PHASE SHIFT ANALYSIS OF ELASTIC SCATTERING OF PROTONS ON TRITIUM NEAR THE THRESHOLD OF THE T(p,n) He³ REACTION

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A phase shift analysis for 990 keV protons elastically scattered on tritium is carried out. By employing the complexity of the angular dependence due to Rutherford scattering and also the data for the threshold anomaly, it has been possible to reduce the number of solutions to two, corresponding to resonance in the $^1S_0$ state.

THE extensive experimental material on interaction between protons and tritium, which can be found in the literature [1,2], is of appreciable interest in connection with the question of the excited states of He^4. Data on elastic scattering are analyzed in some papers [3,4]. However, the most important of these papers [4] makes use of several simplifying assumptions which make the result doubtful. In this connection it is of interest to repeat the analysis, using also the published data on the measurement of the anomalies in the elastic-scattering cross section near the threshold of the $T(p,n)He^3$ reaction [5].

Since there is no noticeable contribution of alpha waves to the reaction and scattering cross sections near threshold, it follows from momentum and parity conservation that the reaction is connected with the absorption of s-protons. Separating the amplitudes of the Rutherford scattering in formula (2.6) of the paper by Baz' [6] and confining ourselves to an examination of s and p waves, we can readily obtain the following expressions for the change in cross section of the elastic scattering near threshold:

$$4\pi\Delta\sigma_S^+ = -(2kR\sin\xi - 1)X + 2kR\cos\xi\, Y - \sigma_r - D\cos\vartheta,$$

$$4\pi\Delta\sigma_S^- = -2kR\cos\xi X - (2kR\sin\xi - 1)Y + B\cos\vartheta. \qquad (1)$$

Here $\Delta\sigma_S^\pm$ is the change in the differential cross section of elastic scattering at an interval ΔE above and below the threshold E_n, in the c.m.s.; σ_r is the total cross section of the reaction at energy $E_n + \Delta E$; k is the wave number of the proton at $E = E_n$; $R = z_1 z_2 e^2/4E_n \sin^2(\vartheta/2)$ is the amplitude of Rutherford scattering; ξ is the phase of Rutherford scattering at $E = E_n$; ϑ is the c.m.s. scattering angle; D and B are quantities that depend on the p-phases;

$$X = {}^1\sigma_r \cos 2\,{}^1\delta_0 + {}^3\sigma_r \cos 2\,{}^3\delta_0,$$
$$Y = {}^1\sigma_r \sin 2\,{}^1\delta_0 + {}^3\sigma_r \sin 2\,{}^3\delta_0; \qquad (2)$$

${}^1\sigma_r$, ${}^3\sigma_r$, ${}^1\delta_0$, and ${}^3\delta_0$ are the cross sections of the reaction¹⁾ and the scattering phase shifts for the states $^1S_0$ and $^3S_1$.

In solving equations (1) we used the values of $\Delta\sigma_S$ at points 30 keV above and below threshold, taken from the paper of Jarmie and Allen [5], and used for the reaction cross section a value of 100 mb as an average of the results of Gibbons and Macklin [8], increased by 15 per cent with allowance for the deviation from the 1/v law, as observed by Bergman et al [9]. The contribution of the p-waves to the scattering amplitude was eliminated using the data for two angles. The coordinates of the vector (2) were found to be $X = (-1.2 \pm 0.5) \times 10^{-25}$ and $Y = (0.1 \pm 0.6) \times 10^{-25}$.

Inasmuch as ${}^1\sigma_r + {}^3\sigma_r = \sigma_{r.\,exp} = 1.15 \times 10^{-25}$, this result can correspond to one of the following solutions:

1) ${}^1\delta_0 \approx {}^3\delta_0 \approx \pi/2$, ${}^1\sigma_r/{}^3\sigma_r$ not determined
2) ${}^3\delta_0 \approx \pi/2$, ${}^1\sigma_r \approx 0$;
3) ${}^1\delta_0 \approx \pi/2$, ${}^3\sigma_r \approx 0$. (3)

¹⁾In these formulas, as is well known, the cross section of the reactions is the first term of the expansion of the scattering amplitude in powers of $k^2\sigma_r$. The validity of such an expansion below the threshold is not obvious. It can be shown, using the resonance formula of one level,[7] that such an expansion is valid also below the threshold subject to the additional condition $\Gamma_r \ll 2|E'_\lambda - E|$, that is, not at resonance. On the other hand, one should expect large threshold anomalies in the cross section only near resonance, where the derivative of the reaction cross section can be large.

Comparison with the results of the phase-shift analysis, given below, enables us to discard the first two solutions.

Thus, the reaction proceeds essentially through one spin state, and the scattering phase in this state is close to resonance. Good agreement is observed between the data of the reaction cross section and the threshold anomaly, but the experimental errors are large (the mean square errors are indicated). The results obtained enable us to carry out more unambiguously a phase shift analysis of elastic scattering near threshold.

In the phase shift analysis it is convenient to write down the expression for the scattering cross section in terms of the following variables:

$$X_0 = \tfrac{1}{4}\cos 2\,{}^1\delta_0 + \tfrac{3}{4}\cos 2\,{}^3\delta_0,$$

$$Y_0 = \tfrac{1}{4}\sin 2\,{}^1\delta_0 + \tfrac{3}{4}\sin 2\,{}^3\delta_0,$$

$$X_1 = \tfrac{1}{4}\cos 2\,{}^1\delta_1 + \tfrac{3}{4}\cos 2\,{}^3\delta_1,$$

$$Y_1 = \tfrac{1}{4}\sin 2\,{}^1\delta_1 + \tfrac{3}{4}\sin 2\,{}^3\delta_1,$$

$$Z = \tfrac{1}{4}\cos 2\,(\psi + {}^1\delta_1 - {}^1\delta_0) + \tfrac{3}{4}\cos 2\,(\psi + {}^3\delta_1 - {}^3\delta_0) \qquad (4)$$

(here ψ is the phase of the Coulomb scattering for $l = 1$). This representation, which allows a determination of the s-phase by simple graphic methods, was proposed by Crysty [10] for the analysis of s-scattering. It also has advantages in the presence of p phases. The analysis reduces to a solution of a system of linear equations in the variables (4). The only values X_0, Y_0, X_1, and Y_1 give two systems of solutions for the s- and p-phases each. If there were no interference term (as in the case of the scattering of identical particles), we would have four systems of solutions. The presence of interferences gives an overdetermined system of equations (4) and enables us in principle to choose the unique solution 2).

The phase-shift analysis was made at an energy $E_p = 900$ keV. The data of Hemmendinger et al [1] and Jarmie and Allen [5] were used. The following four systems of solutions were obtained:

Solution	${}^1\delta_0$	${}^3\delta_0$	${}^1\delta_1$	${}^3\delta_1$
1	90°	−22,5°	−2°	13°
2	28°	−40°	−2°	13°
3	100°	40°	−13°	−5°
4	−5°	60°	−15°	−4°

This multivaluedness can be reduced by using the data for the threshold anomaly. Comparison with the results of an analysis of the threshold anomaly enables us to choose solutions (1) and (3), which correspond to resonance in the ${}^1S_0$ state.

Thus, the use of the threshold anomaly leads to more unique determination of the s-phase of elastic scattering of protons by tritium and confirms the conclusions of Frank and Gammel [4] and Bergman et al [9], and also the recently published data of Werntz and Lefevre [11] concerning the existence of an excited He^4 level with characteristics $J = 0^+$.

Frank and Gamel [4] used the condition ${}^3\delta_1/{}^1\delta_1 = -1$, which was obtained in the Born approximation, and discarded the solutions whose energy dependence did not correspond to any resonances. The results of [4] differ somewhat from ours and the fit to the scattering cross sections is much worse. We note that at other energies, the true values of the phases may differ appreciably from their results [4], inasmuch as Frank and Gammel did not take reaction into account. Yet, even at $E_p = 1050$ keV the cross section for the reaction via the state ${}^1S_0$ amounts to $0.30\,\sigma_{max}$ and at higher energies, reaches probably values which are close to maximum. In addition, the large positive value of the phase ${}^3\delta_1$ causes us to assume the presence of a spin-orbit dependence.

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100

2) Note added in proof (January 10, 1963). Actually, however, there is an almost linear relationship between the coefficients of the variables in (4), so that the number of solutions is increased. The data presented in the table were modified in accordance with this circumstance.