# *APPLICATION OF DISPERSION RELATIONS TO THE COMPTON EFFECT ON THE PROTON*

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The Compton effect on the proton is analyzed by means of a set of six dispersion relations for fixed momentum transfer obtained earlier.[8] The unitarity condition in the one-meson approximation[10] allows to express the imaginary part of the amplitudes in terms of the coefficients of the angular distributions of the photoproduction processes $\gamma + p \rightarrow p + \pi^0$[12] and $\gamma + p \rightarrow n + \pi^+$.[6] The use of the six exact (in the $e_0^2$ approximation) dispersion relations[8] allows to take into account the nucleon recoil (terms linear in the frequency in the dispersion integrals). This improves considerably the agreement with experiment in the subthreshold region[1] ($k_L \lesssim \mu_\pi c^2$) and leads to a better agreement with the newer experiments in the above-threshold region[5,15] as compared to earlier authors[2]. It further improves the accuracy of the estimates of the electric and magnetic polarizabilities of the proton.[1] A number of characteristics of the process (phases, angular distributions, etc.) are computed for $0 < k_L \lesssim 800$ MeV. Some observations are made concerning further possibilities of the analysis of the Compton effect on nucleons.

DURING the last years the nucleon Compton scattering has been investigated quite intensively. A critical review of the basic papers and a detailed list of references is contained in our earlier paper[1].

Despite the fact that by applying two relations for the forward scattering one can satisfactorily describe many features of the process in the subthreshold region[1], it still is clear that this is a rough albeit very simple approximation.

In three papers[2-4] six dispersion relations were applied to the analysis of the nucleon Compton scattering. Jacob and Matthews[3] applied their six dispersion relations (derivation not published) for the purpose of establishing limits of the $\pi^0$-meson lifetime; they added in them mechanically the Low amplitude. Serious objections to this paper have been raised by Lapidus and Chou Kuang-chao (see [1]). Its results also do not agree with the data of Baranov et al.[5]

Akiba and Sato[2] derived a set of six dispersion relations and applied them to the evaluation of $\sigma(k_L)$. They give results for 90° scattering in the center of mass system (c.m.s.) for energies between 180 and 320 MeV and a few angular distributions. We note that these authors derive the dispersion relations in the system $\mathbf{p} + \mathbf{p}' = 0$ ($\mathbf{p}$ and $\mathbf{p}'$ are the nucleon momenta before and after the collision) and then use them in the center of mass system, i.e., they neglect the nucleon recoil. In the given case one has to use with caution the procedure of application of the formulae in a different system than the one in which they have been derived. The point is that in the transformation one has to consider the noncovariant nature of the gauge condition ($e_0 = e_0' = 0$). One thus has to perform an additional gradient transformation (see [1]). This leads to "admixing" of higher frequency terms.

Lapidus and Chou Kuang-chao[4] consider several combinations of amplitudes of the process in the region $k_L \lesssim 300$ MeV. They perform an estimate by means of dispersion relations for these combinations which they derive by differentiating with respect of $\nu_1$ two relations for the forward scattering [see Eq. (1) below] and finally putting $\nu_1 = 0$.[4a] The imaginary parts of the amplitude are expressed in terms of the "pure isospin photoproduction amplitudes"[6a,2] and the total photoproduction amplitudes which are evaluated for $k_L \lesssim 800$ MeV.

Lapidus and Chou Kuang-chao[4] write the cross section in the form

$$\sigma(\theta) = \sum_{l=0}^{3} B_l \cos^l \theta$$

[see Eq. (26) in Sec. 3 of their paper and Table IV]. Even though for $k_L \lesssim 200$ MeV $B_0$ and $B_2$ are rather close to our $\sigma_0$ and $\sigma_2$, their $B_1$ and $B_3 = 0$ differs sharply from the results of the calculations given below.

However, the data of Lapidus and Chou Kuang-chao given below in Figs. 6–9 are closer to our calculations than the results of Akiba and Sato[2] and Jacob and Matthews[3]. Possibly the approximate agreement between the results of Lapidus and Chou Kuang-chao and our results is a consequence of the circumstance that in the amplitudes which determine the quantity $\sigma$ in the subthreshold region ($k_L \lesssim 150$ MeV) the most important role is played by the imaginary and Born parts of the amplitudes. We note that in Fig. 11 (below) is illustrated the remark of Lapidus and Chou Kuang-chao[4] concerning the results of Minami[7] who obtained curves for $\sigma(k_L, 90^\circ_{CM})$ (curves i, ii, iii) by neglecting Re $\Phi_k$ (where $\Phi_k$ are the amplitudes in the c.m.s.[8]), i.e., he made the assumption that for 300–800 MeV the $\gamma$, p scattering is a pure diffraction process (Re $\Phi_k \ll$ Im $\Phi_k$). This is indeed incorrect (see Tables I and II below).

In our opinion the more natural way to obtain dispersion relations in a specific coordinate system is to find relations for the covariant parameters of the process, $\Omega_i(\nu, \nu_1)$, to determine formulae which connect these with the amplitudes in the desired coordinate system (e.g., with the amplitudes in the c.m.s.), and with them establish equations for $\Phi_k(\omega, x)$. This method has been first proposed in the well known paper by Chew, Low, Goldberger, and Nambu.[9]

We recall that

$$\nu = \tfrac{1}{2} PK = k(p + p'), \quad \nu_1 = \tfrac{1}{2} K^2 = kk', \quad p^2 = p'^2 = 1;$$

$$\nu = \omega^2 - 1 - \nu_1 \qquad \nu_1 = \delta^2 (1 - x) \quad \text{for } k^2 = k'^2 = 0. \quad (1)$$

Here $P = p + p'$, $K = k + k'$; k and k′ are the momentum 4-vectors of the photons before and after the scattering; $\omega = \epsilon + \delta$ is the total energy of the photon-nucleon system in the c.m.s.; $x = \mathbf{n} \cdot \mathbf{n}'$ is the cosine of the scattering angle in the c.m.s.; $\hbar = c = M = 1$.

The dispersion relations for $\Phi_k$ obtained in [8] allow naturally to take the nucleon recoil into account in some degree of approximation. In the present paper we shall take into account the recoil in the Born terms to order $\delta^2$ and in the integrands to order $\delta$. We note that one has to take the recoil into account because of two reasons:

1) it will lead in the amplitudes to terms quadratic in $x = \cos\theta$, and in the cross sections to terms $x^l$, $l > 3$ (see below);

2) it provides for "mixing" of the different $\Phi_k$ in the dispersion relations. This can lead to the situation in which the "corrections" (terms of the order $\delta$) to the "basic" integral terms which arise in the "static" limit (see [8]) will be larger than the "basic" terms themselves at experimentally investigated energies ($k_L \approx 300$ MeV; see the discussion in Sec. 2).

## 1. APPLICATION OF THE UNITARITY CONDITION

We have previously[8] obtained formulae which express Im $\Phi_i$ as bilinear combinations of the photoproduction amplitudes:

$$\sum_{i=1}^{6} \operatorname{Im} \Phi_i n_i = \frac{\lambda}{(4\pi)^2} \sum_{\rho} \int do_m \sum_{e,n} F_e^* (\mathbf{n}'\mathbf{m}) F_m (\mathbf{n}\mathbf{m}) f_e' f_n . \quad (2)$$

Here $n_i$ are the six spin structures of the Compton effect[8]; $f_e'$ and $f_n$ are the four spin structures of the photoproduction[11] which contain in addition to the nucleon spin vector $\boldsymbol{\sigma}$ and the meson momentum unit vector $\mathbf{m}$ the polarization vectors $\mathbf{e}$ and $\mathbf{e}'$ and the photon momentum unit vectors $\mathbf{n}$ and $\mathbf{n}'$, where the unprimed and the primed quantities refer to before and after scattering respectively ($f_e'$ contains $\mathbf{e}'$ and $\mathbf{n}'$; $f_n$ contains $\mathbf{e}$ and $\mathbf{n}$);

$$\lambda = 4\pi\delta r = 4\pi\delta \left\{ \left[ 1 - \frac{\mu(2+\mu)}{\nu + \nu_1} \right] \left[ 1 + \frac{\mu(2-\mu)}{\nu + \nu_1} \right] \right\}^{1/2}. \quad (3)$$

The summation in (2) is carried over the isospin index of the photoproduction process $\gamma + p \to p + \pi^0$, $\gamma + p \to n + \pi^+$. Multiplying (2) by 1, $\boldsymbol{\sigma} \cdot \mathbf{n}$ and $\boldsymbol{\sigma} \cdot \mathbf{n}' \times \mathbf{n}$ and taking the sum of the diagonal elements we obtain the formulae which connect Im $\Phi_i$ with $F_{ik} = F_i^*(\mathbf{n}'\mathbf{m}) F_k(\mathbf{n}\mathbf{m})$.[10]

If one restricts oneself to the photoproduction in the S and P states and performs the sum over $\rho$ by means of the Fermi amplitudes X, Y, K (see [6a] where expressions of X, Y, K are given in terms of the photoproduction matrix elements) then one obtains the following expressions for $A_i = \operatorname{Im} \Phi_i$ in terms of the coefficients of the angular distributions of the indicated two photoproduction processes, $A^0$, $A^+$, $C^0$, $C^+$ ($r_0 = e_0^2/Mc^2 \approx 1.53 \times 10^{-16}$ cm):

$$A_1 = \delta r \left\{ \frac{3A^0}{4} + \frac{C^0}{12} - \frac{2C^+}{3} + \left[ \frac{3(A^0 + C^0)(9A^0 + C^0 - 8C^+)}{16} \right]^{1/2} \right\} x = r_0 A_1^1 x,$$

$$A_2 = - r_0 A_1^1,$$

$$A_3 = \delta r \left\{ \frac{3A^0}{4} + \frac{C^0}{12} - \frac{2C^+}{3} - \left[ \frac{3(A^0 + C^0)(9A^0 + C^0 - 8C^+)}{16} \right]^{1/2} \right\} = r_0 A_3^0,$$

$$A_4 = 0,$$

$$A_5 = \delta r\left\{A^+ + C^+ - \frac{A^0+C^0}{2} + x\,\frac{9A^0+13C^0+4C^+}{12}\right.$$
$$= r_0(A_5^0 + xA_5^1),$$
$$A_6 = \delta r\,\frac{4C^+-9A^0-5C^0}{12} = r_0A_6^0. \qquad (4)$$

We note:

$$A_2^0 = -A_1^1, \qquad A_1^1 + A_3^0 + A_5^1 + 3A_6^0 = 0. \qquad (5)$$

We shall use these relations when deriving the approximate equations including recoil in Sec. 2. We also note that for forward scattering there holds

$$\operatorname{Im} f_1 = \operatorname{Im}(\Phi_1+\Phi_3+\Phi_5+\Phi_6)|_{\nu_1=0}$$
$$= \delta r(A^+ + \tfrac{1}{3}C^+ + A^0 + \tfrac{1}{3}C^0),$$
$$\operatorname{Im} f_2 = -\operatorname{Im}(\Phi_5+\Phi_6)|_{\nu_1=0}$$
$$= \delta r[-A^+ - \tfrac{5}{3}C^+ + \tfrac{1}{2}A^0 + \tfrac{1}{6}C^0] \qquad (6)$$

which is the Cini-Stoffelini result (see [1]).

If one takes into account D-state photoproduction ($l = 2$) then there have to appear in (4) coefficients with a $\cos^4\theta$ dependence in the photoproduction angular distribution $E^0, E^+$. The analysis of the photoproduction which has been performed by many authors (see [6,12]) shows that $E^0, E^+$ can be neglected in the region $k_L \lesssim 500$ MeV and play some role but are small and known only with large uncertainties for $500 \text{ MeV} \le k_L \lesssim 800$ MeV and are negligible for $800 \text{ MeV} \lesssim k_L \lesssim 1000$ MeV.

One may hope that the relations (4) are applicable in the region $k_L \lesssim 1000$ MeV. This is, however, only an assumption. We note that for $k_L \gtrsim 600$ MeV inelastic processes of double photoproduction begin to appear: $\gamma + p \to p + \pi^+ + \pi^-$, $\gamma + p \to p + \pi^0 + \pi^0$. Thus Minami remarks[7] that for $k_L \gtrsim 600$ MeV the cross sections for these processes are of the same order as those for the photoproduction.

Thus, in utilizing unitarity[10] to improve on the accuracy of the quantities $A_k$, one has to augment (4) both with higher angular momenta ($l \gtrsim 2$) (this can be easily accomplished; it is however connected with involved computations) and with the double photoproduction process. We assume that one can use (4) in computing the $A_k$, and we shall calculate this way the amplitudes of the Compton effect, $\Phi_j$. The quantities $A_j = \operatorname{Im}\Phi_j$ are given in Table I.

Table I has been compiled on the basis of the data in the literature[6,12]. Hereby beginning at $k_L = 940$ MeV, $A^0, A^+, C^0, C^+$ are extrapolated to the constants $A_k$ ($k_L = 940$ MeV, $\delta \approx 0.579$). The computation thus contains the contributions from the second photoproduction resonance.

## 2. APPROXIMATE EQUATIONS CONTAINING RECOIL. EVALUATION OF THE DISPERSION INTEGRALS

In the relations (12) for $\Phi_j(\omega, x)$ in [8] we change from integrating over $\omega$ to integrating over $\delta = (\omega^2-1)/2\omega$; we expand the integrands in $\delta, y$ ($y$ is the integration variable) and retain in the integrands the terms linear in $\delta, y$ and in the Born terms the terms quadratic in $\delta$. Then the terms $S_i = \operatorname{Re}\Phi_i$ can be written in the form:

$$S_i = d_i + D_i, \qquad D_i = L_i + Q_i. \qquad (7)$$

Here $d_i$ are the Born terms of the amplitudes [see (22) below], $L_i$ are given in [8].

It is convenient for the evaluation of the integrals to introduce the function

$$g_k(y) = A_k(y)/\pi y r_0. \qquad (8)$$

By means of (8) one can write $L_i$ as

$$L_i = r_0\sum_m L_i^m x^m,$$
$$L_1^0 = \int_{\delta_0}^{\infty}\left(\frac{2\delta^2}{y(y+\delta)}g_5^0 - \frac{2\delta^3}{y^3}g_5^1 - \frac{4\delta^2}{y^2}g_6\right)dy,$$
$$L_1^1 = \int_{\delta_0}^{\infty}\left(\frac{2\delta^3}{y(y^2-\delta^2)}g_1 + \frac{2\delta^3}{y^3}g_5^1 + \frac{4\delta^3}{y^2(y+\delta)}g_6\right)dy,$$
$$L_2^0 = \int_{\delta_0}^{\infty}\left(-\frac{2\delta^4}{y^2(y^2-\delta^2)}g_1 + \frac{2\delta^3}{y^2(y+\delta)}g_6\right)dy,$$
$$L_3^0 = \int_{\delta_0}^{\infty}\frac{2\delta^3 g_3}{y(y^2-\delta^2)}dy, \qquad L_5^0 = \int_{\delta_0}^{\infty}\left(\frac{2\delta^3}{y(y^2-\delta^2)}g_5^0 + \frac{2\delta^3}{y^3}g_5^1\right)dy,$$
$$L_5^1 = \int_{\delta_0}^{\infty}\frac{2\delta^5 g_5^1}{y^3(y^2-\delta^2)}dy, \qquad L_6^0 = \int_{\delta_0}^{\infty}\frac{2\delta^3 g_6}{y(y^2-\delta^2)}dy;$$
$$L_i = r_0(L_i^0 + xL_i^1), \qquad i = 1, 5;$$
$$L_j = r_0L_j^0, \quad j = 2, 3, 6; \qquad L_4 = 0 \qquad (9)$$

(the integrals are principal value integrals). We now extract $x = \cos\theta$ from under the integral signs (see [8]) by recalling that the relations for $\Omega_i(\nu, \nu_1)$ and $\Phi_i(x, \omega)$ have been obtained for fixed $\nu_1$. This way we have

$$\int dy\, f(y)\cos\theta' = \int dy f(y)\left[1 - \frac{\nu_1}{y^2}\right]$$
$$= \int dy f(y)\left[1 - \frac{\delta^2(1-x)}{y^2}\right]. \qquad (10)$$

We see that the integrals of the "static type," $L_j^k$, give only a linear dependence on x. Considering (4) and the additional factors x which appear in connection with the structures $n_i$ (see Sec. 3) we find that no higher than the fourth power of x will appear in the cross sections. $L_j^k$ are relatively

**Table I.** Imaginary parts of the amplitudes and phases of the process

| δ | $A_1^1$ | $A_2^0$ | $A_3^0$ | $A_5^0$ | $A_5^1$ | $A_6^0$ | $a_1^{e1}$ | $a_3^{m1}$ | $a_3^{e2}$ | $a_3^{me}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.18 | 0.25 | −0.25 | 0,045 | 0,21 | −0.025 | −0.09 | 0.42 | 0,17 | 0,01 | 0.02 |
| 0.22 | 1.4 | −1.4 | 0.12 | 0.39 | −0,08 | −0,48 | 0.78 | 0.94 | 0,02 | 0,058 |
| 0.26 | 3.21 | −3,21 | 0 | 0.425 | −0,09 | −1.1 | 0.95 | 2.159 | 0.045 | −0,026 |
| 0.3 | 2,1 | −2,1 | 0,16 | 0.56 | −0,34 | −0,64 | 1,12 | 1.37 | −0,09 | 0,145 |
| 0.34 | 1.02 | −1,02 | 0 | 0,52 | 0.03 | −0,35 | 1.04 | 0,685 | 0.015 | −0,009 |
| 0,38 | 0,54 | −0,54 | −0,13 | 0,15 | 0,16 | −0,19 | 1 | 0,325 | 0,015 | −0,084 |
| 0,42 | 0,1 | −0,1 | 0,08 | 1,05 | 0,12 | −0.1 | 2,1 | 0,1 | 0,1 | −0,012 |
| 0.46 | 0.1 | −0,1 | 0,13 | 1,45 | −0,11 | −0.04 | 2.9 | 0,07 | 0.01 | 0,073 |
| 0.5 | 0.28 | −0.28 | 0.33 | 0.81 | −0.25 | −0.12 | 1,62 | 0.166 | 0.04 | 0,168 |

simple and in essence connect Re $\Phi_k$ with Im $\Phi_k$. The "mixing" of the other amplitudes in the dispersion relations is unimportant.

The expressions for $Q_j$ are exceedingly involved; to write them it is useful to introduce the following integrals linear in $g_k(y)$:

$$J_i(g_k) = \int_{\delta_0}^{\infty} \frac{\delta^{2+i}}{y^{1+i}} g_k dy, \qquad i = 1, 2, 3;$$

$$J_{3+i}(g_k) = \int_{\delta_0}^{\infty} \frac{\delta^{2+i}}{y^i (y + \delta)} g_k dy, \quad i = 1, 2, 3, 4;$$

$$J_{8+i}(g_k) = \int_{\delta_0}^{\infty} \frac{\delta^{3+i}}{y^i (y+\delta)^2} g_k dy, \quad i = 0, 1, 2, 3, 4. \qquad (11)$$

The argument in $J_j(g_k)$ indicates which of the $g_k$ to insert in the particular integral.

With (11), (10), and (5) the terms linear in δ in the dispersion relations obtained in [8] can be written after simple but involved algebra in the form

$$Q_j = r_0 \sum_k Q_j^k x^k,$$

$$Q_1^0 = \left[J_4 - \frac{J_5 + J_6}{2}\right](g_1) + [J_4 + 2J_5 - J_6](g_6) + [J_8 + J_{10}](g_5^0)\,[J_4 - J_7](g_5^1),$$

$$Q_1^1 = \left[-J_4 + J_5 + \frac{1}{2} J_9 + J_{10} + J_{11}\right](g_1) + [-4J_4 + 4J_6 + 4J_{11}](g_6)\,[2J_8 - J_9 - J_{10}](g_5^0) + [-J_2 + 2J_3](g_5^1),$$

$$Q_1^2 = \left[-J_3 + J_9 - \frac{1}{2} J_{11}\right](g_1) + [-2J_2 + 4J_9](g_6) + [J_5 - J_6 - J_7)(g_5^1),$$

$$Q_2^0 = [-2J_{10} - J_{11}](g_1) + [3J_4 - 7J_{10} - 5J_{11}](g_6) - 3J_4(g_5^0) + [-J_1 + J_2](g_5^1),$$

$$Q_2^1 = [-2J_5 + J_{11}](g_1) + [-5J_9 - 2J_{10} + 5J_{11}](g_6) + [2J_5 + J_6](g_5^1),$$

$$Q_3^0 = J_9(g_3) + J_4(g_5^0) + J_1(g_6),$$

$$Q_3^1 = J_9(g_3) + J_6(g_5^1) - J_5(g_6),$$

$$Q_4^0 = J_5(g_6) - 2J_5(g_3),$$

$$Q_5^0 = -J_4(g_6) - [J_8 + J_9 + J_{10}](g_5^0) + [-J_4 + J_7](g_5^1),$$

$$Q_5^1 = [2J_9 + J_{10}](g_5^0) + [2J_9 - 3J_{11} - 2J_{12}](g_5^1),$$

$$Q_5^2 = [2J_{11} + J_{12}](g_5^1),$$

$$Q_6^0 = [-J_9 - J_{11}](g_6) - J_4(g_5^0) + [J_2 - J_1](g_5^1),$$

$$Q_6^1 = [2J_9 + J_{10}](g_6) - J_6(g_5^1). \qquad (12)$$

We remark that Eqs. (12) illustrate the importance of keeping the "recoil": $A_1^1$, $A_2^0$, $A_5^0$, $A_6^0$ are big (see Table I) and their contribution to $Q_j^k$ can exceed the small contributions from $A_3^0$ and $A_5^1$; the "mixing" of the $A_k$ becomes important when taking into account the recoil [see (12) and (9)]. Besides, the $Q_j^k$ lead to a quadratic dependence on x. Together with the various interference terms from the $S_i$, (7), appearing in the expression for the cross section [see (26)], this will give the cross section $\sigma(x, k_L)$ which will be discussed below (Sec. 3).

The integrals (9) and (12) were evaluated with mean values[1)] of $A_k$ in the interval 150 MeV ≤ $k_L$ ≤ 940 MeV (0.139 ≤ y ≤ 0.579) by numerical integration for 12 values of the parameter δ (see Table II) corresponding to the interval 60 MeV ≲ $k_L$ ≲ 760 MeV. Corrections from the interval y = 0.579 – ∞ were evaluated by extrapolating all $g_k$ to constants from their values at y = 0.579; they are small: the biggest correction for $k_L$ ~ 760 MeV is of the order of or smaller than 0.3 $r_0$. The influence of the recoil is particulary important for $\Phi_k$ with k = 3, 4, 5, 6. It is also rather large for $Q_1^2$ and $Q_2^1$ and enters entirely

1)The accuracy of the results given in the tables is determined only by the accuracy of the computations.

**Table II.** Dispersion integral contributions to the amplitudes

| $\delta$ | $D_1^0$ | $D_1^1$ | $D_1^2$ | $D_2^0$ | $D_2^1$ | $D_3^0$ | $D_3^1$ | $D_4^0$ | $D_5^0$ | $D_5^1$ | $D_5^2$ | $D_6^0$ | $D_6^1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.06 | 0,054 | 0,009 | 0 | −0,008 | 0 | 0 | 0 | 0 | 0,006 | 0 | 0 | −0,004 | 0 |
| 0.1 | 0,125 | 0,027 | 0.001 | −0,046 | −0.006 | 0,004 | 0 | −0,001 | 0.023 | 0 | 0,001 | −0,022 | 0 |
| 0.14 | 0.237 | 0.175 | 0.013 | −0.15 | −0.024 | 0.015 | 0.001 | −0,005 | 0,134 | −0,012 | 0,003 | −0.096 | 0.002 |
| 0.18 | 0.385 | 0.426 | 0.024 | −0,527 | −0.045 | 0.047 | 0,004 | −0,013 | 0,118 | −0.033 | 0,006 | −0,273 | 0,004 |
| 0,22 | 0,556 | 1.141 | 0.04 | −1,002 | −0,09 | −0.06 | 0,008 | −0,024 | 0,067 | 0,05 | 0.01 | −0,287 | 0,001 |
| 0.26 | 0,736 | 0.398 | 0,076 | 0,284 | −0.159 | −0,303 | 0,015 | −0.043 | −0,075 | 0.052 | 0,019 | −0,863 | 0.004 |
| 0.3 | 0,957 | −2.415 | 0.116 | 3,273 | −0.247 | −0,026 | 0,026 | −0,069 | −0,213 | 0.192 | 0,022 | 0.748 | 0 |
| 0.34 | 1,185 | −3,056 | 0,171 | 3.288 | −0,39 | −0,162 | 0,04 | −0,102 | −0,158 | 0,435 | 0.032 | 0,759 | −0,008 |
| 0.38 | 1.535 | −3,398 | 0.316 | 3,168 | −0,561 | −0,094 | 0.061 | −0,147 | 0.133 | 0,261 | 0.041 | 0,85 | −0.013 |
| 0.42 | 1,589 | −3,637 | 0,318 | 3.21 | −0.833 | −0,052 | 0.094 | −0,207 | 0,311 | 0.020 | 0,053 | 0,879 | −0.034 |
| 0.46 | 1.724 | −3,502 | 0,401 | 3,005 | −1,051 | 0,062 | 0,113 | −0,259 | 0,272 | 0.255 | 0,07 | 0,861 | −0.037 |
| 0,5 | 2.084 | −3.644 | 0.583 | 3,076 | −1,398 | 0,099 | 0.164 | −0.358 | −0.079 | 0,306 | 0,085 | 0.677 | −0.06 |

by way of the dependence of $D_1$ on $x^2$ and $D_2$ on x. (see $D_1^2$ and $D_2^1$ in Table II). The contributions of the dispersion integral to the amplitudes $D_j^k = L_j^k + Q_j^k$, evaluated by means of (9) and (12), are given in Table II.

A comparison of $D_j^k = L_j^k + Q_j^k$ with the corresponding Born term contributions to the amplitudes [8]

$$d_j = r_0 d_j^k x^k,$$
$$d_1^0 = -[(1-\delta+\tfrac{1}{4}\delta^2)\tau_p + (\tfrac{1}{2}\delta - \tfrac{5}{4}\delta^2)\lambda\tau_p - \tfrac{1}{2}\lambda^2\delta^2],$$
$$d_1^1 = \tfrac{1}{2}(1-\delta)\delta(\tau_p+\lambda)^2 + \tfrac{1}{4}\delta^2\lambda(\lambda+\tau_p),$$
$$d_1^2 = \tfrac{1}{2}\delta^2(\lambda+\tau_p)^2,$$
$$d_2^0 = \tfrac{1}{2}(1-\delta)\delta(\tau_p - 2\lambda\tau_p - \lambda^2) + \tfrac{1}{4}\lambda(\lambda+\tau_p)\delta^2,$$
$$d_2^1 = \tfrac{1}{2}\delta^2(-\tau_p + 2\lambda\tau_p + \lambda^2), \quad d_3^0 = -\tfrac{1}{4}\delta^2(\tau_p + 3\lambda\tau_p),$$
$$d_3^1 = -\tfrac{1}{2}(1-\delta)\delta(\tau_p+\lambda\tau_p) + \tfrac{1}{4}\delta^2\lambda(\tau_p+\lambda),$$
$$d_3^2 = \tfrac{1}{2}\delta^2(\tau_p+\lambda\tau_p), \qquad d_4^0 = -d_3^1, \qquad d_4^1 = -d_3^2,$$
$$d_5^0 = \tfrac{1}{2}(1-\delta)\delta\lambda\tau_p - \tfrac{1}{4}\delta^2(\tau + \lambda\tau_p + \lambda^2), \quad d_5^1 = \tfrac{1}{2}d_3^2,$$
$$d_6^0 = -\tfrac{1}{2}(1-\delta)\delta(\tau_p+\lambda)^2 + \tfrac{1}{4}\delta^2(\tau_p + \lambda\tau_p - \lambda^2),$$
$$d_6^1 = \tfrac{1}{2}\delta(1-\delta/2)(\tau_p+\lambda\tau_p), \qquad d_6^2 = -d_3^2,$$
$$\lambda = \lambda_p\tau_p + \lambda_n\tau_n = 1{,}79\,(1+\tau_3)/2 - 1{,}91\,(1-\tau_3)/2 \quad (13)$$

shows that the contribution of $D_j^k$ to the real part of the amplitudes is already important for $k_L \gtrsim 100$ MeV:

$$D_1^k \sim d_1^k, \; D_2^k \sim d_2^k, \; D_4^0 \sim d_4^0, \quad D_6^0 \sim d_6^0, \quad D_5^k > d_5^k. \, (14)$$

Various "interferences" occur between the $D_j^k$ and $d_j^k$; evidently they cannot be obtained from any phenomenological considerations (which are mentioned, e.g., in the introduction of [1]).

## 3. DIVERSE CHARACTERISTICS OF THE PROCESS $\gamma + p \rightarrow \gamma' + p'$

Using the values of

$$\mathrm{Im}\,\Phi_k = A_k, \;\; \mathrm{Re}\,\Phi_k = S_k = d_k + D_k, \;\; \Phi_k = S_k + iA_k$$

[see Tables I and II and Eq. (13)] we evaluate the real and imaginary parts,

$$S(\omega, J, j, \Pi_j, j', \Pi_{j'}) = r_0(\delta_{2J}^{\Pi_j j} + i a_{2J}^{\Pi_j j})$$

the "phases" of the Compton effect.[1,8] In complete analogy to the method of the calculation of the "phases" in [1] we use the formulae of the "generalized phase analysis" from [8]. All $S(\omega, J, \ldots)$ with $j > 3$ will appear "singly" for $x^l (l > 2)$ and since $A_k$ and $S_k$ contain $x^n (n \leq 2)$ they turn out to vanish in our approximation. For the "phases" with j = 1, 2, 3 we obtain the system of equations

$$\sum_{J_j \Pi_j} C_m^n (J_j \Pi_j j' \Pi_{j'}) S(\omega J \ldots) = r_0 \Phi_m^n = r_0 [S_m^n + iA_m^n],$$
$$m = 1, 2, \ldots, 6, \quad n = 0, 1, 2, \qquad (15)$$

where $C_m^n(J \ldots)$ are numerical coefficients, and $S_m^n = d_m^n + D_m^n$ and $A_m^n$ are given by Tables I, II and by (13). The system can be easily solved and we obtain the $\delta_{2J}^{\Pi jj}$, $a_{2J}^{\Pi jj}$ given in Tables I and III.

In connection with the fact that in the S and P wave approximation of the photoproduction the $A_k$ are expressed in terms of the four coefficients $A^0$, $C^0$, $A^+$, $C^+$ only four phases have $a_{2J}^{\Pi jj} \neq 0$ (see Table I). Hereby $a_1^{e1}$ and $a_3^{m1}$ are large (see Fig. 1) and for $k_L < 150$ MeV their behavior qualitatively follows the behavior of $a_1^{e1}$, $a_3^{m1}$ of the dipole model (see Fig. 1 of [1] and Fig. 1 of this paper).

The real parts of the dipole phases $\delta_1^{e1}$, $\delta_1^{m1}$, $\delta_3^{e1}$, $\delta_3^{m1}$ dominate according to the above (see Table III and Fig. 1). However, the contribution of the other "phases" to $\Phi_j$ turns out to be considerable. This explains the rather strong difference of $\delta_1^{e1}$, $\delta_1^{m1}$, $\delta_3^{e1}$, $\delta_3^{m1}$ from the corresponding quantities of the dipole model [1] for $k_L \gtrsim 150$ MeV. This is due both to the consideration of all six dispersion relations and to the inclusion of the recoil

**Table III.** Real parts of the phases of the Compton effect on the proton

| $k_L$ | $\delta_1^{e1}$ | $\delta_1^{m1}$ | $\delta_3^{e1}$ | $\delta_3^{m1}$ | $\delta_3^{e2}$ | $\delta_3^{m2}$ | $\delta_3^{em}$ | $\delta_3^{me}$ | $\delta_5^{e2}$ | $\delta_5^{m2}$ | $\delta_5^{e3}$ | $\delta_5^{em}$ | $\delta_5^{me}$ | $\delta_7^{e3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 60.7 | −0.523 | −0.309 | −0.615 | 0.132 | 0,011 | 0,002 | −0,044 | −0,002 | 0,012 | 0,002 | 0,002 | 0.002 | 0 | 0.002 |
| 103.5 | −0.373 | −0,511 | −0.545 | 0.235 | 0.014 | 0.008 | −0,071 | −0,003 | 0,015 | 0,003 | 0,007 | 0,003 | 0 | 0.005 |
| 151 | 0,001 | −0.936 | −0,429 | 0.394 | 0.028 | 0.023 | −0.09 | −0,001 | 0.031 | 0,008 | 0,01 | 0,005 | −0.001 | 0.01 |
| 201.5 | 0.177 | −1,059 | −0,281 | 0,68 | 0.013 | 0,042 | −0.106 | 0.009 | 0,012 | 0,016 | 0,014 | 0.008 | −0.001 | 0,015 |
| 259 | 0.233 | −0.816 | −0,096 | 1.133 | 0,05 | 0.064 | −0.119 | −0,052 | 0,066 | 0,027 | 0,021 | 0,013 | −0.002 | 0,021 |
| 316 | 0.077 | −1.833 | 0.115 | 1.474 | 0,041 | 0.103 | −0.124 | −0.131 | 0,181 | 0,044 | 0.027 | 0.018 | −0,004 | 0,028 |
| 378 | 0.031 | −1,375 | 0,36 | −1,348 | 0,285 | 0.151 | −0.125 | −0.1 | 0.221 | 0.061 | 0,032 | 0,025 | −0,005 | 0,036 |
| 445 | 0.542 | −1.733 | 0,632 | −1,496 | 0,213 | 0.19 | −0.116 | −0.221 | 0,101 | 0,088 | 0.037 | 0,031 | −0.008 | 0.042 |
| 517 | 1.617 | −1,772 | 1.01 | −1,532 | 0,067 | 0.257 | −0.108 | −0.163 | 0,013 | 0.126 | 0,046 | 0.037 | −0.012 | 0,054 |
| 592 | 2.207 | −2.207 | 1,196 | −1.636 | −0.076 | 0.324 | −0,081 | −0.096 | −0,022 | 0,156 | 0,039 | 0.046 | −0,014 | 0.05 |
| 677 | 2.304 | −2.01 | 1,452 | −1.431 | 0,081 | 0.402 | −0.059 | −0.145 | −0,032 | 0,195 | 0,042 | 0,056 | −0,018 | 0,056 |
| 758 | 2.187 | −2.483 | 1,891 | −1.46 | 0.1 | 0.511 | −0.021 | −0.167 | −0,041 | 0.253 | 0,046 | 0.062 | −0.024 | 0.064 |

terms $Q_j$ in them. The quantities $\delta_3^{e1}$ and $\delta_1^{e1}$ differ particularly strongly. At $k_L \sim 100$ MeV the contribution of $\delta_3^{e1}$ to $S_1^0 = -0.829$ is $-0.817$, i.e., it is overwhelming compared to the remaining "phases". On the other hand at $k_L \sim 260$ MeV the contribution of $\delta_3^{e1}$ equals $-0.144$ compared to $-0.104$ from the other phases; at $k_L \sim 320$ MeV $\delta_3^{e1}$ contributes 0.172 and the other phases $-0.164$. An approximately similar situation holds for $\delta_1^{m1}$, $\delta_1^{e1}$, $\delta_3^{m1}$.

The change of the amplitudes $\Phi$ due to the contribution of the dispersion integrals for $k_L \sim 60$ MeV can be given in the form

$$\Delta\Phi = \hat{\Phi} - \hat{\Phi}^B = (D_1 + xD_2 + x^2D_4 + D_5)\,\mathbf{ee'} + (D_6 - xD_4 - D_2)\,[\mathbf{e'n'}]\,[\mathbf{en}] + D_5\, i\sigma\,[\mathbf{e'e}] + D_6\, i\sigma\,[[\mathbf{e'n'}]\,[\mathbf{en}]]\; r_0\,\{0.06\,\mathbf{ee'} + 0.004\,[\mathbf{e'n'}]\,[\mathbf{en}] + 0.006\, i\sigma\,[\mathbf{e'e}] - 0.004\, i\sigma\,[[\mathbf{e'n'}]\,[\mathbf{en}]]\}. \quad (16)^*$$

Thus the change of the amplitude of the term $\mathbf{e}\cdot\mathbf{e'}$ is relatively large, namely $0.06\,r_0$. The connection of the coefficients of $\mathbf{e}\cdot\mathbf{e'}$ and $[\mathbf{e'}\times\mathbf{n'}]\cdot[\mathbf{e}\times\mathbf{n}]$ with the electric and magnetic polarizability of the proton, $\alpha$ and $\beta$,[1] yields the following estimates:

$$\alpha \approx 104.31\cdot 10^{-44}\,\mathrm{cm}^3, \qquad \beta \approx 6.95\cdot 10^{-44}\,\mathrm{cm}^3 \quad (17)$$

Remembering that $\alpha$ is the sum of the actual polarizability $\alpha_0$ and of $\frac{1}{3} r_0 \langle r_e^2 \rangle \approx 32.6 \times 10^{-44}$ cm$^3$ (see [1]) one has

$$\alpha_0 \approx 71.71\cdot 10^{-44}\,\mathrm{cm}^3 \quad (18)$$

This estimate agrees well with the experimental data of Gol'danskiĭ and collaborators (for references see [1]):

$$\alpha_e \approx (90 \pm 20)\cdot 10^{-44}\ \mathrm{cm}^3$$
$$\beta_e \approx (20 \pm 20)\cdot 10^{-44}\,\mathrm{cm}^3, \quad (19)$$

*[en] = e × n.

Compared with the dipole model[1] we find a change of 9.5% for $\alpha$ and of 530% for $\beta$; these changes lead to an improvement with the experimental data (19). Similarly as in [1] $\beta$ has changed rather considerably even though like previously still holds $\beta \ll \alpha$ ($\alpha/\beta \approx 15$). One may scarcely expect a considerable improvement in the above estimate for $\alpha$ and $\beta$.

In addition to the improvement in the determination of $\alpha$ and $\beta$ we find that at $k_L \sim 60$ two new

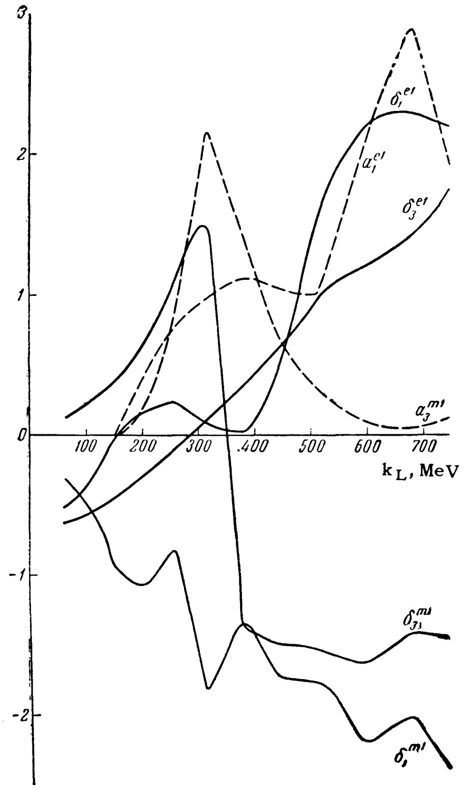


FIG. 1. Dipole "phases" of the Compton effect, $r_0^{-1} S_k (J_j \Pi_j j' \Pi_{j'}) = \delta^{\Pi_j j}_{2J} + i\alpha^{\Pi_j j}_{2J}$. Full lines show $\delta_{2J}^{\Pi_j j}$, dashed lines show $\alpha_{2J}^{\Pi_j j}$.

constants appear with the structures $i\sigma\cdot[\mathbf{e}'\times\mathbf{e}]$ and $i\sigma\cdot[\mathbf{e}'\times\mathbf{n}']\times[\mathbf{e}\times\mathbf{n}]$:

$$\alpha_1 \approx 0.006\ r_0 \qquad (\sim 10.43\cdot 10^{-44}\ \mathrm{cm}^3),$$
$$\beta_1 \approx -0.004\ r_0 \qquad (\sim -6.95\cdot 10^{-44}\mathrm{cm}^3). \qquad (20)$$

The possibility that new constants may appear was considered by V. S. Barashenkov and G. Kaiser (private communication) who found, however, that these constants equal zero.

The differential scattering cross section for unpolarized photons by unpolarized nucleons in the c.m.s. is obtained by the usual formula [13]

$$\frac{d\sigma}{do'} = \frac{1}{2s+1}\sum_{\mathbf{e},\mathbf{e}'} \mathrm{Sp}_{s's}\,(|\hat{\Phi}|^2)_{s's}; \qquad (21)$$

we sum over the photon polarizations $\mathbf{e}$ and $\mathbf{e}'$, $\Phi = \Phi_i n_i$ is the amplitude in the c.m.s. [8], $s = 1/2$. The averaging over the polarizations can be easily performed by means of the formula given in [14]:

$$\sum_\lambda e_r^{(\lambda)} e_s^{(\lambda)} = \delta_{rs} - n_r n_s. \qquad (22)$$

$\mathrm{Sp}_{s's}(|\hat{\Phi}|^2)_{s's}$ is calculated according to the usual rules (see, e.g., [14]).

After performing the indicated operations we obtain the following expression for the cross section:

$$\frac{d\sigma}{do'} = \frac{1+x^2}{2}\widetilde{\Phi}_1^2 + 2x\Phi_{56} + \frac{3-x^2}{2}(\Phi_5^2+\Phi_6^2) +$$
$$+ (1-x^2)\Big\{-x\widetilde{\Phi}_{12} + \frac{1-x^2}{2}\widetilde{\Phi}_2^2 + \frac{1+x^2}{2}\Phi_3^2$$
$$+ \frac{(1-x^2)^2}{2}\Phi_4^2\, x\,(1-x^2)\,\Phi_{34} + x\Phi_{35}$$
$$+ \Phi_{36} - (1-x^2)\,\Phi_{45}\Big\},$$
$$\Phi_i^2 = (S_i)^2 + (A_i)^2,$$
$$\Phi_{ij} = \Phi_i^*\Phi_j + \Phi_j^*\Phi_i = 2\,(S_iS_j + A_iA_j). \qquad (23)$$

In obtaining (23) it turned out to be convenient to redetermine $\Phi_{1,2}$ changing over to $\widetilde{\Phi}_{1,2}$:

$$\widetilde{\Phi}_1 = \Phi_1 + x\Phi_3 + \Phi_5 + x\Phi_6,$$
$$\widetilde{\Phi}_2 = \Phi_2 + x\Phi_4 - \Phi_6. \qquad (24)$$

Then $\hat{\Phi}$ can be written as

$$\hat{\Phi} = \widetilde{\Phi}_1 \mathbf{e}\mathbf{e}' + \widetilde{\Phi}_2\,(\mathbf{e}'\mathbf{n})\,(\mathbf{e}\mathbf{n}') + \Phi_3 i\sigma\,[\mathbf{n}'\mathbf{n}]\,(\mathbf{e}'\mathbf{e})$$
$$+ \Phi_4 i\sigma\,[\mathbf{n}'\mathbf{n}]\,(\mathbf{e}'\mathbf{n})\,(\mathbf{e}\mathbf{n}') + \Phi_5 i\sigma\,[\mathbf{e}'\mathbf{e}]$$
$$+ \Phi_6 i\sigma\,[[\mathbf{e}'\mathbf{n}']\,[\mathbf{e}\mathbf{n}]]. \qquad (25)$$

With the results obtained in Sec. 2 for $S_i = d_i + D_i$ and $A_i$ one obtains after heavy algebra the following expression for $\sigma(x, k_L) = d\sigma/do'$:

$$\sigma\,(x, k_L) = r_0^2 \sum_{i=0}^{7} \sigma_i(k_L)x^i. \qquad (26)$$

The quantities $\sigma_i(k_L)$ are given in Table IV.

A number of experimentally observable quan-

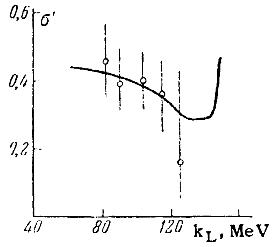


FIG. 2. Differential cross section $\sigma' \equiv \frac{1}{r_0^2(1+x_L^2)}\frac{d\sigma}{d\Omega}(k_L, \theta_L = 50°)$ for scattering angle 50° in the lab system as function of $k_L$. For the experimental points see [1].

tities of the process have been computed (cross sections at fixed angle $\sigma(\bar{x}, k_L)$, angular distributions $\sigma(x, \bar{k}_L)$, etc.) The results are given in Figs. 2–12; where possible, experimental data are also plotted (see [5,15] and also references and graphs in [1]). In the subthreshold region ($k_L \lesssim 150$ MeV) where a reasonable amount of data exists the agreement between theory and experiment is overall satisfactory. The agreement is considerably better than with the dipole approximation [this is particularly so for $\sigma(k_L, \theta_L \approx 50°_L)$, $\sigma(x_L, k_L \approx 60$ MeV) and $\sigma(k_L, \theta_L \approx 135°_L)$] (Figs. 2, 5, 6). The difference between the present results and the dipole model reaches 30% for energies $k_L \lesssim 140$ MeV (see Figs. 2–4 and the corresponding figures in [1]).

The evolution of the angular distributions in going from $k_L < 200$ MeV to $k_L > 200$ MeV is shown in Figs. 5–9. While at energies $k_L < 200$ MeV the behavior of the angular distributions is similar to that of the dipole approximation (Figs. 6, 7), at energies $k_L > 200$ MeV where the recoil is important (see Table IV) it is considerably changed. Figure 8 shows the angular distribution for $k_L \sim 245$ MeV. The agreement with experiment is not as good as in the region $k_L \sim 200$ MeV but it is considerably better than in the papers [2-4] discussed in the introduction.

At resonance energy ($k_L \sim 320$ MeV) the angular distribution is weakly asymmetrical (curve 2 in Fig. 9) and differs considerably from the corresponding Akiba and Sato curves 2′(c = 1) and 2″(c = 0). [2] As $k_L$ increases the cross section for scattering into the back hemisphere (c.m.s.) increases sharply; see curves 3–5, Fig. 9.

If one characterizes this asymmetry by the quantity

$$\mu\,(k_L) = \frac{\sigma\,(k_L, 180°) - \sigma\,(k_L, 0°)}{\sigma\,(k_L, 90°)}, \qquad (27)$$

we find for $\mu$ the following numbers:

$$\mu\,(320) \approx 0.15, \qquad \mu\,(378) \approx 0{,}72,$$
$$\mu\,(445) \approx 2.1, \qquad \mu\,(517) \approx 4{,}5. \qquad (28)$$

In Fig. 10 the curves $\sigma(k_L, \theta_L = 0°_L)$ and $\sigma(k_L, \theta_L = 180°_L)$ are shown. As previously said

**Table IV**

| δ | $k_L$ | $\sigma_0$ | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma_5$ | $\sigma_6$ | $\sigma_7$ |
|---|---|---|---|---|---|---|---|---|---|
| 0,06 | 60,7 | 0,472 | —0,018 | 0,349 | —0,056 | —0,025 | 0,005 | 0,005 | 0 |
| 0,1 | 103,5 | 0,485 | —0,13 | 0,188 | —0,056 | —0,039 | 0,002 | 0,002 | 0 |
| 0,12 | 128 | 0,529 | —0,357 | 0,13 | —0,05 | —0,057 | 0,006 | 0,003 | 0 |
| 0,13 | 140,5 | 0,623 | 0,477 | 0,008 | —0,072 | —0,040 | 0,012 | 0,003 | 0 |
| 0,14 | 151 | 0,85 | —0,455 | —0,029 | —0,097 | 0,021 | 0,014 | 0,008 | 0 |
| 0,18 | 201,5 | 1,313 | —0,757 | —0,009 | —0,035 | 0,137 | 0,004 | 0,02 | 0 |
| 0,212 | 245 | 2,338 | —0,684 | 0,106 | 0,383 | 0,362 | 0,015 | 0,033 | 0,001 |
| 0,22 | 259 | 2,753 | —0,613 | 0,536 | 0,045 | 0,071 | 0,47 | 0,037 | 0,001 |
| 0,26 | 316 | 9,826 | —0,251 | —1,007 | —0,631 | 0,282 | 0,224 | 0,086 | 0,002 |
| 0,3 | 378 | 5,209 | —1,094 | —1,047 | —1,575 | 1,08 | 0,52 | 0,094 | 0,003 |
| 0,34 | 445 | 4,972 | —3,294 | 1,104 | —2,063 | 0,98 | 0,35 | 0,143 | 0,004 |
| 0,38 | 517 | 4,87 | —7,114 | 5,108 | —3,91 | 1,86 | 0,104 | 0,211 | 0,008 |
| 0,42 | 592 | 6,938 | —9,827 | 9,076 | —4,81 | 2,54 | —1,186 | 0,2 | 0,014 |
| 0,46 | 677 | 8,757 | —9,919 | 8,128 | —6,045 | 2,88 | 0,218 | 0,244 | 0,021 |
| 0,5 | 758 | 7,263 | —16,201 | 10,839 | —0,187 | 3,86 | —2,366 | 0,3 | 0,114 |

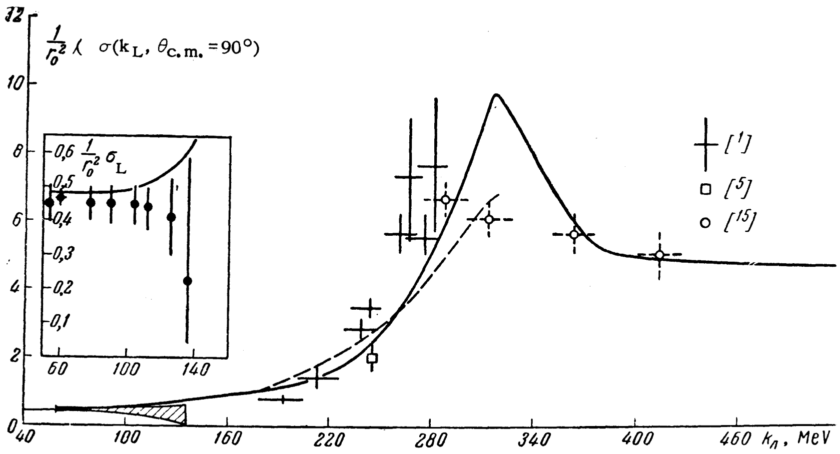


FIG. 3. Differential cross section for 90° c.m.s. In the insert the cross section for 90° lab system is shown (it corresponds to the cross-hatched region in the main drawing). The dashed line is taken from [2].

they show a nonmonotonic behavior. A minimum appears at $k_L \sim 140$ MeV (see the insert in Fig. 10) where we have $\sigma(140, 0°_L) \simeq 0.08\, r_0^2$ (see [1] and [4]).

We rote that in the range 300—350 MeV all characteristics of the process depend rather strangely on the energy (see, e.g., curves 1 and 2, Fig. 9). In this region our numerical estimates have to be considered to be qualitative only. De Wire et al [15] give one experimental point at $k_L \approx 725$—775 MeV and $\theta_{c.m.} = 60°$ as $\sigma_E(\theta_{c.m.}, k_L) = (1.28 \pm 1.28)\, r_0^2$. Table IV gives $\sigma(\theta_{c.m.} = 60°, k_L \sim 758) = 1.83\, r_0^2$.

The behavior of the cross section $\sigma(k_L, \theta = 90°)$ for $300 < k_L < 800$ MeV has been investigated by Minami [7] with rather crude assumptions (see above). The curves i, ii, iii of Fig. 11 show his results and give $\sigma(k_L, \theta = 90°)$ for different assumptions on the connection between the double photoproduction and the (½, ³⁄₂) resonance of the pion-pion system. Here

$$\text{a)}\ \sigma_{13}(2\pi) = 2\sigma(\pi^+\pi^-)\ (\text{curve } i),$$
$$\text{b)}\ \sigma_{13}(2\pi) = \sigma(\pi^+\pi^-)\ (\text{curve } ii),$$
$$\text{c)}\ \sigma_{13}(2\pi) = 0\ (\text{curve } iii). \quad (29)$$

Our results for $\sigma(k_L, x = 0)$ (see Fig. 11) agree qualitatively with the assumptions a) and b). It could be determined which of these cases applies by improving precision for the ratio of the heights of the first and the second resonance in $\sigma(x=0, k_L)$.

The threshold anomalies are illustrated in Figs. 1, 2, 4, 10 and they give as previously an unimportant contribution to the total cross section of the process (see Fig. 12). We note that using the results for the phases [Table III and Eq. (13)] one could improve on the estimates for the threshold anomalies given by Ustinova [16]. She used for the numerical work the imprecise results for the phases given by Gell-Mann, Goldberger, and Thirring (see the discussion in [1]).

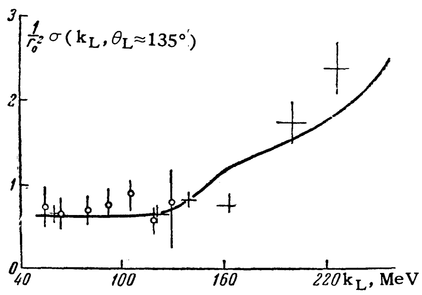


FIG. 4. Differential cross section for 135° lab system. For the experimental points see [1].

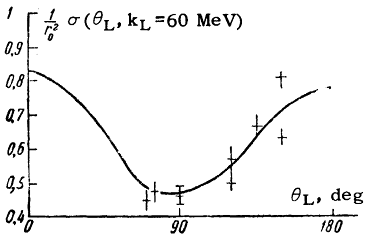


FIG. 5. Angular distributions for $k_L \approx 60$ MeV. For the experimental points see [1].

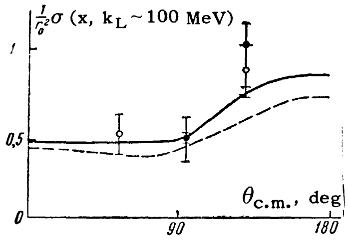


FIG. 6. Angular distributions for $k_L \sim 100$ MeV. Dashed curve results of Lapidus and Chou Kuang-chao.[4b] For the experimental points see[1].

In summary, one may draw the following conclusions:

1. The dispersion relations obtained in [8] allow an all-round description of the Compton scattering on nucleons and agree well with the available experimental data.

2. In the low energy region where the coefficients $A^0$, $C^0$, $A^+$, $C^+$ are known with good accuracy [6,12] the main problem appears to be the maximum improvement of the numerical data needed in the evaluation of the dispersion integrals, i.e., the establishment of an error corridor. Here naturally all six dispersion relations have to be used.

The same problem exists also in the resonance region 300 MeV $\lesssim k_L \lesssim$ 350 MeV. There, however, the recoil has to be taken into account rather accurately. With that are connected considerable computational difficulties.

3. For $k_L \gtrsim$ 400 MeV one may attempt to include in the unitarity condition the contributions

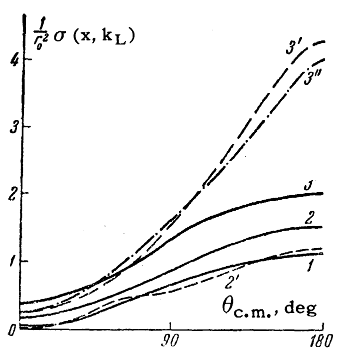


FIG. 7. Angular distributions for $k_L \sim 140$ MeV (curve 1), $k_L \sim 150$ MeV (curve 2) and $k_L \sim 200$ MeV (curve 3). The curves 3′ and 3″ are after Akiba and Sato[2] for c = 1 and c = 0 respectively. Curve 2′ is the result of Lapidus and Chou Kuang-chao[4b] for $k_L \sim 150$ MeV.

from the photoproduction states with $l \geq 2$, and also the contributions from the double photoproduction process.

4. The amplitudes $\Phi_i = S_i + iA_i$ obtained above allow the evaluation of polarization effects which have been discussed by Lapidus and Chou Kuang-chao [4].

5. An important and complicated problem seems to be the establishment of the double (Mandelstam) dispersion relations and their use in computing the

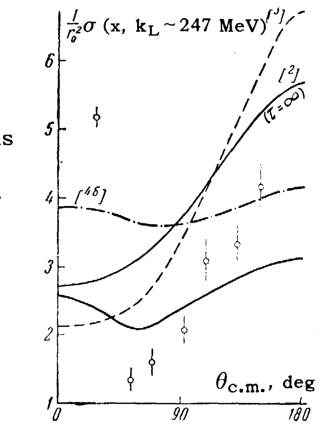


FIG. 8. Angular distributions in the c.m.s. for $k_L \sim 245$ MeV (full line). The dashed and dot-dashed lines are from the indicated references. The experimental points are from Baranov et al.[5]

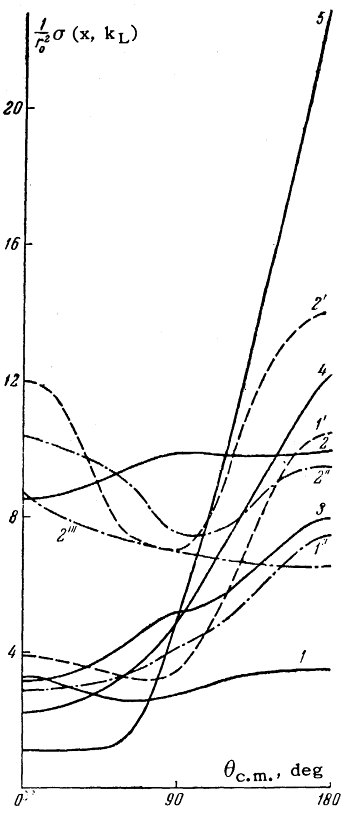


FIG. 9. Angular distributions for $k_L$ equal to 260, 316, 378, 445, 517 MeV (curves 1, 2, 3, 4, 5). Curves 1′ and 1″ for $k_L \sim 260$ MeV, 2′ and 2″ for $k_L \sim 320$ MeV (c = 1 and c = 0 respectively) are taken from[2]. Curve 2‴ for $k_L \sim 300$ MeV is taken from [4b].

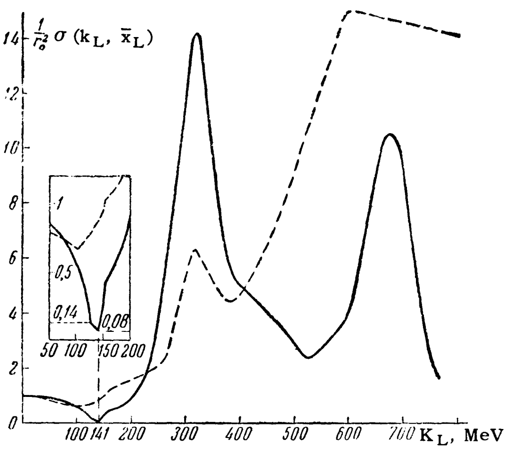


FIG. 10. Differential cross section for forward and backward scattering in the lab system as function of $k_L$: full line for $\theta_L = 0°$, dashed line for $\theta_L = 180°$. In the insert is illustrated the threshold anomaly (see[4] and[1]).

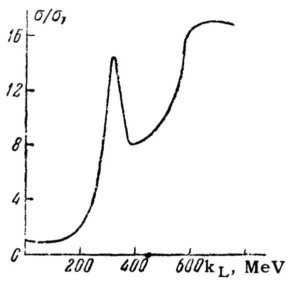


FIG. 12. Total cross section as function of $k_L$ (in units $\sigma_T = 8\pi r_0^2/3$).

different observable quantities. We note that two papers have recently appeared devoted to this problem which are rather similar in ideas and results, by Hearn and Leader[17] and by Contogouris.[18] In these papers a careful analysis of the reactions $\gamma + N \rightarrow \gamma' + N'$, $\gamma + \gamma' \rightarrow \bar{N} + N$ was performed. From the Mandelstam representation with one subtraction they obtain dispersion relations for a fixed angle. They show that in addition to the Low diagram whose sign depends on the number of subtractions, to the dispersion relations contribute the S-phase of the interaction and the partial pion-nucleon scattering cross sections. In these papers no numerical analysis was performed.

6. One can apply the relations from [8] to the evaluation of the Compton effect on the neutron and the deuteron. In particular one evidently can estimate the polarizability of the neutron. However, here a preliminary analysis has to be performed of the two photoproduction processes on the neutron, $\gamma + n \rightarrow n + \pi^0$, $\gamma + n \rightarrow p + \pi^-$.

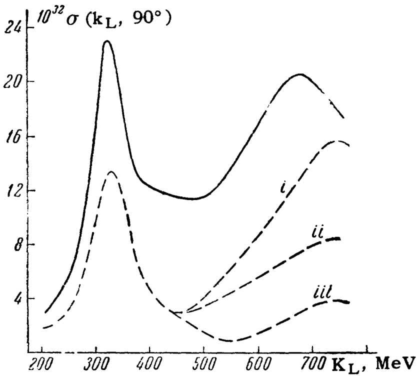


FIG. 11. Differential cross section for scattering at 90° c.m.s. Curves i, ii, iii are taken from Minami[7].

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