SOVIET PHYSICS JETP VOLUME 17, NUMBER 2 AUGUST, 1963

SIZE DISTRIBUTION OF EXTENSIVE AIR SHOWERS

Yu. A. FOMIN and G. B. KHRISTIANSEN

Nuclear Physics Institute, Moscow State University

Submitted to JETP editor August 13, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 666-675 (February, 1963)

An abrupt change in the power exponent of the spectrum of extensive air showers with respect to number of particles has been observed for $N \sim 10^5 - 10^6$ at sea level in a number of investigations. It is shown that in order to explain this change it is sufficient to assume that the distribution of magnetic clouds in the galaxy with respect to the parameter lH (l is the size of the magnetic cloud and H is the magnetic field intensity in it) is such that the diffusion coefficient D for ultrahigh energy cosmic rays changes from D = const to $D \sim E^{\alpha}$ (E is the energy), where $\alpha > 0.5$ when the energy changes by an order of magnitude or possibly even less. For a number of reasons this explanation seems to be the most probable. The composition of the primary radiation that produces showers with $N \sim 10^7$ particles is discussed. An analysis of the experimental data shows that the primary radiation does not consist at any rate of heavy nuclei exclusively.

A few years ago it was observed [1] that the exponent of the spectrum of extensive air showers (EAS) with respect to the number of particles N at sea level exhibits rapid variation in the range $N \sim 10^5 - 10^6$. This fact was later confirmed in many other investigations carried out at sea level. Thus, for example, Fukui et al [2] found that the exponent of the integral shower spectrum at $N \sim 10^5 - 10^6$ varied from $\gamma = 1.4 \pm 0.1$ to $\gamma = 2.0 \pm 0.2$. Allan et al [3] also observed a change from $\gamma = 2.3 \pm 0.1$ to $\gamma = 3.0 \pm 0.15$ in the exponent of the differential particle-number spectrum of showers at $N \sim 5 \times 10^5$. Figure 1 shows the integral spectrum $F(>N)$ at sea level as given by [1-3]. In the present time a similar singularity in the spectrum $F(>N)$ is apparently observed in experiments carried out at mountain altitudes [4-6]. The data obtained at 730 g/cm² [6] also offer evidence in favor of the existence of this singularity.

Such a singularity in the $F(N)$ spectrum can be explained a priori by assuming either 1) a rapid change in the exponent of the energy spectrum of the primary cosmic radiation 1), or 2) the sharp change in the form of the cascade curve at sufficiently large primary-particle energy, which can be connected with sharp changes in the characteristics of the nuclear-cascade process.

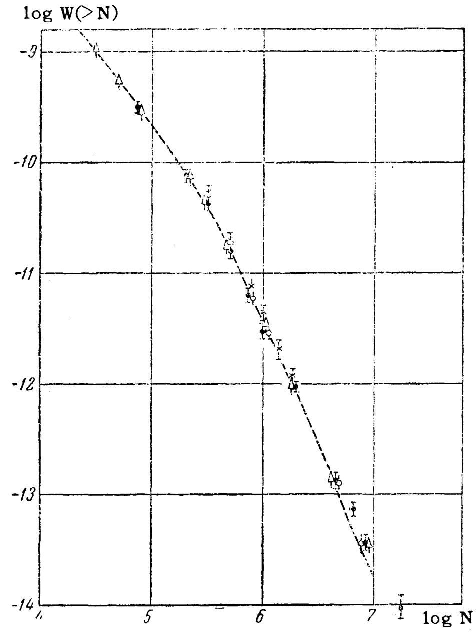

FIG. 1. Integral spectrum of EAS by number of particles, obtained by different authors: ● –[1c], Δ –[2], × –[3], the dashed curve has been calculated in the present paper assuming $\varphi(A)dA = A^{-1.5}dA$.

The second possibility seems little likely to us, for no singularities whatever were observed in numerous experiments on electron-photon, nuclear-active, and muon components of EAS with $N \sim 10^5 - 10^6$ particles at sea level.

1)The expression "rapid change" is used in the sense that, as will be shown below, it is necessary to assume that the exponent γ of the energy spectrum changes by ~ 0.5 when E changes by a factor ~ 10. At the same time it is known that when E varies, for example, over the range $10^9 - 10^{15}$, that is, by six orders of magnitude, the variation of γ likewise does not exceed 0.5.

The sharp change in the energy spectrum of the primary cosmic radiation can, generally speaking, be connected either with singularities in the galactic cosmic-ray sources or with features of their diffusion in galactic space. Since there are grounds for assuming that cosmic rays of ultrahigh energies, exceeding by at least one order the range of energies of interest to us, are produced in supernova shells, we consider primarily the primary cosmic radiation energy-spectrum singularities which can result for diffusion of the cosmic radiation in the galaxy.

It can be shown that if the magnetic clouds in the galaxy have a δ-function distribution with respect to the parameter lH (where l are the dimensions and H the magnetic field intensity of the clouds), then the diffusion coefficient D of the cosmic rays is constant up to a certain critical energy $E_{cr} = 300lHz$, after which it becomes proportional to E^2 [7]. On the other hand, if there is a variance in the distribution of the magnetic clouds with respect to lH, we can obtain different dependences of the diffusion coefficient D on the energy, weaker than $D \sim E^2$.

If we assume that the distribution of the magnetic clouds with respect to the intensity H and dimensions l in the galaxy is such that the values of lH smaller than the most probable $(lH)_\nu$ occur with very low probability, then we can obtain a rapid variation of the diffusion coefficient from D = const when $E < E_{cr} \approx 300\,(lH)_\nu z$ to $D = D(E)$ when $E > E_{cr}$.[2)]

The rapid variation of the diffusion coefficient of the charged particles with changing energy leads to a sharp change in the form of the energy spectrum of the particles in the galaxy. Indeed, in the simplest case of acceleration of nuclei with definite atomic numbers A in the source, the energy spectrum of these nuclei in the galaxy is determined by the product of the energy spectrum at f(E) produced by the source and the accumulation factor k(E).

The value of k, as is well known, has an order of magnitude $\sim Rc/2D$, where R is the dimension of the galaxy, c the velocity of motion of the particle, and D the diffusion coefficient. Therefore in the case under consideration, when $E < E_{cr}$, the energy spectrum of the particles in the galaxy is determined only by the function f(E). However, when $E > E_{cr}$ we have $D = D(E)$ and the energy spectrum has the form $f(E)/D(E)$.

The critical energy for primary nuclei with different mass numbers A varies and is connected with the critical energy of the proton by the equation $E_{cr\,A} \approx (1/2)AE_{cr\,p}$, and therefore the change in the index of the energy spectrum will occur at different energies for the different nuclei contained in the primary cosmic radiation. This circumstance causes a rapid change in the spectrum index at an energy $\sim E_{cr}$ for protons to be likewise observed in the energy spectrum of all the primary particles, but the further behavior of the energy spectrum will be determined not only by the character of the function D(E, A), but also by the form of the spectrum of the primary radiation with respect to the mass numbers A.

It is obvious that when $E < E_{cr\,p}$ the distribution over A can be determined in the limiting cases only by the character of the acceleration of the nuclei in the cosmic-ray sources themselves, or else by their fragmentation upon interaction with the interstellar gas during the course of their diffusion in the galaxy. In the latter case the distribution over A for $E > E_{cr\,p}$ changes not only because E_{cr} depends on A, but also owing to the decrease in the path within the limits of the galaxy, and thus also owing to the reduction in the fragmentation probability, which becomes a decreasing function of E/A.

If we consider the first case and denote by $\varphi(A)\,dA$ the spectrum of the cosmic ray sources averaged over A, then, allowing for the distribution of A, the energy spectrum of the cosmic rays in the galaxy has the form

$$F(E) = \int f(E)\,\varphi(A)\,dA/D(E/A).$$

In the second case the distribution over A cannot be represented as a product of two functions when $E > E_{cr\,p}$. However, assuming, as before, that the diffusion coefficient varies rapidly with the energy at $E \sim E_{cr}$, it is obviously possible to obtain as before, upon suitable choice of the function $D = D(E/A)$ for $E > E_{cr}$, a sharp variation in the primary energy spectrum[3)].

We consider henceforth only the first case. It is thus of interest to ascertain whether we can ob-

---

[2)]It is easy to see that the connection between the distribution $w(lH)\,d(lH)$ and the function D(E) is determined by the relation

$$\frac{1}{D(E)} \sim \int_{lH=E/300z}^{\infty} l^2 w(lH)\,d(lH),$$

which yields

$$w(lH)\,d(lH) = \frac{1}{D^2(E)}\frac{dD(E)}{dE}\Big|_{E=300\,lHz}$$

under the condition that $\int l^2 d(lH)$ does not depend on lH.

[3)]The difference between the first and second case is to be expected if one is interested in the distribution over A when $E > E_{cr}$.

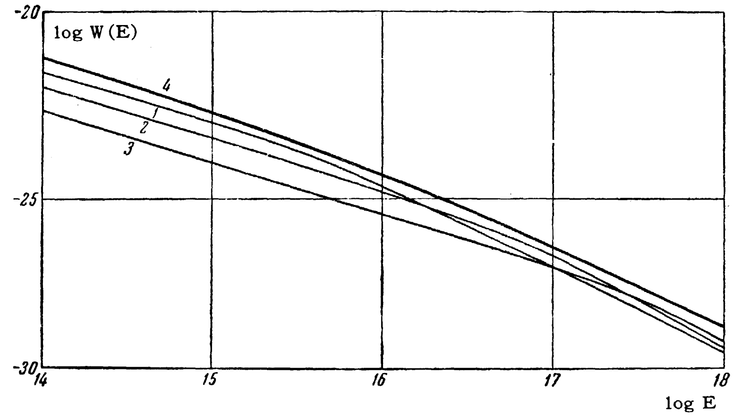

FIG. 2. Partial energy spectrum of the primary nuclei assumed in the calculation: 1 – with A = 1; 2 – with A = 10; 3 – with A = 56; 4 – summary energy spectrum of the primary cosmic radiation with $\varphi(A)dA = A^{-1.5}dA$.

tain the experimentally-observed particle-number spectrum of the showers through a suitable sensible choice of the spectra f(E) and $\varphi(A)$ as well as of the diffusion coefficient, with allowance for the fluctuations in the development of the cascade of shower particles and for possible differences in the character of the development of the shower from the primary proton and the primary nucleus.

We present here our assumptions concerning f(E), $\varphi(A)$, D(E), the character of shower development, and the results of the calculation of the particle-number spectrum of the showers made under these assumptions.

Figure 2 shows the energy spectrum for different groups of nuclei with A = 1, 10, and 56, which we assume to have the form $E^{-\gamma}$. The values of γ in the different intervals of E are as follows:

E:	$<\frac{1}{3}E_{cr}$	$\frac{1}{3}E_{cr}-E_{cr}$	$E_{cr}-2E_{cr}$	$2E_{cr}-3E_{cr}$	$>3E_{cr}$
γ:	1.5	1.75	2.0	2.25	2.5

Actually we propose a transition from D = const to D ~ E as E changes by one order of magnitude. The critical proton energy is assumed to be 5×10^5 eV. Figure 2 also shows the summary energy spectrum of the primary cosmic rays, obtained by adding the partial spectra with weights corresponding to a mass-number distribution of the primary particles in the form $A^{-1.5}dA$ [8]. The spectrum obtained in this manner yields in the region E ~ 10^{16} eV a change in the index $\Delta\gamma = 0.5$.

In principle, to change over from the energy distribution of the primary cosmic radiation to the distribution of the number of particles observed at a definite depth in the atmosphere it is necessary to know the entire pattern of shower development. In our calculation we used a simplified pattern, based on the following two assumptions: 1) the fluctuations occur only at the point of the first interaction, that is, at the place where the shower is produced, and 2) the cascade curve has a definite form, and account is taken of its dependence on the atomic number of the primary nucleus producing the shower.

The number of primary particles with energy E_0, which initiate an extensive air shower at a depth x_0 from the top of the atmosphere, is determined by the expression

$$w(E_0, x_0)\, dE_0 dx_0 = \text{const}\cdot E_0^{-(\gamma+1)}\, dE_0 e^{-x_0/\lambda}\, dx_0/\lambda, \quad (1)$$

where λ is the interaction range, which depends on the atomic number of the primary particles; λ is determined from the formula

$$\lambda = \frac{A_{\text{air}}}{6\cdot 10^{23}\,\pi\,[1.45\cdot 10^{-13}(A_i^{1/3} + A_{\text{air}}^{1/3} - 1.17)]^2}. \quad (2)$$

The interaction ranges in air obtained from this equation, for particle groups with atomic numbers lying in definite intervals, are listed in Table I (L—group of light nuclei, M—medium, H—heavy, VH—very heavy).

In going from (1) to the particle-number spectrum of the showers for a given A, it is assumed that all the showers at the observation level (sea level) are beyond the maximum of their development, and the cascade curve has the form

$$N(E_0, x_0) = kE_0 \exp\left\{-\frac{x-(x_m+x_0)}{\Lambda}\right\}, \quad (3)$$

where k is a constant determined by the relation $kE_0 = 2.5 \times 10^6$, x is the observation level ($x = 1030$ g/cm^2), Λ is the absorption range of the shower particles, assumed equal to 200 g/cm^2, and x_m is the level of the maximum of the shower, which depends on the atomic number of the primary particles [4)]:

4)Wishing primarily to illustrate with the aid of the presented calculation the possibility of obtaining the singularities in the particle-number spectrum of the showers if a singularity exists in the primary energy spectrum, we are making some simplifying assumptions, in particular, that $N \sim E_0$. Actually $N \sim E_0^{\beta}$, where β is larger than unity at sea level

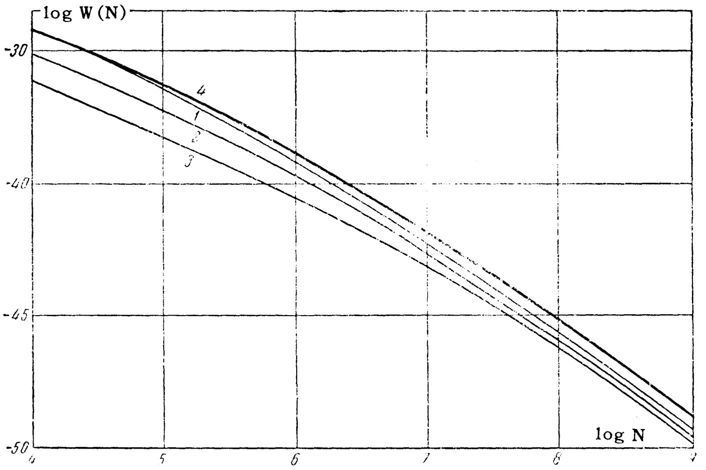

FIG. 3. Partial differential particle-number spectra of extensive air showers, calculated for $\varphi(A)dA = A^{-1.5}dA$: 1 – proton spectrum, L – light-particle spectrum; 3 – very heavy particles; 4 – summary particle-number EAS spectrum.

Table I

Group of nuclei	Z	$\bar{A}$	λ, g/cm²	Percentage content for $E < E_{cr}$	
				$\varphi(A)\,dA = A^{-1.5}dA$	$\varphi(A)\,dA = A^{-2}dA$
p	1	1	80	51	68
α	2	4	44.5	12.6	13.5
L	3–5	10	32	18.4	11.8
M	6–9	14	27	7.5	3.6
H	⩾10	31	18.5	7.4	2.25
VH	⩾20	56	14.6	3.1	0.85

$$x_m = b\ln(E_0/A), \qquad b = 13 \text{ g/cm}^2.$$

Thus, it has been assumed that when the primary nucleus collides with an atom of air, all the nucleons of the colliding nucleus participate in the interaction and the shower that is produced thereby is similar to A showers from primary protons with energy E_0/A, but on the average the heavier the nucleus, the closer to the top of the atmosphere the shower is. The larger A, the closer the position of the maximum of the shower at a given primary-nucleus energy is shifted to the top of the atmosphere.

From (1) and (3) we find that the number of showers at a given level x with a number of particles in the interval N, N + dN, for each atomic number A of the primary nucleus is equal to

$$W_A(N, x)\,dN = \frac{\text{const}\cdot\Lambda}{\Lambda + b - \gamma\lambda} e^{x\gamma/(\Lambda+b)} A^{-\gamma b/(\Lambda+b)} \left(\frac{N}{k}\right)^{-[(\gamma+1)\Lambda+b]/(\Lambda+b)} \times \left\{1 - \left[\left(\frac{N}{k}\right)^b e^{-x} A^{-b}\right]^{[\Lambda+b-\gamma\lambda]/\lambda(\Lambda+b)}\right\} dN. \quad (4)$$

This expression with a value $\gamma = 1.5$ holds true up to $N = k(E_{cr}/3)\exp[-(x - x_m)/\Lambda]$. When $N > 3kE_{cr}$, the differential particle-number spectrum of the showers acquires the same form, but now with a value $\gamma = 2.5$. For values of N in the interval from $k(E_{cr}/3)\exp[-(x - x_m)/\Lambda]$ to $3kE_{cr}$, the differential spectrum has a more complicated form.

The obtained differential spectra $W_A(N)\,dN$ for different A are shown in Fig. 3. The calculation shows that a contribution to a given N can be made by primary particles with energy lying in the interval from $E_{0\,min} = (N/k)$ (when the first interaction occurs at such an altitude that the shower has a maximum at the observation level) to $E_{0\,max} = (N/k)\exp[(x - x_m)/\Lambda]$ (when the first interaction occurs at the top of the atmosphere). However, the interval of the significant values of E_0 depends on the A of the primary nucleus and decreases with A.

The obtained differential particle-number spectra of the showers produced by primary nuclei with atomic number A were summed with suitable weights (see Table I) for two types of primary-radiation atomic-number spectra, $A^{-1.5}dA$[8] and $A^{-2}dA$[7], with a given energy per particle. We obtain the integral spectrum $F(>N)$ (Fig. 1) from the summary differential spectrum $W(N)\,dN$ (Fig. 3).

The obtained spectra are such that when $N \sim 5 \times 10^5$ a rapid change is observed in the index (see

(β = 1.1 – 1.3). The index $\beta\varkappa$ of the primary energy spectrum will in this case be larger than the assumed 1.5. Evidence in favor of this is offered by direct experimental data on the value of $\varkappa$ for showers observed at mountain altitudes, where $\varkappa$ is apparently larger than for showers initiated by a primary particle of the same energy, but observed at sea level. The assumption made does not change the final conclusion, which follows from the calculation. This is seen from formula (4).

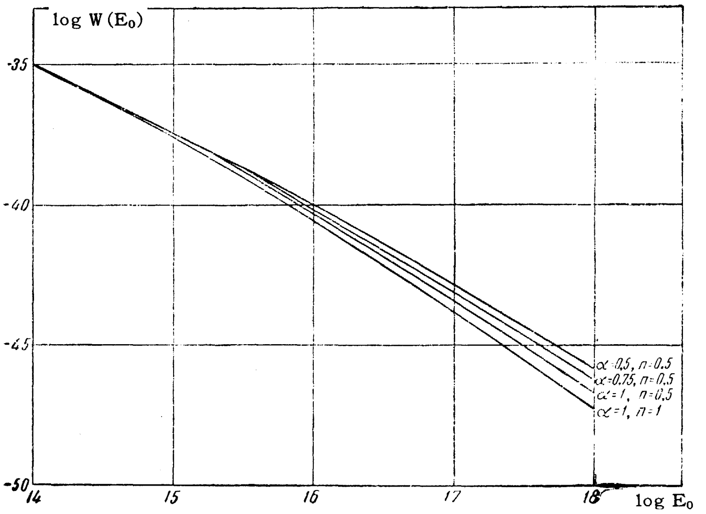

FIG. 4. Differential primary energy spectrum constructed under different assumptions concerning the distribution of $\varphi(A)dA = dA/A^{n+1}$ and the dependence $D(E) \sim E^{\alpha}$ for $E > E_{cr}$.

Table II). Figure 4 and Table II show for comparison data on the primary energy spectrum, constructed under different assumptions for the distribution $\varphi(A)dA$ and for the dependence of $D(E)$ at $E > E_{cr}$ [$\varphi(A)dA = dA/A^{n+1}$, $d(E) \sim E^{\alpha}$ for $E > E_{cr}$]. We calculated the particle-number spectrum for $n = 0.5$, $\alpha = 1$ and $n = 1$, $\alpha = 1$.

Table II

	Calculated values of $\varkappa$		Experimental values of $\varkappa$
	$\varphi(A)\,dA = A^{-1,5}dA$	$\varphi(A)\,dA = A^{-2}dA$	
Differential spectrum			
$N < 10^5$	2.44	2.44	—
$10^5 - 4\cdot10^5$	2.52	2.52	2.3±0.1
$4\cdot10^5 - 10^6$	2.76	2.84	—
$10^6 - 4\cdot10^6$	2.82	3.00	—
$4\cdot10^6 - 10^7$	2.85	3.10	3.1±0.15
$10^7 - 10^8$	2.95		
Integral spectrum			
$2\cdot10^5$	1.48	1.48	1.43±0.1
$2\cdot10^5 - 6\cdot10^5$	1.73	—	—
$6\cdot10^5 - 10^6$	2.12	—	—
$10^6 - 10^7$	2.12	2.10	2.0±0.07

From a comparison of Table III and the experimental data we see that more accurate experimental data are needed, particularly data on the differential particle-number spectrum of the showers, if a choice is to be made between the various assumptions [5]. It must be noted that the

Table III. Variation of energy-spectrum index with energy for different assumptions concerning the functions $\varphi(A)dA = dA/A^{n+1}$ and $D(E) \sim E^{\alpha}$ for $E > E_{cr}$

E_0	Calculated values of $\varkappa$			
	$\alpha = 0,5$, $n = 0,5$	$\alpha = 0,75$, $n = 0,5$	$\alpha = 1$, $n = 0,5$	$\alpha = 1$, $n = 1$
Differential spectrum				
$<10^{15}$	2.5	2.5	2.5	2.5
$10^{15} - 5\cdot10^{15}$	2.57	2.6	2.9	3.0
$10^{16} - 5\cdot10^{16}$	2.8	2.9	3.1	3.2
$>10^{17}$	2.95	3.15	3.35	3.4
Integral spectrum				
$<10^{15}$	1.5	1.5	1.5	1.50
$10^{15} - 5\cdot10^{15}$	1.55	1.58	1.5	1.88
$10^{16} - 5\cdot10^{16}$	1.78	1.91	2.0	2.1
$>10^{17}$	1.95	2.15	2.3	2.35

calculated differential particle-number spectrum of the shower and the differential energy spectrum of the primary radiation differ in form. In view of the inclusion of the fluctuations, the variation of the index κ is slower than the variation of the index γ. In integral spectra, this difference is smaller.

Calculation shows that when the energy E_0 increases and exceeds the critical proton energy, the role of the heavy nuclei in the production of EAS with specified number of particles N begins to increase. However, this increase is much smaller than the increase in the fraction of heavy nuclei in the primary radiation of given energy (Table IV). The contributions of different nuclei to the primary radiation of given energy (above critical) is determined by the change in the index of the primary energy spectrum. The contribution of the protons to the number of showers, even with

[5] In [1c] an attempt was made to obtain the observed singularity of the particle-number spectrum of the showers under the assumption that $D(E) \sim E^2$ for $E > E_{cr}$. The present calculation shows that agreement with experiment is obtained, within the limits of error, when $D(E) \sim E$ and in general when $D(E) \sim E^{\alpha}$ where $\alpha \geq 0.5$.

Table IV. Comparison of the contribution of different nuclei to the primary radiation of a given energy and in an EAS with given number of particles for a primary-mass-number spectrum $A^{-1.5}dA$ and for $D(E) \sim E$

Group of nuclei	Primary radiation at $E = 10^{17}$, eV, %	EAS with $N = 10^7$ for $E > E_{cr}$, %
p	13	38
L	23,7	15
VH	12.5	5

$N \sim 10^8$ particles, is still large. This is a manifestation of the fluctuations in the development of the shower, and also the consequence of the assumed dependence $x_m \sim \log(E_0/A)$. We recall that, as already mentioned before, the contribution of the heavy nuclei to the primary radiation for $E > E_{cr}$ and in the EAS can be appreciably larger when fragmentation plays an important role in the formation of the spectrum over A.

Returning to the other possibility of a sharp change in the primary energy spectrum, mentioned in the start of the article, namely the existence of a limiting energy up to which acceleration in cosmic-ray sources is possible, we must note the following. As is well known [7], the existence of a limiting energy is apparently connected with the diffusion of the accelerated particles beyond the limits of the region where the acceleration is possible. The dependence of the diffusion coefficient on the energy of the accelerated particles will be determined by the variance of the distribution of the parameter lH, which is characteristic of a given source. This variance can hardly be less than the variance considered above in the scales of the galaxy. Recognizing that there are indications in favor of assuming that different sources (supernovas) generate cosmic rays with essentially different spectra [9], it is reasonable to assume that in the cosmic-radiation energy spectrum averaged over all the sources one can hardly expect the appearance of the observed singularities as a result of the existence of limiting energies.

Thus, the rapid change in the particle-number index of the shower spectrum, observed in [1-3], is most probably to be ascribed to the corresponding change in the index of the primary energy spectrum. The latter finds a most natural explanation in the specific character of the distribution of the magnetic clouds with respect to the parameter lH in the galaxy.

The authors thank I. P. Ivanenko and S. I. Syrovat-skiĭ for a discussion of the problem.

APPENDIX

ON THE COMPOSITION OF PRIMARY COSMIC RADIATION IN THE REGION OF SUPERHIGH ENERGIES

New experimental data on the fluctuations of muon fluxes of high energy in showers with a fixed number of particles $N \sim 10^7$ have been obtained recently [10, 11]. These data allow us to consider the question of the composition of the primary cosmic radiation in the region of ultrahigh energies.

The distribution $f(n_\mu/\bar{n}_\mu)$ should be sensitive to the composition of the primary radiation, since nuclear particles with different A can correspond to different forms of the distribution $f(n_\mu/\bar{n}_\mu)$.

Table V lists the results of the calculations of $f(n_\mu/\bar{n}_\mu)$, made under the limiting assumption that the fluctuations in the development of the shower are determined by the fluctuations in the height of its generation. It was assumed here that $n_\mu = k_\mu (E/A)^\alpha A$. The values of α and k_μ were determined from the known experimental data on the muon component of the shower, within the framework of the model under consideration: according to the experimental data $n_\mu \sim N_e^\alpha$ ($\alpha = 0.75 - 0.85$),

Table V

$\Delta\left(\frac{n_\mu}{\bar{n}_\mu}\right)$	$f_p = (n_\mu/\bar{n}_\mu)$, % Experiment [11]	$f_p = (n_\mu/\bar{n}_\mu)$, % $\gamma = 2$, $\Lambda = 170$, $\beta = 1.1$, $\alpha = 0.9$, $\lambda_p = 96$	$f_p(n_\mu/\bar{n}_\mu)$, % $\gamma = 2.5$, $\Lambda = 220$, $\beta = 1.3$, $\alpha = 0.7$, $\lambda_p = 64$	$f_{VH}(n_\mu/\bar{n}_\mu)$, %
0—1/3	6	13	0	0
1/3—2/3	20	27	9,5	0
2/3—1	34	17	30,5	50
1—1.5	26	18.5	60	50
1.5—3	14	24	0	0
3	0	0	0	0

$k\bar{N}_e = \bar{E}$ ($k = 5 \times 10^9 - 10^{10}$). The remaining specific assumptions were the same as in the calculation of the particle-number spectrum of the showers.

Under these assumptions, the partial distribution $f(n_\mu/\bar{n}_\mu)$ corresponding to a given A has the form

$$f(n_\mu)\,dn_\mu = \frac{\text{const}}{\alpha(N/k)}\,\frac{\Lambda}{\lambda}\left(\frac{N}{k}\right)^{-\beta\Lambda/\lambda} e^{-x/\lambda} A^{-b/\lambda} A^{-(1-\alpha)[(\beta\Lambda+b)/\lambda-\gamma]} \times \left(\frac{n_\mu}{k_\mu}\right)^{[(\beta\Lambda+b)/\lambda-\gamma-\alpha]/\alpha} dn_\mu.$$

Calculation shows the following: 1) it is possible to obtain agreement with experiment for large N by suitable modification of the parameters Λ, β, and ($\beta\Lambda$ + b) within definite limits, and 2) the primary radiation does not consist entirely of nuclei of the H (heavy) and VH (very heavy) groups.

---

[1]a) G. V. Kulikov and G. B. Khristiansen, Nuovo cimento Suppl. 8 (1958). b) G. V. Kulikov and G. V. Khristiansen, JETP **35**, 635 (1958), Soviet Phys. JETP **8**, 441 (1959). c) Vernov, Khristiansen, Atrashkevich, Belyaeva, Dmitriev, Kulikov, Nechin, Solov'eva, Fomin, and Khrenov, Materials of the Tokyo International Conference on Cosmic Rays, 1961.

[2] Fukui, Hasegawa, Matano, Miura, Oda, Suga, Tanahashi, and Tanaka, Progr. Theor. Phys. Suppl. **16**, 1 (1960).

[3] Allan, Beamish, Glencross, Thomson, and Wills, Preprint, (1962).

[4] Kulikov, Nesterova, Nikol'skiĭ, Solov'eva, Khristiansen, and Chudakov, Trans. Moscow International Conference on Cosmic Rays, IUPAP, **2**, AN SSSR, 1959.

[5] G. Clark and I. Escobar, Preprint, (1961).

[6] Miyake, Hinotani, Katsumata, and Kaneko, N. Ito. J. Phys. Soc. Japan **16**, 847 (1961).

[7] V. L. Ginzburg and S. I. Syrovat-skiĭ, UFN **71**, 411 (1960), Soviet Phys. Uspekhi **3**, 504 (1961).

[8] B. Peters, Trans. Moscow International Conference on Cosmic Rays, IUPAP, **3**, 1959.

[9] B. Peters, Nuovo cimento **22**, 800 (1961).

[10] Vernov, Solov'eva, Khrenov, and Khristiansen, JETP **41**, 340 (1961), Soviet Phys. JETP **14**, 246 (1962).

[11] Matano, Miura, Oda, Tanahashi, Tanaka, Hasegawa, Higashi, Kitamura, Mishima, Miyamoto, Shibato, and Watase, Phys. Rev. Lett. **8**, 284 (1962).

Translated by J. G. Adashko
108