TEMPERATURE DEPENDENCE OF THE GAP IN A SUPERCONDUCTOR

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It is shown that the temperature dependence of the energy gap in superconductivity theory is valid without any assumption on the weakness of the interaction if the gap is small in comparison with the Debye frequency.

As is well known, the most important characteristic of a superconductor is the gap in the spectrum of elementary excitations. The temperature dependence of the gap in the case of weak interaction of the electrons was obtained by Bardeen, Cooper, and Schrieffer. [1] This dependence can be represented in the following form:

$$\int_0^\infty dx \left\{ \frac{\operatorname{th}(\sqrt{x^2+\Delta^2}/2T)}{\sqrt{x^2+\Delta^2}} - \frac{1}{\sqrt{x^2+\Delta_0^2}} \right\} = 0. \qquad (1)^*$$

Here Δ is the gap at temperature T; $\Delta_0 \equiv \Delta|_{T=0}$.

As is seen from (1), there exists between Δ/Δ_0 and the reduced temperature T/T_0 (T_0 is the critical temperature) a universal connection which does not depend on the details of the interaction. It will be shown below that this takes place in the case of any interaction provided only that the gap changes only slightly as a function of frequency and momentum in the intervals Δ_0 and Δ_0/v (v = the Fermi velocity), respectively.

Let us consider an isotropic superconducting Fermi system. In accord with Gor'kov, [2] we introduce the functions

$$\begin{aligned} G(p, \tau_1-\tau_2) &= \langle T_\tau a_{p\sigma}(\tau_1)\, a^+_{p\sigma}(\tau_2)\rangle, \\ F(p, \tau_1-\tau_2) &= \langle T_\tau a_{p^1/_2}(\tau_1)\, a_{-p-^1/_2}(\tau_2)\rangle, \\ F^+(p, \tau_1-\tau_2) &= \langle T_\tau a^+_{p^1/_2}(\tau_1)\, a^+_{-p-^1/_2}(\tau_2)\rangle, \end{aligned} \qquad (2)$$

$$a_{p\sigma}(\tau) = e^{H\tau} a_{p\sigma} e^{-H\tau}, \qquad a^+_{p\sigma}(\tau) = e^{H\tau} a^+_{p\sigma} e^{-H\tau},$$

where $a_{p\sigma}$ and $a^+_{p\sigma}$ are the operators of annihilation and creation of a particle with momentum p and spin σ; $H = \mathscr{H} - \mu N$ ($\mathscr{H}$ = Hamiltonian of the system, N = operator of the number of particles, μ = chemical potential). The symbol $\langle \ldots \rangle$ denotes averaging over the statistical ensemble:

$$\langle A \rangle = \operatorname{Sp}\{A \exp[(\Omega - H)/T]\}.$$

The Dyson equations for the Green's function (2) are very simple in the "discrete frequency representation." [3,4] We denote the Green's function in this representation by means of

$$G(p\ \varepsilon_n), \qquad F(p, \varepsilon_n), \qquad F^+(p, \varepsilon_n)$$

$$(\varepsilon_n = (2n+1)\pi T).$$

Then the Dyson equations are written in the following fashion:

$$\begin{aligned} G(p, \varepsilon_n) &= G_0(p, \varepsilon_n)\,[1 + \Sigma_1(p, \varepsilon_n)\, G(p, \varepsilon_n) \\ &\quad - \Sigma_2^+(p, \varepsilon_n)\, F(p, \varepsilon_n)], \\ F(p, \varepsilon_n) &= G_0(p, -\varepsilon_n)\,[\Sigma_1(p, -\varepsilon_n)\, F(p, \varepsilon_n) \\ &\quad - \Sigma_2(p, \varepsilon_n)\, G(p, \varepsilon_n)], \end{aligned} \qquad (3)$$

where G_0 is the Green's function in the absence of the interaction. In the diagram technique, the functions Σ_1, Σ_2, Σ_2^+ correspond to certain combinations of irreducible graphs. That is, the function Σ_1 corresponds to graphs with one entrance and one exit, the function Σ_2 to graphs with two exits, the functions Σ_2^+ to graphs with two entrances.

We denote the characteristic dimensions of Σ_1, Σ_2 in momentum by q_0 and in frequency by ω_0 (for example, in the case of a metal, q_0 is of the order of the Fermi momentum p_0, ω_0 is of the order of the Debye frequency). We assume that the following condition is satisfied:

$$\Delta_0 \ll \lambda \equiv \min(vq_0, \omega_0). \qquad (4)$$

In this case, Eqs. (3) are considerably simplified.

In the estimates, we use for F the expression obtained in the weak coupling approximation: [2]

$$F(p, \varepsilon_n) = \Delta/(\omega_p^2 + \varepsilon_n^2),$$

where $\omega_p = \sqrt{\eta_p^2 + \Delta^2}$, η_p is the energy of the free particle, measured from the Fermi surface. The simplest graph containing F is drawn in Fig. 1. Here the line with two arrows corresponds to the function $F(q, \epsilon_n)$. The dashed line in the case of

*th = tanh.

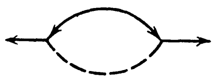

FIG. 1

pair interaction of electrons corresponds to the Fourier component of this interaction $V(|\mathbf{p}-\mathbf{q}|)$ (we shall assume that the exit momentum is $p = p_0$).

The contribution of the graph under consideration has the form

$$T\sum_n \int \frac{d^3q}{(2\pi)^3} V(|\mathbf{p}_0-\mathbf{q}|)\frac{\Delta}{\omega_q^2+\varepsilon_n^2}.$$

This expression is equal to the following in order of magnitude:

$$\Delta \ln(\Delta_0/q_0^2)\, p_0 V(0)$$

(if q_0 differs strongly from p_0, then some small factors appear). It is then seen that in the fundamental region the summation and integration the function F makes a contribution of the order of

$$\Delta q_0^{-4}\ln(\Delta_0/q_0^2),$$

which contains the small ratio Δ/q_0^2. The same is also valid for graphs of higher order.

In the estimate, some assumptions were made on the character of the interaction (two-particle interaction was assumed). It is clear that even in the case of other types of interactions the estimate does not change appreciably, since only the satisfaction of the condition (4) is of importance. Thus one can confirm the fact that in the graphs for Σ_1 and Σ_2, each F-line introduces a small quantity $\sim\Delta_0/\lambda$ (we note that the G-function does not contain such a small quantity). This makes it possible to simplify Eq. (3).

Actually, account is not taken in Σ_1 of graphs with F-lines. In the remaining graphs, one can everywhere replace G by the Green's function of the normal state $\mathcal{G}$, inasmuch as they differ appreciably only in a narrow layer of the order of Δ_0. As a result, Eqs. (3) take the form

$$G(p,\varepsilon_n) = \mathcal{G}(p,\varepsilon_n)[1-\Sigma_2^+(p,\varepsilon_n)F(p,\varepsilon_n)],$$
$$F(p,\varepsilon_n) = -\mathcal{G}(p,-\varepsilon_n)\Sigma_2(p,\varepsilon_n)G(p,\varepsilon_n). \quad (5)$$

We then obtain

$$F(p,\varepsilon_n) = \frac{-\Sigma_2(p,\varepsilon_n)}{\mathcal{G}^{-1}(p,\varepsilon_n)\mathcal{G}^{-1}(p,-\varepsilon_n)-\Sigma_2(p,\varepsilon_n)\Sigma_2^+(p,\varepsilon_n)}. \quad (6)$$

The functions $G(p,\epsilon_n)$ and $F(p,\epsilon_n)$ are given in the complex plane by the set of discrete points $\epsilon = i\epsilon_n$. By analytic continuation of these functions in the complex plane ϵ, one can, as is well known, obtain functions of two types. We denote by the index R functions that are regular in the upper half-plane ϵ and by the index A, functions which are regular in the lower half-plane of ϵ. The poles G_R and F_R lying in the lower half-plane close to the real axis give the energy and the damping of the quasi-particles. The minimum energy of the quasi-particles is that of the gap. Let us find how it is related to Σ_2. For this purpose, we write out the analytic continuation of (6) on the real axis:

$$F_R(p,\varepsilon) = \frac{-\Sigma_{2R}(p,\varepsilon)}{\mathcal{G}_R^{-1}(p,\varepsilon)\mathcal{G}_A^{-1}(p,-\varepsilon)-\Sigma_{2R}(p,\varepsilon)\Sigma_{2R}^+(p,\varepsilon)}. \quad (7)$$

Close to the Fermi surface, the function $\mathcal{G}_R$ has the form

$$\mathcal{G}_R(p,\varepsilon) = \frac{a}{v(p-p_0)-\varepsilon-i\delta} \qquad (\delta\to+0)$$

(the finiteness of the damping δ is not important). Making use of this, we write out Eq. (7) for ϵ, $v|p-p_2| \ll \lambda$:

$$F_R(p,\varepsilon) = \frac{-a^2\Sigma_{2R}(p_0,0)}{v^2(p-p_0)^2-a^2\Sigma_{2R}(p_0,0)\Sigma_{2R}^+(p_0,0)-(\varepsilon+i\delta)^2}. \quad (8)$$

It will be shown in the Appendix that the following relation holds:

$$\Sigma_{2R}^+(p,\varepsilon) = -\Sigma_{2R}^*(p,-\varepsilon).$$

Taking this into account in (8), we find for the gap $\Delta = a|\Sigma_{2R}(p_0,0)|$. Denoting $\epsilon_p = \sqrt{v^2(p-p_0)^2+\Delta^2}$, we rewrite (8) in the form

$$F_R(p,\varepsilon) = \frac{-a^2\Sigma_{2R}(p_0,0)}{(\varepsilon_p-\varepsilon-i\delta)(\varepsilon_p+\varepsilon+i\delta)}. \quad (9)$$

We note that in the denominator of (7) the term $\Sigma_{2R}\Sigma_{2R}^+$ can be taken for $\epsilon = 0$, $p = p_0$. Actually, $\Sigma_{2R}\Sigma_{2R}$ is of the order of Δ^2 and varies appreciably upon change of its arguments over a range $\sim\lambda$. But for ϵ, $v|p-p_0| \sim \lambda$, the product $\mathcal{G}_R^{-1}\mathcal{G}_A^{-1}$ contains terms $\sim\lambda^2$, and the quantity $\sim\Delta^2$ can be neglected. As a result, Eq. (7) takes the form

$$F_R(p,\varepsilon) = \frac{-\Sigma_{2R}(p,\varepsilon)}{\mathcal{G}_A^{-1}(p,-\varepsilon)\mathcal{G}_R^{-1}(p,\varepsilon)+(\Delta/a)^2}. \quad (10)$$

We have considered only graphs in Σ_2 with single F-lines (Fig. 2). This corresponds to the equation

$$\Sigma_2(p,\varepsilon_n) = T\sum_m \int \frac{d^3q}{(2\pi)^3} K(\varepsilon_n,\varepsilon_m;\mathbf{p},\mathbf{q})F(q,\varepsilon_m). \quad (11)$$

The function K represents the contribution of irreducible graphs which do not contain F-lines, with two exits and two entrances (the square in Fig. 2). Each of these graphs cannot be separated into two parts which are united only by two lines directed in the same sense. We note that, just as in Σ_1, one can make the substitution $G \to \mathcal{G}$ everywhere in K, so that K does not depend on Δ.

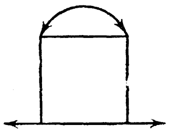

FIG. 2

In the continuation of Eq. (11) on the real axis, we assume that the irreducible vertex K has the same analytic properties as the complete four-vertex. It was shown by Éliashberg [5] that the complete vertex $\Gamma(\epsilon_1, \epsilon_2; \epsilon_3, \epsilon_4)$ (ϵ_1, ϵ_2 are the exit and ϵ_3, ϵ_4 the entrance frequencies) is an analytic function with cuts

1) $\mathrm{Im}\,\varepsilon_1 = \mathrm{Im}\,\varepsilon_2 = \mathrm{Im}\,\varepsilon_3 = \mathrm{Im}\,\varepsilon_4 = 0$;
2) $\mathrm{Im}\,(\varepsilon_1 - \varepsilon_3) = \mathrm{Im}\,(\varepsilon_1 - \varepsilon_4) = 0$;
3) $\mathrm{Im}\,(\varepsilon_1 + \varepsilon_2) = 0$.

In our case, $\epsilon_3 = -\epsilon_4 = \epsilon_n$, $\epsilon_1 = -\epsilon_2 = \epsilon_m$ so that there will be singularities only of the type 1) and 2).

We rewrite Eq. (11), writing out the arguments explicitly for which K has singularities:

$$\Sigma_2(p, \varepsilon_n) = T\sum_m \int \frac{d^3q}{(2\pi)^3} Q(\varepsilon_n, \varepsilon_m, \varepsilon_n - \varepsilon_m, \varepsilon_n + \varepsilon_m; \mathbf{p}, \mathbf{q})\, F(q, \varepsilon_m), \tag{12}$$

where $Q(\epsilon_n, \epsilon_m, \epsilon_n - \epsilon_m, \epsilon_n + \epsilon_m; \mathbf{p}, \mathbf{q}) \equiv K(\epsilon_n, \epsilon_m; \mathbf{p}, \mathbf{q})$.

As usual, we carry out analytic continuation of (12) along the real axis (see, for example, [5]). That is, with the aid of the function $\tanh(\omega/2T)$, we replace the summation by integration over contours enclosing the imaginary axis in the complex plane ω. We then transform to integration along the cuts and direct $i\epsilon_n$ along the real axis. As a result, (12) takes the form

$$\Sigma_{2R}(p, \varepsilon) = \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{4\pi i} \left\{ \mathrm{th}\frac{\omega}{2T} Q_1(\varepsilon, \omega; \mathbf{p}, \mathbf{q}) + \mathrm{cth}\frac{\omega - \varepsilon}{2T} Q_2(\varepsilon, \omega; \mathbf{p}, \mathbf{q}) \right\} F_R(q, \omega), \tag{13*}$$

$$Q_1(\varepsilon, \omega) = Q(\varepsilon + i\delta, \omega + i\delta, \varepsilon - \omega + i\delta, \varepsilon + \omega + i\delta) + Q(\varepsilon + i\delta, -\omega - i\delta, \varepsilon + \omega + i\delta, \varepsilon - \omega + i\delta), \tag{14}$$

$$\begin{aligned} Q_2(\varepsilon, \omega) = {} & Q(\varepsilon + i\delta, \omega + i\delta, \varepsilon - \omega - i\delta, \varepsilon + \omega + i\delta) \\ & - Q(\varepsilon + i\delta, \omega + i\delta, \varepsilon - \omega + i\delta, \varepsilon + \omega + i\delta) \\ & - Q(\varepsilon + i\delta, -\omega - i\delta, \varepsilon + \omega + i\delta, \varepsilon - \omega + i\delta) \\ & + Q(\varepsilon + i\delta, -\omega - i\delta, \varepsilon + \omega + i\delta, \varepsilon - \omega - i\delta). \end{aligned} \tag{15}$$

*cth = coth.

The momentum dependence in (14) and (15) has been omitted for brevity. The infinitely small contributions $\pm i\delta$ to the arguments of the function Q show along what boundary of the cut this function is taken.

Equation (13) determines Σ_{2R} with account of the expression for F_R (10). We recall that the functions Q_1, Q_2, and $\mathcal{G}$, entering into Eq. (13) are completely determined by the normal state, i.e., do not depend on Σ_{2R}. These functions, are slightly different from their values for $T = 0$ at sufficiently low temperatures $T \ll \lambda$. In Eq. (13) the temperatures $T \lesssim \Delta_0 \ll \lambda$ are considered, such that Q_1, Q_2, and $\mathcal{G}$ can be taken at zero temperature.

It is evident from (15) that Q as a function of $(\epsilon - \omega)$ is the difference of the retarded and advanced functions. For this reason, Q_0 vanishes for $\epsilon - \omega = 0$, that is, one can write for $|\epsilon - \omega| \ll \lambda$:

$$Q_2(\varepsilon, \omega; \mathbf{p}, \mathbf{q}) = f(\varepsilon; \mathbf{p}, \mathbf{q})(\omega - \varepsilon)/\lambda. \tag{16}$$

Making use of this property, we shall show that one can make the following substitution in Eq. (13) in the term with Q_2:

$$\begin{aligned} & \mathrm{cth}\,[(\omega - \varepsilon)/2T] \to \mathrm{sign}\,(\omega - \varepsilon), \\ & F_R(q, \omega) \to -\Sigma_{2R}(q, \omega)\, \mathcal{G}_R(q, \omega)\, \mathcal{G}_A(q, -\omega). \end{aligned} \tag{17}$$

Let us consider the case $\epsilon = 0$. In Eq. (13) we separate in the integral with Q_2 the interval $-\omega_1 < \omega < \omega_1$ ($\Delta_0 \ll \omega_1 \ll \lambda$). The functions $\coth(\omega/2T)$ and $\mathrm{sign}\,\omega$ are essentially different only in the narrow interval $\omega \lesssim T \ll \omega_1$ while in the expression (10) Δ in the denominator plays a role in a layer $\sim \Delta_0/\omega_1$. Therefore, outside the interval $(-\omega_1, \omega_1)$, one can make the substitution (17) with accuracy up to terms $\sim \Delta_0/\omega_1$.

We write the integral with Q_2 in the interval $(-\omega_1, \omega_1)$ (we omit the constants) in the form

$$\int d^3q f(0; \mathbf{p}, \mathbf{q}) \int_{-\omega_1}^{\omega_1} d\omega \frac{\omega}{\lambda} \frac{\mathrm{cth}(\omega/2T)}{(\varepsilon_q - \omega - i\delta)(\varepsilon_q + \omega + i\delta)},$$

where Eqs. (16) and (9) are used. One can establish the fact that this integral is independent of T and Δ with accuracy up to terms $\sim \Delta_0/\omega_1$. Thus for $\epsilon = 0$, the substitution (17) is valid. Making use of the property (16), we can show that this is also true for arbitrary ϵ.

Making the substitution (17) in Eq. (13), we get

$$\Sigma_{2R}(p, \varepsilon) = -\int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega}{4\pi i} \Sigma_{2R}(q, \omega) P(\varepsilon, \omega; \mathbf{p}, \mathbf{q}; T), \tag{18}$$

$$P(\varepsilon, \omega; \mathbf{p}, \mathbf{q}; T) \equiv \frac{\mathrm{th}(\omega/2T)\, Q_1(\varepsilon, \omega; \mathbf{p}, \mathbf{q})}{\mathcal{G}_R^{-1}(q, \omega)\, \mathcal{G}_A^{-1}(q, -\omega) + (\Delta/a)^2} + \mathrm{sign}\,(\omega - \varepsilon)\, Q_2(\varepsilon, \omega; \mathbf{p}, \mathbf{q})\, \mathcal{G}_R(q, \omega)\, \mathcal{G}_A(q, -\omega). \tag{19}$$

It is seen from (19) that the function P differs essentially from its value at T = 0 in a narrow region ω, $v\,|\,q - p_0\,| \lesssim \Delta_0$. It is therefore expedient to rewrite Eq. (18) in the form

$$\Sigma_{2R}(p,\varepsilon)=\int\frac{d^3q}{(2\pi)^3}\int\frac{d\omega}{4\pi i}\Sigma_{2R}(q,\omega)\{P(\varepsilon,\omega;\mathbf{p},\mathbf{q};0)-P(\varepsilon,\omega;\mathbf{p},\mathbf{q};T)\}-\int\frac{d^3q}{(2\pi)^3}\int\frac{d\omega}{4\pi i}\Sigma_{2R}(q,\omega)P(\varepsilon,\omega;\mathbf{p},\mathbf{q};0). \quad (20)$$

Computing the first term on the right in this equation, we get

$$\Sigma_{2R}(p,\varepsilon)=-\Sigma_{2R}(p_0,0)\,\varphi(T,\Delta)\,\chi(p,\varepsilon)-\int\frac{d^3q}{(2\pi)^3}\int\frac{d\omega}{4\pi i}\Sigma_{2R}(q,\omega)P(\varepsilon,\omega;\mathbf{p},\mathbf{q};0); \quad (21)$$

$$\chi(p,\varepsilon)=\frac{a^2p_0}{vp}\int\limits_{|p-p_0|}^{p+p_0}\frac{p_1dp_1}{(2\pi)^2}Q_1'(\varepsilon;p,p_1),$$

$$Q_1'(\varepsilon;p,p_1)\equiv Q_1(\varepsilon,0;\mathbf{p},\mathbf{p}_0)\qquad(p_1\equiv|\mathbf{p}-\mathbf{p}_0|),$$

$$\varphi(T,\Delta)=\int\limits_0^\infty dx\left\{\frac{\operatorname{th}(\sqrt{x^2+\Delta^2}/2T)}{\sqrt{x^2+\Delta^2}}-\frac{1}{\sqrt{x^2+\Delta_0^2}}\right\}. \quad (22)$$

Let us make clear the structure of Eqs. (20) and (21). As has been noted, the difference $P(\epsilon, \omega; \mathbf{p}, \mathbf{q}; T) - P(\epsilon, \omega; \mathbf{p}, \mathbf{q}; 0)$ is important in the narrow range ω, $v\,|\,q - p_0\,| \lesssim \Delta_0$, which makes it possible to compute the first term on the right in (20). As is seen from the result [Eq. (21)], this term is a product of factors of two types: the function $\chi(p, \epsilon)$, which does not depend on T, and the function $\Sigma_{2R}(p_0, 0)\,\phi(T, \Delta)$, which does not depend on p, ϵ. It then follows that the dependences on T and p, ϵ are eliminated in Σ_{2R}, that is, Σ_{2R} has the form

$$\Sigma_{2R}(p,\varepsilon)=(\Delta/\Delta_0)\,\Sigma_{2R}^0(p,\varepsilon), \quad (23)$$

where $\Sigma_{2R}^0(p, \epsilon) = \Sigma_{2R}(p, \epsilon)\,|_{T=0}$. Substituting (23) in (21), we have

$$\varphi(T,\Delta)=0. \quad (24)$$

Equation (24) determines the gap as a function of temperature and is identical with the equation obtained in the weak coupling approximation. This can be made clear in the following manner. Upon satisfaction of the condition (4), the reduction of the gap with increasing temperature is evidently associated only with the diffusion of the distribution of quasiparticles (which are well defined at low temperatures). Therefore there is no difference from the result obtained in the weak coupling approximation.

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APPENDIX

In superconductivity theory, according to Bogolyubov, [6] one makes use of the Hamiltonian

$$H-\nu\sum_{\mathbf{p}}(a^+_{\mathbf{p}^1/_2}a^+_{-\mathbf{p}-^1/_2}+a_{-\mathbf{p}-^1/_2}a_{\mathbf{p}^1/_2})\qquad(\nu\to 0). \quad (A\text{-}1)$$

If the interaction does not depend on the spin, then this Hamiltonian is invariant relative to the substitution

$$a_{\mathbf{p}\sigma}\to a_{\mathbf{p},-\sigma}\operatorname{sign}\sigma,\qquad a^+_{\mathbf{p}\sigma}\to a^+_{\mathbf{p},-\sigma}\operatorname{sign}\sigma. \quad (A\text{-}2)$$

Making use of the invariance of the Hamiltonian (A-1) relative to the substitution (A-2), we get

$$\langle T_\tau a_{\mathbf{p}^1/_2}(\tau_1)\,a_{-\mathbf{p}-^1/_2}(\tau_2)\rangle=-\langle T_\tau a_{\mathbf{p}-^1/_2}(\tau_1)\,a_{-\mathbf{p}^1/_2}(\tau_2)\rangle. \quad (A\text{-}3)$$

Direct confirmation of the validity of the equation

$$\langle T_\tau a_{\mathbf{p}-^1/_2}(\tau_1)\,a_{-\mathbf{p}^1/_2}(\tau_2)\rangle=-\langle T_\tau a_{-\mathbf{p}^1/_2}(-\tau_1)\,a_{\mathbf{p}-^1/_2}(-\tau_2)\rangle \quad (A\text{-}4)$$

can be established. But by comparing (A-3) and (A-4) and recalling the definition of the F-function (2), we get

$$F(p,\tau)=F(p,-\tau). \quad (A\text{-}5)$$

Similar behavior exists for F^+. In the frequency representation, (A-5) takes the following form:

$$F(p,\varepsilon_n)=F(p,-\varepsilon_n);\qquad F_R(p,\varepsilon)=F_A(p,-\varepsilon). \quad (A\text{-}6)$$

We write down the spectral decompositions of F and F^+:

$$F(p,\varepsilon_n)=\int dE\,I(p,E)\frac{1+e^{-E\beta}}{E-i\varepsilon_n},$$

$$F^+(p,\varepsilon_n)=-\int dE\,I^*(p,E)\frac{1+e^{-E\beta}}{E-i\varepsilon_n},$$

$$I(p,E)=\sum_{nm}e^{(\Omega-E_n)\beta}(a_{\mathbf{p}^1/_2})_{nm}(a_{-\mathbf{p}-^1/_2})_{mn}\,\delta(E-E_m+E_n), \quad (A\text{-}7)$$

where $\beta = 1/T$, E_n is the eigenvalue of H. It is seen from (A-7) that there exists the following connection between F and F^+:

$$F^*(p,\varepsilon_n)=-F^+(p,\varepsilon_n),\quad F_A^*(p,\varepsilon)=-F_R^+(p,\varepsilon). \quad (A\text{-}8)$$

With the help of Eq. (5), one can satisfy oneself that relations of the type (A-6) and (A-8) also hold for Σ_2, Σ_2^+. In particular, $\Sigma_{2R}^+(p, \epsilon) = -\Sigma_{2R}^*(p, -\epsilon)$.

[1] Bardeen, Cooper and Schrieffer, Phys. Rev. **108**, 1175 (1957).

[2] L. P. Gor'kov, JETP **34**, 735 (1958), Soviet Phys. **7**, 505 (1958).

[3] Abrikosov, Gor'kov and Dzyaloshinskiĭ, JETP **36**, 900 (1959), Soviet Phys. JETP **9**, 636 (1959), E. S. Fradkin, JETP **36**, 1286 (1959), Soviet Phys. JETP **9**, 912 (1959).

[4] Abrikosov, Gor'kov and Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheckoĭ fizike (Methods of Quantum Field Theory in Statistical Physics) Fizmatgiz, 1962.
[5] G. M. Eliashberg, JETP **41**, 1241 (1961), Soviet Phys. JETP **14**, 886 (1962).
[6] N. N. Bogolyukov, Kvayzisrednie v zadachakh staticheskoĭ mekhaniki (Quasi-averages in Problems of Statistical Mechanics) (preprint, Joint Inst. Nuc. Res., 1961).

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