AMPLITUDE DISTRIBUTION OF BURSTS PRODUCED BY HIGH-ENERGY MUONS UNDER THICK FILTERS

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A trial and error method is employed to calculate the amplitude distribution of bursts produced by a monoenergetic flux of high energy (10^{13} and 10^{14} eV) μ mesons under thick lead filters. Creation of bursts as a result of direct production of electron-positron pairs by the μ mesons and μ-meson bremsstrahlung is taken into account. The calculated burst distribution obtained in very broad (Figs. 2 and 3). The possibility of determining the μ-meson energy on the basis of the large and small bursts created by them under thick filters is discussed.

RECENTLY, in connection with investigations of the interaction between high-energy muons and matter, and also in connection with the problem of muon generation, interest is exhibited again in the study of the energy spectrum of muons in the high-energy region.

Appreciable and perfectly reliable results on the energy spectrum of muons at energies $\lesssim 10^{12}$ eV have been obtained by various methods [1-3]. On the other hand, the gathering of information on the energy spectrum of muons at high energies encounters great difficulties [4]. However, the efficiency of registration of high-energy muons can be increased by using apparatus in which it is possible to select small bursts produced by muons as a result of production of electron-positron pairs. The point is that the differential cross section of pair production in the energy region $v = 10^{-3}$–10^{-4} (v is the pair energy expressed as a fraction of the muon energy) is practically two orders of magnitude larger than the differential cross section of bremsstrahlung. This thought was expressed in a paper by Alekseev and Zatsepin [5], where it was proposed to study the muon energy spectrum relative to small bursts with the aid of arrays consisting of several rows of detectors (ionization chambers, scintillation counters), separated by thick layers of matter. The statement was made that such a system will make it possible to determine the energy of the muon in each individual case by measuring the number of particles passing through each row of detectors.

Taking into account the timeliness of new methods of muon-energy determination, we have calculated the distribution of muon bursts produced in such an array, at a fixed muon energy [1)]. In the calculation, which was made by the Monte Carlo method, we considered bursts from muons of two fixed energies (10^{13} and 10^{14} eV) under a layer of lead 15 cm thick or a multiple of this thickness.

We have assumed that the muons produce bursts only because of pair production and bremsstrahlung in the absorber. The contribution of the "nucleon" interactions of the muons to the number of bursts of given value is in the case of a lead filter at least one order of magnitude lower than the contribution of the bremsstrahlung [3]. As regards the formation of muons by delta electrons, their contribution can be noticeable only at very small values of the bursts n, when only a few relativistic particles pass through the chambers. In addition, by virtue of other assumptions (see below), our calculation is valid only in the region $n > 10$ relativistic particles.

The differential cross sections of the indicated processes, calculated for lead for a muon energy 10^{13} and 10^{14} eV, are shown in Fig. 1. In the calculations we used the formulas for the differential muon-induced bremsstrahlung and pair-production cross sections, given in [7]. The cross section for the production of δ-electrons, which we show for comparison, was taken from [8]. From these data we obtain the following values of the average muon energy losses in lead for each cascade unit (equal to 5.25 g/cm^2): 1) pair-production losses $(dE/dt)_p = 5.3 \times 10^{-5} E_0$ eV; 2) bremsstrahlung losses $(dE/dt)_r = 4.8 \times 10^{-5} E_0$ eV. It follows therefore

1)These calculations were stimulated by a lecture given by É. V. Gedalin at the conference at Bakuriani (April, 1962).[6]

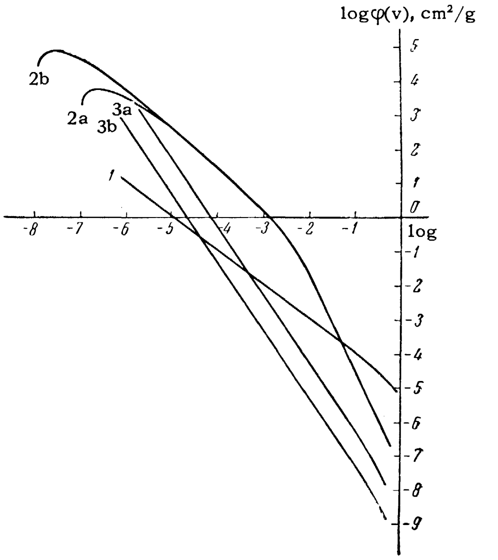

FIG. 1. Differential cross sections of different processes for muons in lead: curve 1 – bremsstrahlung ($E_\mu = 10^{13}$ and 10^{14} eV); curve 2 – pair production; curve 3 – δ-electron production (a – $E_\mu = 10^{13}$ eV, b – $E_\mu = 10^{14}$ eV).

that either process should make approximately the same contribution to the average value $\bar{n}$ of the size of the burst.

The next assumption, which was made in the calculation, consisted of neglecting the bursts produced by the electron-positron pairs with energy $< 6 \times 10^8$ eV. The muon energy losses (at the energies considered) to pair production with energy $< 6 \times 10^8$ eV amount to approximately 20 MeV per cascade unit (that is, < 2% of the total energy loss). The main contribution will be made by the secondary particles of the indicated energies to bursts of size n < 10 particles. Therefore the characteristics of the burst distribution given below are hardly influenced by our assumption.

We notice also that the energy of the muon passing through the filter is considered constant. This is true, since the maximum considered filter thickness (150 cm lead or ~1700 g/cm^2) is much less than the total range of the muons of the energies under consideration (~3×10^{-5} g/cm^2). In addition, the calculation did not disclose a single event in which a muon lost more than 10 per cent of its energy as a result of interaction.

It has been found further that the probabilities of muon interaction per cascade unit with energy loss $\geq 6 \times 10^8$ eV are respectively 0.045 and 0.090 for muon energies 10^{13} and 10^{14} eV. We note that the probabilities of such an interaction, calculated with account of only the pair-production process, have values 0.044 and 0.089.

The calculation was carried out in the following manner. For each passage of a muon through the filter, random tests were used to determine the locations of interaction and the energies of the produced electron-positron pairs and gamma quanta. The 15-cm lead layer was broken up into thirty-three layers of one cascade unit each. The behavior of the muon passing successively through each layer was tried (that is, passage without interaction or with interaction in a given layer). Further, for each case of interaction, pair production or the production of gamma quanta with energy amounting to a certain fraction of the primary muon energy was tried. (The entire range of energy of the electron-positron pairs and the gamma quanta was broken up into intervals of equal logarithmic width: 6×10^8–1.2×10^9 eV; 1.2×10^9–2.4×10^9 eV, etc.) The size of the burst n in the detector was determined from the muon interaction depths and from the produced-particle energies obtained in this manner. The cascade curves for the lead were used by us in the form borrowed from [9].

Altogether 300 cases were tried, for each passage of muons with energies 10^{13} and 10^{14} eV through a filter of thickness 33 t-units. (To find the burst distribution under large filter thicknesses, we used the same 300 events, from which we obtained 150, 100, etc passages of the muons through a filter of thickness 66, 99, etc. t-units, respectively.) The burst distributions obtained as a result of such a trial are shown in Figs. 2 and 3.

For the distributions corresponding to 33 t-units, we calculated the average size of the burst $\bar{n}$ and the square root of the relative variance of the distribution, that is, of the quantity $\Delta = \sigma/\bar{n}$. They turn out to be $\bar{n} = 130 \pm 30$ and $\Delta = 3.5$ for 10^{13} eV muons and $\bar{n} = 1440 \pm 300$ and $\Delta = 3.4$ for 10^{14} eV muons. From the data presented it is seen that within the limits of calculation error the average burst size increases in proportion to the energy.

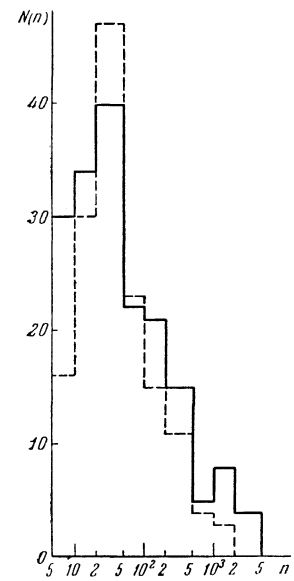

FIG. 2. Amplitude (in the number of relativistic particles) distribution of bursts obtained with account of both pair production and bremsstrahlung (solid line), and with account of pair production only (dashed line). The energy of the muon is 10^{13} eV, the filter thickness 33 t-units.

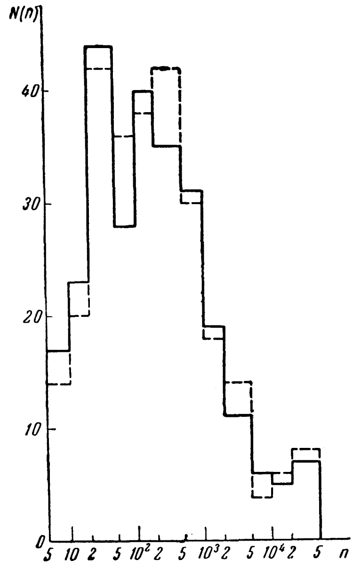

FIG. 3. Burst distribution obtained with account of both pair production and bremsstrahlung. The muon energy is 10^{14} eV, the filter thickness 33 t-units (solid line) and 66 t-units (dashed line). The area of the histogram for 66 t-units is normalized to the histogram for 33 t-units.

It must be noted that the burst distribution was found to be quite broad.

In the calculation we did not take into account fluctuations in the development of the electron-photon cascades. An account of this factor will lead to a certain increase in the distribution width, although apparently an insignificant one.

Figure 2 shows also the burst distribution obtained under the assumption that the 10^{13} eV muon loses energy only to pair production. The square root of the relative variance of the distribution is in this case $\Delta = 2.8$ at an average burst size $\bar{n} = 53 \pm 10$. Although the small number of radiation bursts does influence the foregoing values of $\bar{n}$ and Δ, the distribution remains quite broad nonetheless.

Figure 3 shows the distributions of the bursts produced by muons with energy 10^{14} eV for filters of 33 and 66 t-units. The distributions differ from one another only slightly. This means that for the range of burst sizes under consideration and within the limits of the calculation accuracy attained, the electron-photon cascades accompanying the muons are in equilibrium with the generating component, in any case at depths $\sim$ 30 t-units.

The obtained burst distributions (Figs. 2 and 3) show that a sufficiently accurate determination of the energy of the individual muon by the method proposed in [5], even at such high energies as $10^{13}-10^{14}$ eV, is hardly possible with the aid of the realized installations. Arrays consisting of several rows of ionization chambers or scintillation counters through which high-energy muons pass, will not yield a unique result, owing to large fluctuations in the burst sizes from event to event. If the number of rows of the recording system is appreciably increased, the distribution of the observed burst sizes $n_1, n_2, \ldots, n_k$ (k is the number of rows) will approach the distributions shown in Figs. 2 and 3 (continuous curves). In this case the average burst turns out to be proportional to the muon energy and is connected with it uniquely, that is, this makes it possible to determine the muon energy at sufficiently large values of k.

Some idea concerning the minimum value of k needed to determine the muon energy E_μ with a certain prescribed accuracy can be gained in the following fashion. The summary ionization produced in several rows of the recording system is subject to lesser fluctuations than the ionization produced in one row. With increasing number of rows, the width of the distribution of the summary ionization decreases. It was found in the calculations that for 1, 3, 5, and 10 rows the square root of the relative variance Δ_k has values 3.5, 1.9, 1.4, and 1.1, respectively. In general, as the number of rows increases the fluctuations in the total sum of the bursts vary in such a way that $\Delta_k = \Delta_1/\sqrt{k}$. This is a simple consequence of the mutual independence of the burst sizes $n_1, n_2, \ldots, n_k$, which obtains at the filter thicknesses under consideration.

However, in order to solve the problem of the minimum value of k, it is necessary in general to solve not only the problem of the magnitude of the fluctuations for a specified energy E_μ, but also of the distribution of the probabilities for the true values of E_μ, a distribution corresponding to the observed experimental combination of burst sizes $n_1, n_2, \ldots, n_k$. For this distribution we can write, by the Bayes theorem, the following expression:

$$\begin{aligned} f_1(E_\mu \mid n_1, n_2, \ldots, n_k)\, dE_\mu \\ \sim \varphi(E_\mu)\, dE_\mu \cdot F_2(n_1, n_2, \ldots, n_k \mid E_\mu) \\ = \varphi(E_\mu)\, dE_\mu \cdot f_2(n_1 \mid E_\mu) \cdot f_2(n_2 \mid E_\mu) \ldots f_2(n_k \mid E_\mu), \end{aligned}$$

where $\varphi(E_\mu)$ is the energy spectrum of the muons incident on the array $[\varphi(E_\mu)\, dE_\mu \sim E^{-(\gamma+1)}\, dE_\mu]$; $f_2(n_i \mid E_\mu)$ is the probability of producing a burst n_i in the i-th row by a muon with energy E_μ.

We see therefore that the minimum value of k is determined not only by the burst distribution curves $f_2(n \mid E_\mu)$ which we calculated, but also by the exponent γ of the energy spectrum of the muons incident on the array. In particular, owing to the drooping character of the spectrum $\varphi(E_\mu)$ and the finite probability of bremsstrahlung at sufficiently low k, the combination of the burst sizes $n_1, n_2, \ldots, n_k$ can generally speaking correspond to a muon of relatively low energy. At sufficiently large k the behavior of the function $f_1(E_\mu \mid n_1, n_2, \ldots, n_k)$ will be determined essentially by the behavior of the function $f_2(n_1, n_2, \ldots, n_k \mid E_\mu)$. However, in this case the minimum value of k is apparently still larger than the value that follows from an examination of the distribution of the bursts without account of the muon spectrum.

Although the burst distribution which we calculated points out the very great difficulty in realizing arrays for the measurement of the energies of individual muons, nevertheless arrays of this type can apparently be used to measure the energy spectrum of the muons. In this case the conversion from the burst spectrum to the muon energy spectrum can be made in principle by a way similar to that described in [3]. In addition to the greater efficiency, this method has compared with [3] still another advantage in that small bursts are known to be produced by pure electromagnetic muon interactions.

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