac{R(p_0)}{2p_0^0} D^{(+)}(p_1)\,dp_1 \ldots D^{(+)}(p_\nu)\,dp_\nu G_\tau \delta \times \Big(p_0 + l - \sum_{i=1}^{\nu} p_i\Big), \tag{3.6}$$

$$L_{\sigma\tau} \sim \sum \int \bar{G}_\sigma D^{(+)}(p_0)\,dp_0 \ldots D^{(+)}(p_\nu)\,dp_\nu G_\tau \delta \times \Big(p_0 + l - \sum_{i=1}^{\nu} p_i\Big). \tag{3.7}$$

Here $R(p_0) = p_0 \pm m_0$ if p_0 is a fermion, and $R(p_0) = 1$ if p_0 is a boson, while the summation extends over all polarization states of the particles k_α, so that L and T are numbers.

The generalized $L_{\sigma\tau}$ and $G_{\sigma\tau}$ diagrams are similar to the $M_{\sigma\tau}$ diagrams introduced in [11]

$$M_{\sigma\tau} \sim \int \bar{G}_\sigma D^c(p_0)\,dp_0 \ldots D^c(p_\nu)\,dp_\nu\, G_\tau \delta \Big(p_0 + l - \sum_{i=1}^{\nu} p_i\Big). \tag{3.8}$$

Unlike the customarily employed generalized $T_{\sigma\tau}$ diagrams, the phase volume Q in the $L_{\sigma\tau}$ diagrams is infinite.

We note also that by virtue of (2.3) the integrals in (3.3)–(3.8) contain only B_σ and B_τ. We can therefore rewrite (3.6) and (3.7) in the form

$$T_{\sigma\tau} \sim \Sigma \bar{A}_\sigma \int \bar{B}_\sigma \frac{R(p_0)}{2p_0^0} D^{(+)}(p_1)\,dp_1 \ldots D^{(+)}(p_\nu)\,dp_\nu B_\tau \delta \times \Big(p_0 + l - \sum_{i=1}^{\nu} p_i\Big) A_\tau = \Sigma \bar{A}_\sigma \mathscr{T}_{\sigma\tau} A_\tau, \tag{3.6a}$$

$$L_{\sigma\tau} = \Sigma \bar{A}_\sigma \mathscr{L}_{\sigma\tau} A_\tau. \tag{3.7a}$$

Here $\mathscr{L}_{\sigma\tau}$ is a strongly connected generalized diagram.

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1)Here, as in [11], the unperturbed Green's function of the corresponding field is D(p) for Bose particles and S(p) for Fermi particles.

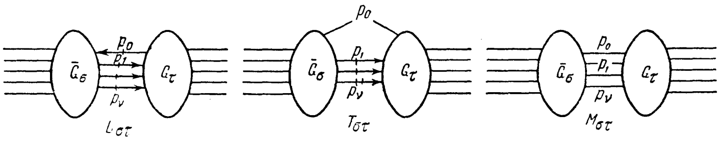

FIG. 1

B. We now determine, in analogy with the procedure used by Bogolyubov and Shirkov [12], the degree of growth or the index $\omega(\mathscr{L}_{\sigma\tau})$ of the diagram $\mathscr{L}_{\sigma\tau}$ in the following manner.

If we multiply all the s_ρ by a factor a, fixing all the remaining parameters then we have as $s \to \infty$

$$\mathscr{L}_{\sigma\tau}(as_\rho) \to a^{\omega(\mathscr{L}_{\sigma\tau})}\mathscr{L}_{\sigma\tau}(s_\rho). \tag{3.9}$$

Let further

$$\mathscr{T}_{\sigma\tau} = s^{c_{\sigma\tau}} H_{\sigma\tau}(s);$$

$$\int |H_{\sigma\tau}(s)|^2 \frac{ds}{s} s^\varepsilon = \begin{cases} \infty, & \varepsilon > 0 \\ < \infty, & \varepsilon < 0. \end{cases} \tag{3.10}$$

Since $\mathscr{T}_{\sigma\tau}$ is finite, it follows from (3.3), (3.5), (3.9), and (3.10) that

$$c_{\sigma\tau} = \omega(\mathscr{L}_{\sigma\tau}) - 3. \tag{3.11}$$

We shall be interested everywhere below in the quantity $c_{\sigma\tau}$. It separates with power-law accuracy the contributions from different diagrams in T. Relation (3.11) makes it possible to replace an investigation of the quantity $c_{\sigma\tau}$ by a study of $\omega(\mathscr{L}_{\sigma\tau})$. The latter seems to us to be technically somewhat simpler.

4. DEGREES OF GROWTH OF THE $\mathscr{L}_{\sigma\tau}$ DIAGRAMS

A. We choose among all the G_σ diagrams of given topology (b_σ, f_σ) those diagrams whose high-energy parts B_σ correspond to the first nonvanishing approximation B_σ^0 of perturbation theory. In such diagrams B_σ there are no closed cycles. Here

$$\mathscr{L}_{\sigma\tau}^0 \sim \int \bar{B}_\sigma^0 D^{(+)}(p_0)\, dp_0 \ldots D^{(+)}(p_\nu)\, dp_\nu B_\tau^0 \delta\left(p_0 + l - \sum_{i=1}^{\nu} p_i\right). \tag{4.1}$$

In Sec. 2 we have shown with power-law accuracy that in all the topologically equivalent diagrams the value of B_σ increases not faster than B^0 as $s \to \infty$. Therefore the estimate (4.1) of the degree of growth of the $\mathscr{L}_{\sigma\tau}^0$ diagram is the majorant of this estimate for all diagram pairs topologically equivalent to B_σ^0 and B_τ^0.

We denote the momenta on the internal lines of the $\mathscr{L}_{\sigma\tau}^0$ diagram by q_j ($\{p_\alpha\} \in \{q_j\}$). Since the integration in (4.1) goes only over the crossed lines, to which the $D^{(+)}(p_\alpha)$ corresponds, we can carry out the integration with respect to p_α^0. We are then left only with three-dimensional integrals of the type (2.5). Here

$$\mathscr{L}_{\sigma\tau}^0 \sim \int \bar{B}_\sigma^0 \prod_\alpha R(p_\alpha) B_\tau \frac{|\mathbf{p}_0|^2 d|\mathbf{p}_0|}{p_0^0} \theta(p_0 l) \frac{d|\mathbf{p}_2|}{|\mathbf{p}_1 + \mathbf{p}_2|} \prod_{i=3}^{\nu} \frac{|\mathbf{p}_i|^2 d|\mathbf{p}_i|}{p_i^0}$$
$$\times\ \theta(1 - |x_i|)\, dx_i. \tag{4.2}$$

Integration in (4.2) is over the region

$$\sum_{i=3}^{\nu} p_i^0 (1 - x_i) \leqslant \mu_0 u.$$

B. Let $u > 0$ in (1.12a). Then, by virtue of (1.5) and (1.11) we have generally speaking $p_\alpha p_\beta$, $p_\alpha l_\sigma = s_\rho \sim s$. This means that a contribution $\sim 1/s$ corresponds to each Bose line of the $\mathscr{L}_{\sigma\tau}^0$ diagram, while the corresponding contribution to each Fermi line is $\sim \hat{q}_j/s$.

If $\mathscr{L}_{\sigma\tau}^0$ contains closed Fermi cycles, then the products of pairs of factors $\hat{q}_j$ in the numerators of such cycles give quantities on the order of s. The function $f_\sigma + f_\tau$ is even, and if any Fermi line enters in $\mathscr{L}_{\sigma\tau}$, it must go out of it. On such a Fermi line there are vertices in which a small momentum is transferred to the boson (if the boson is external with respect to $L_{\sigma\tau}$), and vertices in which a large momentum is transferred to the boson (if the boson is internal in $L_{\sigma\tau}$), as well as vertices from which a slow fermion emerges ("extreme").

In the vertices of the first of these types, in $L_{\sigma\tau}$, the contribution $\sim s$ gives a pair of factors $\hat{q}_j$, as well as on the lines belonging to the closed cycles. In the remaining vertices, each of the $\hat{q}_j$ makes a contribution $\sim s$ only if the composition of the corresponding group of slow k_α particles is not too "poor." Otherwise the contribution $\sim s$ is no longer made by each $\hat{q}_j$, but their number (of contributions $\sim s$) is larger than in the first case ($\hat{q}_i\hat{q}_j \sim s$).

Owing to kinematic limitations, the integration in (4.2) is essentially two-dimensional. Therefore the index of the vertex μ, in which the Fermi line belongs to the cycle closed in $\mathscr{L}_{\sigma\tau}$, is $\omega(\mu) = 0$. This is also the index of a vertex where a small momentum is transferred to the boson. The indices of the remaining vertices μ are $\omega(\mu) = 1$. The summation of the factors s is made more complicated in the general case by the presence of vertices with $\omega(\mu) = 1$ and also by virtue of the dependence of $\omega(\mu)$ on the composition of the external lines.

For each specific set (b, f), (b′, f′) we can readily obtain all the $\omega(\mathscr{L}_{\sigma\tau})$ for different sets (b_σ, f_σ), (b_τ, f_τ). Thus, for sufficiently large f′, the maximum value of $[\omega(\mathscr{L}_{\sigma\tau})]_{max} = \omega_m$ is realized for the following sets $(f_\sigma + f_\tau, b_\sigma + b_\tau) = (\varphi\beta)$:

b, f	ω_m		Exceptions for small b		
	Quantity	(φ, β)	b	ω_m	(φ, β)
$f=0,\ b\geqslant 3$	$b-1$	(4.0)	3	2	(4.0), (2.1), (0.2)
$f=1,\ b\geqslant 2$	b	(4.0)	2	2	(4.0), (2.1), (6.0),
$f=2,\ b\geqslant 1$	$b+1$	(4.0)	1	2	(4.0), (2.1), (0.2)
$f=3,\ b\geqslant 0$	$b+1$	(6.0)	0	2	(4.0)
			1	2	(2.0), (4.0), (6.0)
$f=4,\ b\geqslant 0$	$b+2$	(4.0)	0	2	(4.0), (2.1), (0.2)

(4.3)

etc. If f′ = 0 then, by virtue of (2.2), $f_\sigma = f_\tau = 0$; if f′ = 2, b′ = 0, then, by virtue of (2.2), f_σ is equal to 0 or 2. In both cases the maximum value of $\omega(\mathscr{L}_{\sigma\tau}) = 2$ is attained when

$$b_\sigma = b_\tau = 1, \quad f_\sigma = f_\tau = 0. \tag{4.4a}$$

If f′ = 1, then $f_\sigma = 1$ by virtue of (2.2) and the maximum value of $\omega(\mathscr{L}_{\sigma\tau}) = 1$ is attained when

$$b_\sigma = b_\tau = 0, \quad f_\sigma = f_\tau = 1. \tag{4.4b}$$

It is also easy to see that at fixed (b′, f′) and sufficiently large (b, f), the maximum value of $\omega(\mathscr{L}_{\sigma\tau})$ is attained at the minimum possible $n_\sigma = b_\sigma + f_\sigma$; $(b_\sigma, f_\sigma = b_\tau, f_\tau)$.

In all these cases it is possible to arrange the terms in the sum (3.4) such that the change in $n_\sigma + n_\tau$ will correspond generally speaking to a reduction in $\omega(\mathscr{L}_{\sigma\tau})$, that is, as well as in $c_{\sigma\tau}$ in (3.1) and (3.11).

5. EXPANSION IN POWERS OF 1/s AND PERIPHERALITY

A. We have shown above that the terms $T_{\sigma\tau}$ in the sum (3.2) for the differential cross section, which depend differently on s, correspond to diagrams G_σ and G_τ of different topology. At the same time, the terms which depend in the same manner on s (with power-law accuracy), correspond to diagram pairs G_σ and G_τ, of a few identical topologies. In particular, in each of the orders of perturbation theory, as $s \to \infty$ the quantity $T_{\sigma\tau}$, increases generally speaking, most rapidly for the pair of diagrams G_σ and G_τ with the simplest topology, considered in the preceding section. Such a diagram $T_{\sigma\tau}$ makes the main contribution to T in each order of perturbation theory. It is therefore natural to assume that their sum will also make the main contribution to T as $s \to \infty$, regardless of the value of the coupling constant.

Additional account of some symmetries between different topologically equivalent diagrams can lead sometimes to changes in this hierarchy of the diagram classes. These changes can be taken into account by again considering the diagrams of the first vanishing orders of perturbation theory and then using a procedure similar to that carried out in Secs. 2 and 3.

Let us consider the sum of all the diagrams of topologically equivalent G_σ:

$$\sum_{b_\sigma f_\sigma = \text{const}} G_\sigma(b_\sigma, f_\sigma) = \Gamma_\sigma. \tag{5.1}$$

Then

$$\sum \int \bar{\Gamma}_\sigma \frac{R(p_0)}{2p_0^0} D^{(+)}(p_1)\, dp_1 \ldots D^{(+)}(p_\nu)\, dp_\nu \Gamma_\tau \delta \times \left(p_0 + l - \sum_{i=1}^{\nu} p_i\right) = \sum_{\substack{(b_\sigma, f_\sigma) = \text{const} \\ (b_\tau, f_\tau) = \text{const}}} T_{\sigma\tau} = V_{\sigma\tau}. \tag{5.2}$$

In Γ_σ and $V_{\sigma\tau}$ as well as everywhere below, σ and τ number only the topology of the diagram. All the terms depend in identical fashion on s in these sums.

We can now write sums that are already finite [by virtue of (2.2)]

$$G = \sum \Gamma_\sigma, \tag{5.3}$$

$$T = \sum V_{\sigma\tau}. \tag{5.4}$$

By grouping in suitable fashion the terms in these sums we obtain in (5.3) and (5.4) finite sums, in which the succeeding terms vanish compared with the preceding ones as $s \to \infty$. If we assume that the $V_{\sigma\tau}$ differ from one another by the same power of s as their terms $T_{\sigma\tau}$ (this means that they are identically summed in some sense), then the sums (5.3) and (5.4) represent a certain expansion in powers of 1/s. The dimensionless parameter of the expansion should in this case, of course, be one of the quantities

$$\varepsilon = |t| s^{-1}, \quad s^{-1} \sum m_\alpha^2 \ll 1 \qquad (5.5)$$

or, what is more probable, the largest of them. (Here and throughout m_α are the masses of the fast particles.)

Thus, as $s \to \infty$ the principal terms in T can be represented in the form of one of the diagrams of the type of Fig. 2. We denote by $\mathcal{G}_{A_i}$ the Green's functions of the blocks of slow particles of the diagram of Fig. 2. (For convenience we include in them also the propagation functions of slow virtual particles, which connect these blocks with the fast-particle block B.) Then

$$G \to \Pi \mathcal{G}_{A_i} \times \mathcal{G}_B. \qquad (5.6)$$

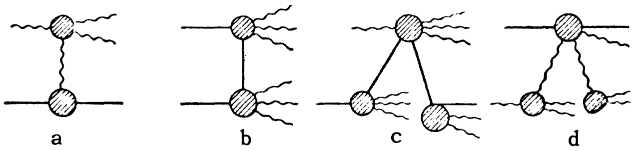

FIG. 2

B. It is easy to understand that when $p_0 l = o(s)$ the situation is likewise analogous to that considered above. The corresponding analysis is more cumbersome than in Sec. 4, and will not be presented here. Generally speaking, however, one should pick out not one or two Γ_σ diagrams, but several. This is precisely why the usual somewhat naive picture of peripheral interactions at high multiplicity is customarily replaced by a crude hydrodynamic or statistical model. From our point of view, the interactions should be peripheral at high energies. This means that only a small part of the Γ_σ diagrams need by considered, generally speaking. For the analysis of these latter diagrams we can then use the results obtained at lower energies and lower multiplicity.

C. We thus arrive at the well known peripheral model, according to which the main contribution of the cross section at large energies is made by the Γ_σ diagrams with the "simplest" topology. In particular, for a whole series of processes the main contribution to the cross section is made by diagrams corresponding to the exchange of one meson between groups of fast and slow particles. Such processes are, for example, the processes $(\gamma)\pi + n \to 2\pi + n$, if the nucleons are slow, and the bosons fast. In other processes, such an $n + \bar{n} \to k\pi + k_1 K$, the principal term corresponds to exchange of one nucleon. In processes where the group of slow particles is sufficiently large and contains nucleons, while the group of fast particles contains none, the principal term is the one corresponding to the exchange of two nucleons, etc. The corresponding Γ_σ diagrams are shown in Fig. 2.

To find $\mathcal{G}_{A_i}$ and $\mathcal{G}_B$ we can use our knowledge of the simpler processes of lower multiplicity and lower energy. Thus, if the single-meson term is "principal," then to investigate the process $(\gamma)\pi + n \geq n + k\pi$, which for some breakdown into fast and slow particles can be represented in the form of the diagram of Fig. 3a, it is sufficient to know the matrix elements of the processes of Figs. 3b—d.

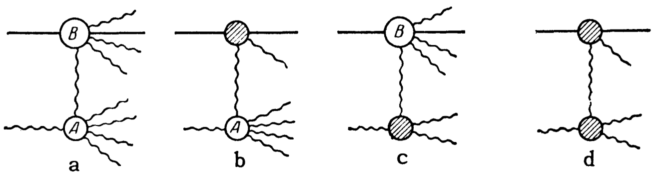

FIG. 3

We then have asymptotically

$$G_a G_d \approx G_c G_b. \qquad (5.7)$$

For a consistent execution of such a program it is necessary to know the phases of three out of four processes in Fig. 3. In practice, however, it becomes necessary to make some assumptions concerning the dependence of $\mathcal{G}_B$ on t. The simplest is the assumption that by virtue of (5.5) $\mathcal{G}_B$ depends little on t. Then the contribution from the diagram B can be replaced, accurate to ϵ, by the total cross section of the process

$$p_0 + l = \sum_{i=1}^{\nu} p_i. \qquad (5.8)$$

Here $p_0 l = su$, and $l^2 = \mu_i^2$.

However, the considerations connected with the summation of infrared singularities (see Sec. 2 and [14]) and with the analytic properties of the S matrix (compare, for example, with [15]), force us to regard such an assumption as far from reality. On the other hand, the matrix elements $\mathcal{G}_{A_i}$ can also depend on t in a different fashion merely because an estimate of the type (5.5) does not always hold for them. To study these elements we may need to know the matrix elements of processes of the type of Fig. 3b.

Usually [3,4] the criterion of applicability of the one-meson approximation [the particular case (5.6)] is considered to be smallness of the quantity

$$\varkappa = (t - \mu_\pi^2) / M_n^2. \qquad (5.9)$$

It is assumed here that $\mathscr{G}_A$ and $\mathscr{G}_B$ can be approximated by diagrams which correspond to real processes. The region of applicability of such an approximation is a priori not large (since the region where κ is small is not large). The available experimental data apparently confirm the correctness of such a one-meson approximation for several processes (see, however, [1]). This is a particular case of our result (5.6), the actual smallness parameter being here not κ but ϵ, and quite probably even $\kappa\epsilon$.

The experimental data on interactions up to 25—30 BeV (see, for example, the reviews [3,9]) indicate that the contribution of the peripheral interactions to the cross section actually increases with the energy. In particular, it is shown in [7,8] that the experimental data on nucleon-nucleon collisions at 1—10 BeV can be well explained by the peripheral model in a region which is incomparably broader than that in which κ is small. Inasmuch as large scattering angles are of low probability and the criterion (5.5) is satisfied almost everywhere, this result appears to be perfectly natural.

The proposed model with smallness parameter ϵ can also be used to describe other experimental data at $\geq 10^3$ BeV [16], which cannot be explained by means of the peripheral model with smallness parameter κ [17]. From our point of view, the increase in the role of two-center cases with increasing energy, as discussed in [16], is quite natural.

D. At sufficiently high energies, the differential cross sections of all the inelastic processes should be described by such a peripheral model. The criterion for regarding the energy as high is apparently the satisfaction of the inequality

$$\delta = (\Sigma m_\alpha^2 + \Sigma \mu_\alpha^2) / s \ll 1. \qquad (5.10)$$

Inasmuch as the multiplicity increases with energy not faster than $s^\alpha (\alpha < 1)$, all the interactions can be regarded as peripheral at sufficiently high energies. Therefore, at medium energies (5—25 BeV) the peripheral model describes poorly interactions with high multiplicity. For such interactions the value of δ is not small.

In the described model it is difficult to draw any conclusions concerning the total cross section ($n' = b' + f' = 1$, $\nu' = 0$). We can, however, attempt to obtain an estimate for the dependence of the cross section on the energy, postulating some connection between the power-law dependences of $\Gamma_\sigma(s)$ and $G_\sigma(s)$. If it is assumed that with power-law accuracy $\Gamma_\sigma(s) \sim G_\sigma(s)$ as $s \to \infty$, then the cross sections of all the inelastic processes decrease as $1/s$. Assumptions of this type establish some connection between the form of the Lagrangian and the degree of growth of the cross section as $s \to \infty$ (cf. [18]).

Let us assume that the $V_{\sigma\tau}$ for the same process differently subdivided into groups of fast and slow particles differ from one another in the same power of s as their terms $T_{\sigma\tau}$. Then, comparing the different terms $T_{\sigma\tau}(s)$ with one another, we can obtain certain "selection rules" for the most "convenient" subdivisions into groups of fast and slow particles in the given process. These will make the main contribution to the total cross section at high energies.

E. We have shown above, under certain assumptions concerning the character of the summation of the perturbation-theory series, that at sufficiently high energies we can confine ourselves in the analysis of strong interactions to only a few diagrams of the simplest topology. The selection of these essential diagrams is determined by the character [which is unique! (1.5), (1.15), (1.16)] of the subdivision of the particles into groups of fast and slow particles. To carry out this subdivision it is sufficient to examine diagrams of different topologies, in which the high-energy part is the diagram of the lowest nonvanishing order in perturbation theory with $\nu + 1 + n_\sigma$ ends. In particular, for some sets of particles, the peripheral diagrams of the type 2a and b predominate, while for others, the predominating types are of type 2c and d—peripheral with "isobars," etc. The order of smallness of the terms discarded in the sums (5.1) and (5.2) is determined by the value of ϵ. The region of applicability of the obtained results is therefore larger than the usual one defined by the parameter κ (see, for example, [1,4]).

This result has been obtained in renormalized theory with zero vertex index. It is easy to understand how it varies if the index of the vertex is not zero. In either case, $\omega(\mu) > 0$ or $\omega(\mu) < 0$, the coupling constants are dimensional, but the result (2.7) and (2.5) remains valid with a natural change in the power of N_σ in accord with the dimensionality of the constant g_i, which in the diagrams of the first orders of perturbation theory corresponds to a change in $\omega(\mu)$.

In both cases the results of Secs. 2—5 remain in force for each order of perturbation theory. When $\omega(\mu) > 0$, however, the diagrams of more "complicated" topology increase in each succeeding order of the perturbation theory no more slowly than the diagrams of the "simpler" topology in the preceding order of perturbation theory. Therefore in such—nonrenormalized—theories any conclusions concerning the most essential terms of Γ_σ and $V_{\sigma\tau}$ are very risky. If $\omega(\mu) < 0$, to the contrary, the estimates made in Secs. 2—5 only become stronger with increasing order of perturbation theory. The general results of Sec. 4 are even more likely in this case than when $\omega(\mu) = 0$.

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Note added in proof (January 23, 1963). Recently the author became acquainted with[19], where the experimental conditions were close to those considered in Sec. 1. In accord with the considerations advanced above, it turns out that at the investigated energies (12–17 BeV) the role of the peripheral interactions increases compared with the experiments carried out at lower energies.

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87