THE EFFECT OF THE FREQUENCY OF SOUND ON ITS ABSORPTION BY A METAL IN A MAGNETIC FIELD

M. S. SVIRSKIĬ

Chelyabinsk State Pedagogical Institute

Submitted to JETP editor August 4, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 628-632 (February, 1963)

The question of gigantic quantum oscillations of sound absorption by a metal in a magnetic field due to variation of the ultrasonic frequency is considered. The possibility of experimental observation of such an effect is discussed.

GUREVICH, Skobov, and Firsov [1] predicted and Korolyuk and Prushchak [2] discovered gigantic quantum oscillations of sound absorption by a metal with increasing magnetic field. It is of interest to consider whether, under the conditions [1,3]

$$\zeta \gg \hbar\Omega \gg kT, \quad \omega\tau \gg 1 \tag{1}$$

(ζ is the Fermi energy; $\Omega = eH/mc$ is the Larmor frequency; τ is the relaxation time) similar gigantic quantum oscillations of sound absorption could take place at a fixed magnetic field intensity H, but with increasing frequency of the sound. Such a possibility is suggested by the fact that already at H = 0, according to the laws of conservation of energy and momentum, electrons whose velocity components v_χ in the direction of the wave vector of the sound wave χ are determined by the expression [4]

$$v_\chi = \omega_p (1 - h\nu / 2m\omega_p^2), \tag{2}$$

where ν is the frequency and ω_p is the phase velocity of the sound (symbolized by u in [4]), play a fundamental role in sound absorption. In [1] (see also [3]) the condition

$$v_\chi = \omega_p, \tag{3}$$

which does not take into account the dependence of v_χ on the sound frequency ν, was stipulated instead of Eq. (2). However, the condition (3) is only an approximate one; it is obtained by limiting the expansion of the electron energy in momentum to only terms of the first order in $\hbar\chi$. If one considers the case of a quadratic electronic dispersion law, it is easy to see that the accurate condition (2) holds.

The equation considered in [1]

$$\hbar\Omega (n + 1/2) + p_z^2/2m + \hbar\omega_\chi = \hbar\Omega (n' + 1/2) + (p_z + \hbar\chi_z)^2/2m, \tag{4}$$

which takes into account the laws of conservation of energy and z-component of the wave vector for $H \neq 0$, corresponds precisely to the case of a quadratic isotropic dispersion law. An approximate solution to this equation was found in [1] with the condition

$$\Omega > \chi_z v_F \tag{5}$$

(v_F is the Fermi velocity) and with the limitation of the energy expansion to terms of first order in $\hbar\chi$. The following expression was thereby obtained for the z component of the wave vector of the electrons participating in sound absorption [1]:

$$p_z^0 = m\omega_p/\cos\theta, \tag{6}$$

where θ is the angle between the vectors χ and H.

It is not difficult, however, to obtain an exact solution to Eq. (4), which, when the condition

$$\Omega > v_z\chi_z + \hbar\chi_z^2/2m \tag{7}$$

is fulfilled, has the form

$$p_z^0(\nu) = m\omega_p/\cos\theta - h\nu\cos\theta/2\omega_p, \tag{8}$$

where, in distinction to Eq. (6), p_z^0 is found to depend on the frequency ν. In particular, for $\theta = 0$ (i.e., $H \parallel \chi$), it follows from Eq. (8) that

$$p_z^0(\nu) = m\omega_p (1 - h\nu/2m\omega_p^2), \tag{9}$$

in agreement with Eq. (2).

The occurrence of a frequency dependence in Eq. (8) leads one to expect a gigantic-oscillation effect in the absorption of sound with change in frequency at a fixed value of H. In fact, maxima in the sound absorption of electrons occur each time that p_z^0 falls in any interval $(\Delta p_z)_n$ corresponding to the projection on the axis p_z of the portion of the parabola

$$\varepsilon_n = \hbar\Omega (n + 1/2) + p_z^2/2m, \tag{10}$$

intersected by a band of width kT centered on the level ζ (see the figure in [1], which displays six such $(\Delta p_z)_n$ for n = 0, 1, 2, 3, 4, 5). Consequently, as the frequency is increased from ν_1 to ν_2 the value of p_z^0 will decrease, in accordance with Eq. (8), from $p_z^0(\nu_1)$ to $p_z^0(\nu_2)$, which will lead to the appearance of as many maxima of sound absorption as there are intervals $(\Delta p_z)_n$ located between $p_z^0(\nu_2)$ and $p_z^0(\nu_1)$.

A consideration of Eq. (16) in [1] for the coefficient of sound absorption Γ, which was obtained for the case of quadratic isotropic electronic dispersion for $\mathbf{H} \parallel \boldsymbol{\chi}$, leads to a similar conclusion. In fact, in going from Eq. (16) to Eq. (18) in [1] the authors actually neglected, not the quantity $(p_z^0)^2/2m$, but the quantity $[p_z^0(\nu)]^2/4mkT$ in the argument of the hyperbolic cosine. Indeed, if we retain this quantity, which is responsible for the frequency effect, then instead of Eq. (18) in [1] we obtain the expression

$$\Gamma = \Gamma_0 \frac{\hbar\Omega}{8kT} \sum_{n, s_z} \mathrm{ch}^{-2} \left\{ \frac{\zeta - \hbar\Omega(n + 1/2) - s_z\mu_0 H - [p_z^0(\nu)]^2/2m}{2kT} \right\}, \qquad (11)^*$$

which shows that for $\hbar\Omega \gg kT$ the absorption coefficient has a maximum each time that the projection on the axis p_z of the intersection of the parabola (10) and the level $\zeta - s_z\mu_0 H$ coincides with $p_z^0(\nu)$ from (9).

The frequency dependence was also omitted in [1] in the consideration of the behavior of the absorption coefficient far from the maxima. Having replaced the δ-function in Eq. (16) by Eq. (20),[1)] the authors[1] then assumed $\hbar\chi^2/2m \ll 1/\tau$, $\omega \ll 1/\tau$, and the frequency dependence correspondingly vanished in Eq. (22). However, the assumption $\omega\tau \ll 1$ corresponds to the classical description of the absorption of a sound wave and not to the quantum case, in which $\omega\tau \gg 1$ (see, e.g., [5]). Moreover, the use of the condition $B \gg 1$ later in Eq. (24) of [1] is incompatible with the assumption $\omega\tau \ll 1$ in the low-temperature region. Hence it is expedient to transform Eq. (16) in [1] without an assumption about the magnitude of the frequency ω. In this case, we obtain

$$\Gamma = \Gamma_0 \frac{\hbar\Omega}{2\pi kT} \int dy \frac{B}{1 + B^2 (y - p_z^0(\nu)/\sqrt{2mkT})^2} \times \sum_{m, s_z} \frac{1}{4} \mathrm{ch}^{-2} \left(\frac{y^2 - A_n}{2} \right) \qquad (12)$$

instead of Eq. (22) of [1] (the symbols are the same as in [1]).[2)]

It can be seen from Eq. (12) that the first function under the integral, in distinction to [1], has a maximum not at y = 0 but at $y = p_z^0(\nu)/(2mkT)^{1/2}$ to which corresponds $p_z = p_z^0(\nu)$. If now the position of one of the maxima of the second integrand function coincides with the position of the maximum of the first, and if the width of the maximum of the first function is much less than the distance between the two maxima of the second function closest to it, then a gigantic oscillation will occur. The latter condition (that the separation between two maxima of the second function should significantly exceed the width of the maximum of the first) has, as in [1], the form

$$B(\hbar\Omega/kT) \gg 1, \qquad (13)$$

and the condition that the width of the maximum of the first function be much less than that of the second is

$$B \gg 1. \qquad (14)$$

Further, an estimate of the ratio $\Gamma_{max}/\Gamma_{min}$ leads to

$$\Gamma_{max}/\Gamma_{min} \sim B(\hbar\Omega/kT)^{3/2}. \qquad (15)$$

However, in distinction to [1], we shall write B in Eqs. (13), (14), and (15), not in its approximate form

$$B = (2kT/m)^{1/2} \chi\tau \sim \chi l (kT/\zeta)^{1/2}, \qquad (16)$$

where use was made of the assumption $v_F \approx (2\zeta/m)^{1/2}$, which is not valid in the presence of a field H, but in its exact form

$$B = (\omega\tau/\omega_p)(2kT/m)^{1/2}. \qquad (17)$$

Then, for example, condition (14) takes the form

$$\omega\tau \gg \omega_p (2kT/m)^{-1/2}, \qquad (18)$$

which at $T \approx 1°K$ means $\omega\tau \gg 1$, in agreement with the aforementioned condition of the quantum treatment of sound absorption as a phonon-electron collision process.

In considering the possibility of experimental observation of this oscillation effect, it should be realized that it will occur only if there is at least one interval $(\Delta p_z)_n$ located between $p_z = 0$ and $p_z^0(0)$.[3)] In the figure in [1] is illustrated the ex-

*ch = cosh.

1)There is an error in Eqs. (20) and (21) of [1]; instead of $\hbar\chi p_z/m$ read $\chi p_z/m$.

2)In Eq. (22) of [1] the exponent 2 on y in the argument of the hyperbolic cosine was omitted.

3)In this discussion of a possible experiment it is assumed that $\nu \lesssim \nu_0 = 2m\omega_p^2/h$, which is evidently already two orders of magnitude higher than the frequencies used in [2].

actly opposite case, when all the intervals $(\Delta p_z)_n$ lie to the right of $p_z^0(0)$. In this case, motion to the left of p_z^0 with increasing ν obviously cannot lead to a maximum in sound absorption. However, by changing the field intensity H (to some other fixed value), it is possible to obtain an interval $(\Delta p_z)_n$ to the left of p_z^0 [and for $kT < (p_z^0)^2/2m$ completely within the region $(0, p_z^0)$]. A shift of the intervals $(\Delta p_z)_n$ from the region $p_z > p_z^0$ to the region $p_z < p_z^0$ is exactly what was accomplished experimentally by Korolyuk and Prushchak [2] in fields of the order of 10^4 Oe at T = 1.9—4.2°K in Zn (judging from the number of maxima in Fig. 1 of [2] there were 31 such shifts). Consequently, from this point of view the conditions necessary for the observation of the presumed effect were completely satisfied.

The range of frequencies capable of significantly reducing the quantity $p_z^0(\nu)$ can furnish some important experimental requirements for comparison with the results of Korolyuk and Prushchak. According to Eq. (8), we have at $\theta = 0$

$$\delta p_z^0(\nu) = -h\delta\nu/2\omega_p, \tag{19}$$

consequently, in order to reduce $p_z^0 = m\omega_p$ to 0.1 of its magnitude, the requirement is

$$\delta\nu = m\omega_p^2/5h, \tag{20}$$

which for $m \sim 10^{-27}$ g and $\omega_p \sim 10^5$ cm/sec gives $\delta\nu \sim 10^9$ sec^{-1}, whereas in [2] a frequency of 220 Mc/sec was used.[4)]

A similar estimate is obtained by comparing the $\delta\nu$ and δH capable of causing an equivalent change δp_z (and hence an equivalent shift according to the curve of Fig. 1 in [2]). From Eqs. (19) and (10) follows

$$\delta p_z = -\frac{h\delta\nu}{2\omega_p} = -\frac{m}{p_z}\hbar\left(n+\frac{1}{2}\right)\delta\Omega, \tag{21}$$

[4)]The above estimate of the mass of the carriers can be obtained on the basis of the experiment of Korolyuk and Prushchak,[2] starting from the relation

$$m \leqslant \frac{1}{\omega_p}\left\{\frac{2e\hbar}{c}\left[\frac{1}{\Delta H^{-1}} - \left(n+\frac{1}{2}\right)H_1\right]\right\}^{1/2},$$

where n is the number of observed maxima and H_1 is the intensity of the field at which the first maximum appeared. Substituting $H_1 \sim 10^4$ Oe, $\Delta H^{-1} \sim 2 \times 10^{-6}$ Oe^{-1}, $\omega_p \sim 3.7 \times 10^5$ cm/sec, we find $m \lesssim 10^{-27}$ g. Thus, in the estimates presented we have used $m \sim 10^{-27}$ g. For smaller values of m the magnitude of $\delta\nu$ in Eq. (2) will obviously be reduced.

whence for $p_z^0 \sim m\omega_p$, considering Eq. (19) in [1], we have

$$\delta\nu \approx \frac{1}{\pi}\frac{e}{mc}\frac{1}{H\Delta H^{-1}}\delta H. \tag{22}$$

With $H \approx 3 \times 10^4$ Oe and $\Delta H^{-1} \approx 0.2 \times 10^{-5}$ Oe^{-1}[2] and the above accepted value of m, we obtain

$$\delta\nu \approx 10^8\,\delta H\ [\text{sec}\cdot\text{Oe}]^{-1}. \tag{23}$$

Therefore, under the conditions of this experiment [2] and with $m \sim 10^{-27}$ g a field change of 10 Oe is equivalent to a frequency change of 10^9 sec^{-1}.

For the values of m and ω_p used above and for $\theta = 0$, a frequency of the order of $\nu_0 \approx 10^{10}$ sec^{-1} will, according to Eq. (9), lead to $p_z^0 \to 0$, so that for such magnitudes of m and ω_p the effect ought to be extremely sensitive to frequency changes in the range 10^9—10^{10} sec^{-1}. Phonons with such frequencies are readily available in the experiment (see, e.g., [6]); hence, from this point of view, the experimental observation of the effect considered above is perfectly feasible. In this connection an increase in frequency ν will permit some lowering of the requirements on the frequency of the sample engendered by the inequality $\omega\tau > 1$. Such an experiment could in particular serve to verify the character of the electronic dispersion law for conduction electrons in metals and semi-metals as well as the magnitudes of the effective masses of the carriers [e.g., from Eq. (22)].

In conclusion, I extend my sincere thanks to Prof. S. V. Vonsovskiĭ for valuable discussions on the problems considered above.

[1] Gurevich, Skobov, and Firsov, JETP **40**, 786 (1961), Soviet Phys. JETP **13**, 552 (1961).

[2] A. P. Korolyuk and T. A. Prushchak, JETP **41**, 1689 (1961), Soviet Phys. JETP **14**, 1201 (1962).

[3] H. N. Spector, Phys. Rev. Letters **6**, 407 (1961).

[4] M. S. Svirskiĭ, FMM **2**, 397 (1956).

[5] Akhiezer, Kaganov, and Lyubarskiĭ, JETP **32**, 837 (1957), Soviet Phys. JETP **5**, 685 (1957).

[6] M. Pomerantz, Phys. Rev. Letters **7**, 312 (1961).

Translated by L. M. Matarrese
103