DETERMINATION OF THE PARAMETERS OF THE GENERALIZED DIFFRACTION MODEL OF THE NUCLEUS AT 660 MeV

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A relation between the nuclear scattering amplitudes and the parameters of the generalized diffraction theory for scattering of high energy particles on atomic nuclei, as developed by Greider and Glassgold [1], is established by taking spin-orbit interaction into account. The parameters of the generalized diffraction model are determined from the values of the amplitudes for scattering of protons on carbon nuclei obtained previously [2]. Spin effects are discussed.

1. INTRODUCTION

To analyze data on the nuclear scattering of strongly interacting particles at high energies, Greider and Glassgold [1] proposed a generalized diffraction model of the nucleus. This model is a further generalization of the "black sphere" model, which corresponds fully to a translucent nucleus. As is well known, the scattering amplitude of nuclear spinless particles on a nucleus with zero spin has the form

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(\eta_l - 1)(2l+1)P_l(\cos\theta),$$

where the amplitude (or scattering coefficient) η_l is connected with the corresponding phase shift δ_l by tne relation $\eta_l = \exp(2i\delta_l)$. The "black sphere' model is characterized by the following assumptions with respect to the scattering coefficient:

$$\eta_l = \begin{cases} 0 \text{ for } l \leqslant L \text{ (maximum absorption)}, \\ 1 \text{ for } l > L \text{ (no perturbation)}. \end{cases}$$

The parameter L thus indicates the number of completely absorbed partial waves.

The characteristic features of the generalized diffraction model can be readily traced by considering the translucency coefficient of the nucleus

$$\beta(l) = 1 - \eta^2(l),$$

where $\eta(l)$ is the modulus of the complex amplitude of the l-th scattered wave:

$$\eta_l = \eta(l)\exp[i\varphi(l)],$$

and $\varphi(l)$ is the phase of the scattering coefficient. It is assumed that $\beta(l)$ is a continuous monotonic function of l, which varies smoothly from a maximum value at the center of the nucleus to zero outside the nucleus (see Fig. 1). In the general case absorption in the middle of the nucleus is not complete; the real part of the scattering amplitude differs from zero.

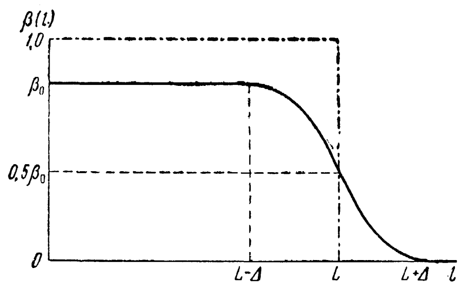

FIG. 1. Dependence of the translucency coefficient $\beta(l)$ on the number of the partial wave l. The continuous line corresponds to the Greider and Glassgold model, the dash-dot line corresponds to the "black sphere" model.

Thus, the generalized diffraction model is characterized by the following parameters: the translucency of the nucleus β_0 at small l; the number of strongly absorbed partial waves L, which correspond to half the height of the distribution $\beta(l)$; the region 2Δ of strong variation of $\beta(l)$, corresponding to the diffuseness of the edge of the nucleus. The phase $\varphi(l)$ is assumed equal to a constant value when $\eta(l) < 1$ (that is, inside the nucleus) and equal to zero when $\eta(l) = 1$ (that is, outside the nucleus). The parameters of the Greider and Glassgold model are determined from a phenomenological analysis of the total cross sections and the absorption cross sections in scattering of high-energy neutrons by different nuclei.

It is of interest to determine the parameters of the generalized diffraction model of the nucleus from data on the differential scattering cross section and the polarization. This approach leads to more accurate parameters for the model, and the use of the data on the polarization makes it possible to consider spin effects. The amplitudes of the nuclear scattering were determined earlier [2] from data on the differential cross section and polarization in the scattering of protons by carbon at 660 MeV. In the present work, using the connection between the amplitudes and phases of nuclear scattering, we determine the parameters of the generalized diffraction model of the nucleus. We take into account here the spin-orbit interaction, which leads to a "splitting" of the translucency coefficient of the nucleus.

2. CONNECTION BETWEEN AMPLITUDES AND PHASES OF NUCLEAR SCATTERING

The scattering matrix of a particle with spin 1/2 on a nucleus with spin 0 has the form

$$M(\theta) = g(\theta) + h(\theta)\sigma_n,$$

where σ_n is the projection of the spin operator on the normal to the scattering plane. The spin-independent and spin-dependent amplitudes of nuclear scattering $g(\theta)$ and $h(\theta)$ are determined by the expressions

$$g(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}\{(l+1)\exp(2i\delta_l^+) + l\exp(2i\delta_l^-) - (2l+1)\}P_l(\cos\theta);$$

$$h(\theta) = \frac{1}{2k}\sum_{l=0}^{\infty}\{\exp(2i\delta_l^+) - \exp(2i\delta_l^-)\}P_l^1(\cos\theta). \quad (1)$$

Here $\delta_l^{\pm}$ are the phase shifts of the partial wave, corresponding to states with total angular momentum $j = l \pm 1/2$.

Expressions (1) are expansions of the nuclear-scattering amplitudes in Legendre functions. Using the orthogonality of the Legendre functions and solving equations (1) relative to the amplitudes of the l-th partial wave, corresponding to two possible orientations of the spin of the scattered particles relative to the orbital momentum, we can obtain

$$\eta_l^+ = \exp\{2i\delta_l^+\} = 1 + ik\int_{-1}^{+1} g(x)P_l(x)\,dx + \frac{k}{l+1}\int_{-1}^{+1} h(x)P_l^1(x)\,dx,$$

$$\eta_l^- = \exp\{2i\delta_l^-\} = 1 + ik\int_{-1}^{+1} g(x)P_l(x)\,dx - \frac{k}{l}\int_{-1}^{+1} h(x)P_l^1(x)\,dx. \quad (2)$$

The nuclear scattering amplitudes $g(\theta)$ and $h(\theta)$ can be represented in the region of small scattering angles θ (smaller than the angle of the first diffraction minimum) in the form

$$g(x) = g(0)\,e^{\alpha_1(x-1)},$$

$$h(x) = h'(x)\sqrt{1-x^2} = h'(0)\sqrt{1-x^2}\,e^{\alpha_2(x-1)}. \quad (3)$$

Here $g(0)$ and $h'(0)$ are the forward scattering amplitudes, $x = \cos\theta$, $\alpha_1 = k^2a_1^2/2$, $\alpha_2 = k^2a_2^2/2$, k is the wave number of the incoming proton in the proton-nucleus c.m.s., while a_1 and a_2 are the radial parameters of the form factors pertaining to the amplitudes of the nuclear scattering.

The overwhelming part of the diffraction scattering is concentrated in the region of angles up to the first diffraction minimum. Therefore relations (3) can be extended without appreciable error over the entire range of angles from 0° to π (according to an estimate, this procedure introduces an error not larger than $1/\alpha$ in the value of η_0).

Substituting expressions (3) in formulas (2), integrating, and also taking into account the fact that $\eta_l^{\pm}$, $g(x)$, and $h(x)$ are in general complex, we obtain

$$\mathrm{Re}\,\eta_l^+ = 1 - k\left\{\frac{1}{\alpha_1}g_{nI}(0)F_l(\alpha_1) - \frac{l}{\alpha_2^2}h'_{nR}(0)F_l(\alpha_2)\right\},$$

$$\mathrm{Im}\,\eta_l^+ = k\left\{\frac{1}{\alpha_1}g_{nR}(0)F_l(\alpha_1) + \frac{l}{\alpha_2^2}h'_{nI}(0)F_l(\alpha_2)\right\},$$

$$\mathrm{Re}\,\eta_l^- = 1 - k\left\{\frac{1}{\alpha_1}g_{nI}(0)F_l(\alpha_1) + \frac{l+1}{\alpha_2^2}h'_{nR}(0)F_l(\alpha_2)\right\},$$

$$\mathrm{Im}\,\eta_l^- = k\left\{\frac{1}{\alpha_1}g_{nR}(0)F_l(\alpha_1) - \frac{l+1}{\alpha_2^2}h'_{nI}(0)F_l(\alpha_2)\right\}, \quad (4)$$

where we introduce the function $F_l(\alpha) = e^{-\alpha}\sqrt{2\pi\alpha}\,I_{l+1/2}(\alpha)$; $I_{l+1/2}(\alpha)$ is a Bessel function of imaginary argument with half-integer index. Analogous expressions can be obtained also in the case when the real and imaginary parts of the amplitudes $g(\theta)$ and $h(\theta)$ are characterized by form factors with their radial parameters. The function $F_l(\alpha)$ satisfies the recurrence relation

$$F_{l+1}(\alpha) = F_{l-1}(\alpha) - \frac{2l+1}{\alpha}F_l(\alpha).$$

At values $\alpha \gg 1$ ($\alpha \sim 70$ at 660 MeV) we have accurate to terms of order $e^{-2\alpha}$

$$F_0(\alpha) = 1,\quad F_1(\alpha) = 1 - 1/\alpha.$$

If there is no spin-orbit interaction, relations (4) simplify to

$$\mathrm{Re}\,\eta_l = 1 - \frac{k}{\alpha}g_{nI}(0)F_l(\alpha), \quad \mathrm{Im}\,\eta_l = \frac{k}{\alpha}g_{nR}(0)F_l(\alpha).$$

When $l = 0$, which in the classical treatment corresponds to the passage of a particle through the central region of the nucleus, using the optical theorem $g_{nI}(0) = k\sigma_t/4\pi$, we obtain the connection between the translucency coefficient for the central region of the nucleus β_0, the total scattering cross section σ_t, and the differential forward scattering cross section

$$\sigma(0) = g_{nR}^2(0) + g_{nI}^2(0):$$
$$\beta_0 = 1 - |\eta_0|^2 = \sigma_t/\pi a^2 - 4\sigma(0)/k^2a^4.$$

For the phase φ_0 of the scattering coefficient η_0 we then obtain the expression*

$$\operatorname{tg}\varphi_0 = \operatorname{Im}\eta_0/\operatorname{Re}\eta_0 = g_{nR}(0)/[a/k - g_{nI}(0)].$$

3. PARAMETERS OF GENERALIZED DIFFRACTION MODEL OF THE NUCLEUS AT 660 MeV

The translucency coefficient of the nucleus and the phase of the scattering coefficient

$$\beta^{\pm}(l) = 1 - |\eta_l^{\pm}|^2, \qquad \operatorname{tg}\varphi^{\pm}(l) = \operatorname{Im}\eta_l^{\pm}/\operatorname{Re}\eta_l^{\pm}$$

were calculated for the following values of the nuclear-scattering amplitudes

$$g_{nR}(0) = (-4.45 \pm 0.39)\cdot 10^{-13}\text{ cm},$$
$$g_{nI}(0) = (13.41 \pm 0.40)\cdot 10^{-13}\text{ cm},$$
$$h'_{nR}(0) = (-8 \pm 10)\cdot 10^{-13}\text{ cm},$$
$$h'_{nI}(0) = (29.1 \pm 7.2)\cdot 10^{-13}\text{ cm},$$

and of the radial parameters

$$a_1 = (2.02 \pm 0.03)\cdot 10^{-13}\text{ cm}, \quad a_2 = (2.32 \pm 0.13)\cdot 10^{-13}\text{ cm},$$

obtained in [2] (the values of the amplitudes are taken in the c.m.s.). Figure 2a shows the dependence of the translucency coefficient $\beta(l)$ on l. The continuous line corresponds to the coefficient $\beta(l)$ calculated without account of the spin-orbit interaction, while the dashed lines correspond to the coefficients $\beta^+(l)$ and $\beta^-(l)$, in the calculation of which the spin-orbit interaction was taken into account. It is seen that an account of the spin-orbit interaction leads to a "splitting" of the translucency coefficient of the nucleus into two coefficients $\beta^+(l)$ and $\beta^-(l)$, corresponding to interactions with different orientations of the spin of the incoming proton relative to the orbital angular momentum. There is no such splitting at values of l close to zero. This corresponds to the illustrative representation that polarization is a surface effect. Protons with a spin parallel to the orbital momentum have a larger radius of interaction with the nucleus than protons with spin anti-parallel to the orbital momentum. They are therefore differently scattered, which leads to polarization. This fact was first noticed by Fermi [3], who described the polarization of the nucleons by introducing in addition to the central potential also additional spin-orbit interactions on the edge of the nucleus.

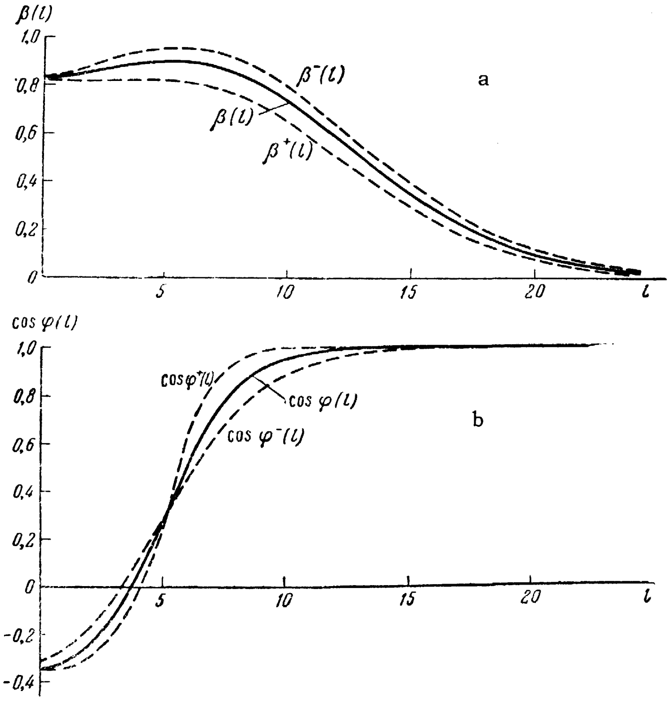

FIG. 2. Translucency coefficient (a) and cosine of the phase of the coefficient η_l (b) as a function of l, calculated for carbon in accordance with the data of [2]. The continuous lines pertain to $\beta(l)$ and $\cos\varphi(l)$, calculated without allowance for the spin-orbit interaction, while the dashed lines include allowance for the spin-orbit interaction.

Figure 2b shows the dependence of $\cos\varphi(l)$ on l and, as before, the continuous line pertains to a case when spin effects were neglected. The phase $\varphi(l)$ varies from $-200°$ at the center of the nucleus to 0° on the edge of the nucleus, and it is close to zero already at the start of the diffuse region of the nuclear edge.

As can be seen from Fig. 2a, $\beta(l)$ decreases to one-half its value at $L \approx 14$. This value of orbital momentum can be set in correspondence in the quasiclassical approximation to the value of the carbon-nucleus radius $R \approx L/k \approx 2.4 \times 10^{-13}$ cm. The diffuse region of the nuclear edge has a width $2\Delta \approx 10$, which corresponds to a surface layer of thickness $t \approx 2\Delta/k \approx 1.8 \times 10^{-13}$ cm. These estimates agree with the data obtained from other experiments [4].

The value of the translucency coefficient β_0 at the center of the nucleus turns out to be 0.84. This value is in good agreement with the value $\beta_0 = 0.89$ obtained in [1] for 700 MeV.

*tg = tan.

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