THERMOMAGNETIC WAVES AND THE EXCITATION OF A MAGNETIC FIELD IN A NON-EQUILIBRIUM PLASMA

L. É. GUREVICH

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor June 29, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 548-555 (February, 1963)

It is shown that in a plasma in which there is a temperature gradient, magnetic fields arise during hydrodynamic motions and are capable of producing resonant acceleration of electrons and ions. In such a plasma special kinds of thermomagnetic waves can exist, the Alfvén wave is split up into two waves with different frequencies, and the spectrum of magneto-acoustic waves is modified.

IN order to explain the production of cosmic-rays and cosmic radio waves, it has been suggested (see [1], for example) that magnetic fields exist in both stellar and interstellar plasmas. Fermi's statistical mechanism for the acceleration of charged particles and bremsstrahlung at radio frequency by relativistic electrons are both based on the existence of such fields. However, the mechanism where by sufficiently strong magnetic fields can be produced has remained vague. In this paper it is shown that hydrodynamic motion in a non-equilibrium plasma, in which there is a temperature gradient ∇T, results in the production of magnetic fields. In such circumstances it is possible for fields for which the Larmor frequency of the ions is comparable to the oscillation frequency to occur in shock waves. In this the conditions for parametric amplification and, possibly, for resonant acceleration of ions are realized. This amplification could provide an injection mechanism for Fermi acceleration.

Parametric resonance can occur for electrons in quite weak acoustic waves. In a hot plasma with high radiation pressure, and also in some other cases mentioned in the paper, much stronger magnetic fields can arise in a turbulent non-equilibrium plasma. In the work it is found that a plasma with a temperature gradient ∇T has oscillatory characteristics noticeably different from a normal plasma. Even in the absence of an external magnetic field and hydrodynamic motion in the plasma, transverse "thermomagnetic" waves are possible, in which oscillations of the magnetic field alone take place. If there is a constant external magnetic field, **H**, then the wave vector of the thermomagnetic wave must be perpendicular to it and lie in the **H**, ∇T plane.

Furthermore, the usual Alfvén wave is split up into "hydrothermomagnetic" waves in which the vectors **v** and **H** are perpendicular to ∇T. Finally, the spectrum of magnetic sound waves can be modified noticeably in the case where the speed of propagation of the thermomagnetic waves is comparable to the velocity of sound and velocity of the Alfvén wave. If a uniform longitudinal magnetic field is applied to a plasma with a temperature gradient in it, then the field is gradually lined-up along the direction of the temperature gradient by the effects of the thermomagnetic fields.

1. MAGNETIC FIELD ARISING FROM HYDRODYNAMIC MOTION

We consider a plasma in which there is a temperature gradient ∇T, which is independent of time and position. We will also assume in the subsequent discussion that the important lengths are much smaller than $L = T/|\nabla T|$, so that the temperature variation is small over these distances. In the presence of a temperature gradient, the plasma can be in a steady state if the pressure p is constant, i.e., $\nabla\rho/\rho = -\nabla T/T$ where ρ is the density of the plasma. Suppose there is a weak magnetic field **H** in such a plasma, and that the Larmor frequency of the electrons Ω_e is small compared with their collision frequency $1/\tau$. In the presence of an electric field **E**, of electron-concentration and temperature gradients of ∇n and ∇T, and of hydrodynamic motion of velocity $\mathbf{v}(\mathbf{r}, t)$, the electrical current density has the form*

$$\mathbf{J} = \sigma\mathbf{E}^* + \sigma' [\mathbf{E}^*\mathbf{H}] - \alpha\nabla T - \alpha' [\nabla T, \mathbf{H}];$$

$$\mathbf{E}^* = \mathbf{E} + \frac{[\mathbf{vH}]}{c} - \frac{T}{e}\frac{\nabla n}{n} \qquad (e > 0).$$

*$[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

Hence,

$$\mathbf{E} = -\frac{[\mathbf{vH}]}{c} - \Lambda'[\nabla T, \mathbf{H}] + \frac{c}{4\pi\sigma}\,\mathrm{rot}\,\mathbf{H} - \frac{c\sigma'}{4\pi\sigma^2}[\mathrm{rot}\,\mathbf{H}, \mathbf{H}] + \frac{T}{e}\frac{\nabla\rho}{\rho} + \Lambda\nabla T. \qquad (1)^*$$

Here

$$\Lambda = \alpha/\sigma, \qquad \Lambda' = (\alpha'\sigma - \alpha\sigma')/\sigma^2.$$

Substituting (1) in the equation $\partial\mathbf{H}/\partial t = -c\,\mathrm{curl}\,\mathbf{E}$ we obtain the equation

$$\frac{\partial\mathbf{H}}{\partial t} = \mathrm{rot}\left\{\left[\left(\mathbf{v} + c\Lambda'\nabla T - \frac{c\sigma'}{4\pi\sigma^2}\mathrm{rot}\,\mathbf{H}\right)\mathbf{H}\right] + \frac{c}{4\pi\sigma}\mathrm{rot}\,\mathbf{H} + \frac{T}{e}\frac{\nabla\rho}{\rho} + \Lambda\nabla T\right\}. \qquad (2)$$

Neglecting the quadratic terms in $\mathbf{H}$ in (2), we can rewrite it in the form

$$\frac{\partial\mathbf{H}}{\partial t} - \nu_m\nabla^2\mathbf{H} - \mathrm{rot}\,[(\mathbf{v} - \mathbf{u}_T + \mathbf{u}_s)\,\mathbf{H}] = -c\,\mathrm{rot}\,\mathbf{E}'.$$

The following symbols are introduced here

$$c\,\mathrm{rot}\,\mathbf{E}' = \frac{cT}{e}\tilde{\Lambda}\left[\nabla\ln T,\ \nabla\frac{\rho'}{\rho}\right],$$

$$\tilde{\Lambda} = \left[2(\gamma-1)\,e\Lambda - \gamma e\rho\frac{\partial\Lambda}{\partial\rho} + 2 - \gamma\right],$$

$\tilde{\Lambda}$ is a dimensionless parameter of order unity in the usual case and of order N/n for large radiation pressure[2]; ν_m is the magnetic viscosity,

$$\mathbf{u}_s = \frac{c^2}{4\pi\sigma}\left[\sigma T\frac{\partial}{\partial T}\left(\frac{1}{\sigma}\right)\right]\frac{cT}{eH_0}\nabla\ln T.$$

We consider the case of longitudinal waves and assume that E′ and v are functions of $\xi = \mathbf{k}\cdot\mathbf{x} - \omega t = \mathbf{k}\cdot(\mathbf{x} - \mathbf{u}t)$ where u is the velocity of propagation of the wave, which in the case of elastic waves corresponds to the velocity of sound s. The vector v is parallel to the x axis, and ∇T has an arbitrary direction. The $H = H(\xi, t)$, where we assume that at the starting time $t = 0$ and $H(\xi, 0) = 0$.

Then,

$$\frac{\partial H_\perp}{\partial t} - \nu_m k^2\frac{\partial^2 H_\perp}{\partial\xi^2} - k\frac{\partial}{\partial\xi}(u - u_T - v + u_s)H_\perp = -c\,\mathrm{rot}_\perp E',$$

$$\frac{\partial H_x}{\partial t} - \nu_m k^2\frac{\partial^2 H_x}{\partial\xi^2} - k\frac{\partial}{\partial\xi}(u - u_T - v)H - ku_s\frac{\partial H_y}{\partial\xi} = 0;$$

$H_\perp$ is the component of $\mathbf{H}$ perpendicular to the wave vector $\mathbf{k}$. We resolve $H_\perp$ into two terms, $H_\perp = H_{\perp\infty}(\xi) + H'_\perp(\xi, t)$, where

$$\nu_m k^2\frac{\partial^2 H_{\perp\infty}}{\partial\xi^2} - k\frac{\partial}{\partial\xi}[(v + u_T - u - u_s)H_{\perp\infty}] = ck\frac{\partial E'}{\partial\xi}, \qquad (3)$$

$$\frac{\partial H'_\perp}{\partial t} - \nu_m k^2\frac{\partial^2 H'_\perp}{\partial\xi^2} - k\frac{\partial}{\partial\xi}[(v + u_T - u)H'_\perp] = 0. \qquad (4)$$

*rot = curl.

We integrate (3) with respect to ξ and set the arbitrary constant equal to zero, which is equivalent to choosing the orgin of coordinates. Then,

$$H_{\perp\infty} = -\frac{c}{\nu_m k}\exp\left[-\int\frac{[u - u_T + u_e - v]}{\nu_m k}d\xi'\right] \times\int_{\xi_0}^{\xi}E'(\xi')\exp\left[\int\frac{u - u_T + u_s - v}{\nu_m k}d\xi''\right]d\xi'. \qquad (5)$$

Setting $H'_\perp(\xi, t) = \exp(\mu t)\,X(\xi)$, we have,

$$\nu_m k^2 X'' + (kv - \omega_T)X' - k\frac{\partial v}{\partial\xi}X - \mu X(\xi) = 0.$$

Introducing the new function

$$X_1 = X\exp\left\{-\frac{1}{2\nu_m k}\int(v + u_T - u)\,d\xi\right\},$$

we obtain for it the following equation

$$X_1'' - \frac{1}{\nu_m k^2}\left[\mu + k\frac{\partial v}{\partial\xi} + \frac{(v + u_T - u)^2}{2\nu_m}\right]X_1 = 0. \qquad (6)$$

If $v(\xi)$ is a periodic function, then (6) is a Mathieu-Hill equation. For $v \ll |u - u_T|$, which is valid in practice when the velocity of the oscillations is much lower than the velocity of sound one can set $X(\xi) \sim e^{i\kappa\xi}$ where,

$$\mu = -i\varkappa(\omega - \omega_T) - \nu_m k^2\varkappa^2. \qquad (7)$$

In this case, the solution of (4) has the form

$$H'(\xi, t) = \int C(\varkappa)\exp[i\varkappa\xi + \mu t]\,d\varkappa.$$

When $t = 0$

$$H'(\xi, 0) = \int C(\varkappa)\,e^{i\varkappa\xi}d\xi = -H_\infty(\xi).$$

From this it follows that $C(\kappa)$ is a Fourier component of $-H_\infty(\xi)$.

Using (7) we find,

$$H(\xi, t) = H_\infty(\xi) - \frac{1}{2\pi}\int_{-\infty}^{\infty}d\varkappa\int_{-\infty}^{\infty}d\xi' H_\infty(\xi') \times\exp\{i\varkappa[\xi - \xi' + (\omega - \omega_T)t - \nu_m k^2\varkappa^2 t]\} = H_\infty(\xi) - (4\pi\nu_m k^2 t)^{-1/2}\int_{-\infty}^{\infty}H_\infty d\xi'\left[-\frac{(\xi - \xi' + \omega t - \omega_T t)^2}{4\nu_m k^2 t}\right].$$

As far as the component H_x is concerned, it proves to be proportional to $(\nabla T)^2$ and is therefore generally less than $H_\perp$.

In order to estimate the intensities of the magnetic fields occurring in a turbulent plasma, we can consider either (5) or else (3) directly. In the most important case of low magnetic viscosity $\nu_m k < |u - u_T + u_s - v|$ we neglect the terms containing it in (3). We then obtain

$$H_{\perp}=\frac{cT\widetilde{\Lambda}\,[\nabla\ln T,\ \mathbf{k}]}{ke}\,\frac{\mathbf{k}\mathbf{v}_0\cos\xi+\mathbf{v}_0\nabla\ln T\sin\xi}{\omega\,(u-u_T-v_0\cos\xi+u_3)}. \qquad (8)$$

In the case $v(\xi) \ll u - u_T + u_s$ we can expand the numerator of (8) into a power series in terms of $v(\xi)$. If $v(\xi)$ is a periodic function, then averaging with respect to its phases, we obtain

$$H_{\perp}=\frac{1}{2}\widetilde{\Lambda}\,\frac{\nabla_z T}{e}\,\frac{c}{s}\left(\frac{v_0}{s}\right)^2,$$

where v_0^2 is the average value of the function $v^2(\xi)$. For $u - u_T + u_s \approx s$ we obtain an estimate of the Larmor frequency for ions

$$\Omega_i \approx (s/L)\,(v_0/s)^2. \qquad (9)$$

Although our estimates relate to the case $v_0 \ll s$ and $\lambda \ll L$, they are valid in order of magnitude up to values $v_0 \approx s$ and $\lambda \approx L$. Under such conditions $\Omega_i \approx s/\lambda \approx \omega$, and for a plasma with a large radiation pressure $\Omega_i \approx N\omega/n$. The indicated limiting values are reached in shock waves. The electron Larmor and oscillation frequencies can even be equal in normal sound waves.

It is possible for much stronger magnetic fields to be produced. For $|v_{max}| > |u - u_T + u_s|$, which is possible if $|u - u_T + u_s| < s$, the average value of (8) can be larger than (9). For parametric resonance ($v \approx \nu_m k$), it is possible according to (6) for a strong magnetic field to be formed. In addition to this effect, the magnetic fields excited by these processes can be amplified further by "entanglement" of the magnetic field lines or by the dynamo effect.

We now estimate the contribution to the damping of the turbulence that arises from the conversion of part of the energy of turbulence into magnetic energy. In the case $\Omega_i \approx \omega$ the ratio of magnetic to kinetic energy is

$$\frac{H^2}{8\pi\rho v^2}=\frac{1}{2}\frac{M}{m}\left(\frac{c}{s}\right)^2\left(\frac{\omega}{\omega_{pe}}\right)^2\ll 1 \text{ for } \omega<\omega_{pe}\frac{s}{c}\sqrt{\frac{2m}{M}},$$

and in the case where $\Omega_e \approx \omega$

$$\omega<\omega_{pe}\frac{s}{c}\sqrt{\frac{2M}{m}}.$$

For Ω_e and Ω_i close to the frequency of oscillation, either electrons or ions can be accelerated as a result of parametric resonance [3]. This can contribute to the production of relativistic electrons and ions in cosmic sources of radio emission, or at least provide an injection mechanism for Fermi acceleration.

The equation of motion of particles in a uniform alternating magnetic field with Larmor frequency $\Omega(t)$ is, with allowance for the induced electric field and damping,

$$\ddot{z}+i\,[\Omega(t)-i\nu/2]\,\dot{z}+2i\dot{\Omega}(t)z=0.$$

Here $z = x + iy$ are the coordinates of the particle in a plane perpendicular to the magnetic field and ν is the damping frequency, which for estimating purposes can be considered to be independent of the particle energy.

Let

$$\Omega(t)=\Omega_0+\Omega_1\cos\omega t=\Omega_0\,[1+h\cos\Omega_0(1+\Delta)\,t],$$

$$\Delta\ll 1.$$

We consider two limiting cases.

1) First the case

$$h\ll 1, \qquad \nu/\Omega_0\ll 1.$$

By the usual procedure (see [4]) we obtain

$$z=e^{-\nu t/2}\exp\left(-\frac{i}{2}\int_0^t\Omega(t')\,dt'\right)\{C_1e^{\mu t}[\cos\Omega_0(1+\Delta)\,t$$
$$+\gamma\sin\Omega_0(1+\Delta)\,t]+C_2e^{-\mu t}[\cos\Omega_0(1+\Delta)\,t$$
$$-\gamma\sin\Omega_0(1+\Delta)\,t]\}.$$

Here

$$\mu=\frac{1}{2}\sqrt{h^2-4(\Delta^2+\nu^2)}, \qquad \gamma=[\Delta-h/2+i\nu]/\mu.$$

After averaging over initial conditions for the coordinates and velocities and over the oscillation period we obtain the following expression for the energy when $(\mu - \nu)t \gg 1$

$$\varepsilon\approx\frac{1}{2}m\,|\dot{z}|^2\approx e^{(\mu-\nu)\Omega_0 t}\,[\varepsilon_{\perp 0}+mr_0^2\Omega_0^2/4]. \qquad (10)$$

In this equation we used the estimate $\gamma \approx 1$ while r_0 and ϵ_0 are the length over which the magnetic field is uniform and the initial energy, respectively.

Our system can release fast particles as a result of collision between accelerated particles (for which the resonance condition is satisfied) and unaccelerated particles.

2) In the second limiting case the magnetic field has no constant component so that $\Omega(t) = \Omega_0 \cos\omega t$. In this case it will be assumed that $\nu \ll \mu$. It is then evident from dimensional considerations that particle acceleration must occur when $\omega \approx \Omega_0$ and that in these circumstances the order of magnitude of the dimensionless increment of oscillations is $\mu = 1$ (see [5]). The energy $\epsilon_{\perp}$ in this case also increases with time as $\exp(\mu\omega t)$.

Examination of these two cases shows that in the frequency range corresponding to ion acceleration the electrons are not accelerated, and vice-versa. The relationship between the frequencies of turbulent pulsation and the Larmor frequencies in interstellar matter, stellar atmospheres, and the earth's ionosphere are much more suitable for the acceleration of ions than of electrons. The acceleration of heavy ions is more probable than acceleration of light ions, and may result in their preferential injection for the formation of cosmic rays.

We now consider the effects of friction experienced by the accelerated particles. We have seen in the presence of friction acceleration takes place only when $\mu > \nu$. In the first case, when $\mu \approx h$, this means that, to an order of magnitude accuracy, we should have

$$\Omega_1 = eH_1/mc > \nu, \tag{11}$$

and in the second case

$$\Omega_0 \gtrsim 10\nu. \tag{12}$$

In such conditions the possibility of upsetting the prevailing state is reduced. If the constant magnetic field in the first case and the amplitude of the magnetic field in the second case are so small that the inequalities above are not satisfied, no particle accelerations can occur with these relaxation times. However, the relaxation time of charged particles in a plasma, is proportional to the cube of their velocities, and therefore for particles with initial transverse energies $\epsilon_\perp > T$ these inequalities can be satisfied. In such circumstances an activation barrier exists for resonant acceleration of particles and only a small fraction of them are actually accelerated. It is only for this condition that the calculation given above for resonant acceleration is valid, for otherwise the magnetic field of the accelerated particles themselves must be taken into account. It is possible that phenomena produced when the inequalities (11) and (12) are satisfied for particles of mean thermal energies, i.e., acceleration of all particles (general heating of plasma according to [3]) without activation, are similar to flares on the sun.

Particle acceleration occurs in regions where the magnetic field is uniform. It can repeat many times and has the characteristics of diffusion in energy space, which leads to a significant acceleration of the particles.

3. HYDROTHERMOMAGNETIC WAVES

In this section we study the variation of the spectrum of magnetohydrodynamic oscillations of a plasma in which there is a temperature gradient such as considered above. We place our plasma in a constant and uniform magnetic field such that

$$\Omega_e\tau < 1, \qquad H < 4\pi neLs/c. \tag{13}$$

We add to (2) the equations of hydrodynamics

$$\rho\partial \mathbf{v}/\partial t = -\nabla p + [\mathrm{rot}\,\mathbf{H},\ \mathbf{H}]/4\pi, \qquad \partial\rho/\partial t + \rho\,\mathrm{div}\,\mathbf{v} + (\mathbf{v}\nabla)\rho = 0. \tag{14}$$

We set $T = T_0 + T'$, $\rho = \rho_0 + \rho'$, $H = H_0 + H'$ (subsequently, the suffix zero will be omitted), $p \sim \rho^\gamma$ and we assume that the primed quantities as well as v are proportional to $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$. We then obtain the dispersion equation

$$(\omega^2 - \omega\omega_T - \omega_A^2)\,\{\omega^4 - \omega^3\omega_T - \omega^2k^2\,(s^2 + v_A^2) + \omega\,[k^2s^2\omega_T + k^2v_A\,(\mathbf{k}\,[\mathbf{u}_1\mathbf{v}_A])] + \omega_s^2\omega_A^2\} = 0. \tag{15}$$

The symbols have the following significance: ω_S is the frequency of sound, $\mathbf{v}_A$ and ω_A are the velocity and frequency of the Alfvén waves, and

$$\mathbf{u}_1 = \frac{cT\widetilde{\Lambda}}{eH_0}\nabla \ln T.$$

In deriving (15) we disregarded dissipative terms. Their magnitudes will be estimated below. Furthermore we have neglected terms containing the small parameter $\Omega_e\tau$ in those cases when they enter along with similar expressions not containing $\Omega_e\tau$, and we have rejected from the left half of (15) a term which is smaller than the remaining one by a factor

$$\frac{s}{u_1}\left\{\frac{u_3}{u_A}\left[T\frac{\partial}{\partial T}\left(\frac{\sigma'}{\sigma^2}\right) - \rho\frac{\partial}{\partial\rho}\left(\frac{\sigma'}{\sigma^2}\right)\right] \Big/ T\frac{\partial}{\partial T}\left(\frac{1}{\sigma}\right)\right\}^2 \approx \frac{s}{u_1}\left[\frac{u_3}{u_A}\Omega\tau\right]^2.$$

This factor is less than unity for the conditions specified in (13).

Let us consider particular cases for the solution of the dispersion equation.

1. Thermomagnetic waves ($v = 0$). This case occurs when $\mathbf{k} \perp \mathbf{H}$ and lies in the $\mathbf{H}$, ∇T plane. Under these circumstances $\mathbf{H}' \perp \mathbf{k}$, $\mathbf{H}' \perp \mathbf{H}$, and $\omega = \omega_T = \mathbf{k}\cdot\mathbf{u}_T$. In particular, transverse thermomagnetic waves $\mathbf{H}' \perp \mathbf{k}$ are possible even in the absence of an external magnetic field.

2. The case, $\mathbf{k} \perp \mathbf{H}$. When $u_A \ll s\sqrt{\Omega\tau}$ the spectrum consists of two branches: a pure thermomagnetic branch, $\omega = \omega_T$, and an acoustic one.

3. In the case $\mathbf{k} \perp \nabla T$, one obtains the normal magnetohydrodynamic spectrum [6].

4. If the conditions, 1, 2, and 3 are not fulfilled, then the hydrothermomagnetic spectrum divides into two branches, one of which resembles Alfvén waves, and the other magnetic-sound waves. The differences from the normal spectrum are only important in the case where one of the frequencies (ω_1, ω_2, ω_3) is comparable to (ω_A, ω_S) or exceeds

them. The factor which corresponds to Alfvén waves when $\nabla T = 0$ yields, in our case, two "quasi-Alfvén" waves of identical structure ($\mathbf{k}\cdot\mathbf{v} = 0$, $\mathbf{H}' \parallel \mathbf{v}$) with the additional limitation that the vectors $\mathbf{v}$ and $\mathbf{H}'$ must now be perpendicular to ∇T as well. In these circumstances,

$$\omega(k) = \frac{1}{2}\left[\sqrt{4\omega_A^2 + \omega_T^2} \pm \omega_T\right].$$

The second factor in (15) determines when $\omega_T = 0$ the frequencies of the magnetic sound waves [6]. In our case no general solution of the corresponding equation is possible and it is necessary to consider various limiting cases. If

$$\omega_A \ll \omega_s \ll \omega_T \quad \text{or} \quad \omega_T < \omega_s, \quad \omega_A < \omega_T^2/\omega_s,$$

then there is a single almost doubly degenerate solution $\omega = \pm\,\omega_s$ and the roots are $\omega \approx \omega_T$, $\omega \approx \omega_A^2/\omega_T$. The inequality $\omega_T \gtrsim \omega_s$ stipulated above is equivalent to the relationship $(l/L) \times \sqrt{M/m} \gtrsim 1$, and for large radiation pressure

$$\frac{\omega_1}{\omega_s} \approx \frac{l}{L}\frac{N}{n}\sqrt{\frac{M}{m}} \gtrsim 1$$

(l is the mean free path of the electrons and N the number of photons in a unit volume).

5. Special attention should also be paid to the case of a very hot, rarefied plasma in which radiation pressure of the gas and the interaction of electrons with photons is much stronger than Coulomb scattering. In [2] it is shown that if the Compton scattering predominates over bremsstrahlung and its corresponding inverse process for electron-ion collisions, then the velocity u_1 in (15) is increased roughly by a factor N/n. If N/n is so large that the velocities s and u_T are much smaller than $(u_1 u_A^2)^{1/3}$ then the magnetic-sound branch has a single near-zero frequency and another root, $\omega \approx k\,(u_1 u_A^2)^{1/3}$, which is threefold degenerate.

We now estimate the dissipative terms. In Eq. (14) it has the form $(k/\omega)\,s^2(\mathbf{v}\nabla \ln T)$. Comparing this to $i\omega v$ we obtain a condition for dissipation to be small, as $kL \gg (s/L)^2$.

In Eq. (2) the dissipative term is in our approximation:

$$\nu_m k^2 H' + \frac{(\gamma-1)\,k^2\,(kh)\,(\mathbf{H}\mathbf{H}')\,cT}{4\pi\rho\,(\omega^2-\omega_s^2)\,e}\left[(\gamma-1)\frac{\partial\Lambda'}{\partial T}T + \rho\frac{\partial\Lambda'}{\partial\rho}\right] = A + B.$$

Comparing A with $\omega_T H'$, we obtain the condition for it to be small, namely $l \gg c/\omega_p$ (ω_p is the plasma frequency). The term B is small compared with A in all cases except when

$$|\omega - \omega_s|/\omega_s \ll 1.$$

When $\omega_A \ll \omega_s$ for the magnetic sound branch this ratio is of the order of $(u_A/S)^2$ if $\mathbf{k}\cdot\mathbf{h} \neq 0$. For this case

$$\frac{B}{A} \approx \left(\frac{M}{m}\frac{s}{c}\,\omega_p\tau\right)^{1/2},$$

i.e., it is independent of the magnetic field and in a sufficiently rarefied and hot plasma can exceed unity. We note that for these conditions the term B as well as the dissipative term in (14) can lead to the growth of oscillations over a length L.

In conclusion we consider the limiting case when the magnetic field changes slightly over the length L, in contrast to the problem considered up to this point. This condition is realized in practice only on an astrophysical scale. We limit ourselves to the case v = 0. Then (2) becomes

$$\frac{\partial\mathbf{H}}{\partial t} = \frac{\partial\Lambda'}{\partial T}[\nabla T[\nabla T, \mathbf{H}]] = \frac{\partial\Lambda'}{\partial T}\{\nabla T(\nabla T, \mathbf{H}) - \mathbf{H}(\nabla T)^2\}.$$

Solution of this equation leads to the conclusion that the component of $\mathbf{H}$ parallel to ∇T does not vary with time, but the perpendicular component is damped by a factor e^{-t/t_0} where

$$t_0^{-1} = \frac{\partial\Lambda'}{\partial T}(\nabla T)^2 \approx \frac{vl}{L^2}$$

(v is the mean electron velocity).

[1] V. L. Ginzburg, UFN **62**, 156 (1957).

[2] L. É. Gurevich and V. I. Vladimirov, JETP **44**, 166 (1963), Soviet Phys. JETP **16**, 118 (1963).

[3] L. I. Rudakov and R. S. Sagdeev, Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and the Problem of Controllable Thermonuclear Reactions) **3**, Atomizdat, 1959, p. 153.

[4] L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics) Gostekhizdat, 1958, p. 98.

[5] N. W. McLachlan, The Theory and Applications of Mathieu Functions, Oxford, 1947.

[6] S. I. Syrovat-skii, UFN, **62**, 342 (1957).

Translated by F. L. Curzon
93