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ON THE INTERPRETATION OF AN EXPERIMENT WITH HIGH-ENERGY NEUTRINOS

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RECENTLY there were published the results of the first experiment with high-energy neutrinos, which was made with the 32 BeV accelerator at Brookhaven.[1] The main results of this important research were: 1) the establishment of the fact that the number of muons produced by neutrinos from the decay $\pi^{\pm} \rightarrow \mu^{\pm} + \nu(\bar{\nu})$ is much larger than the number of electrons (more exactly, with the limited statistics, not one case definitely associated with the production of an electron was observed); 2) an approximate estimate of the cross sections of reactions induced by high-energy neutrinos.

That muons predominated over electrons was evidence of the existence of two kinds of neutrinos, ν_e and ν_μ. In the discussion in [1], and also in papers by Pontecorvo,[2] Markov,[3] and Schwartz [4] (which also suggested that such an experiment be done), it was assumed that in the framework of the one-neutrino hypothesis one should expect equal numbers of muons and electrons in such an experiment.

We wish here to call attention to the fact that even in the framework of the one-neutrino hypothesis the number of muons can exceed the number of electrons, and that additional experiments are needed to settle more reliably the question about muon and electron neutrinos.

The "elastic" processes of interaction with free nucleons

$$\nu + n \rightarrow p + l^- \quad (l = \mu, e), \tag{1}$$

$$\bar{\nu} + p \rightarrow n + l^+ \tag{2}$$

have been theoretically treated earlier by a number of authors.[5-8] Strong-interaction effects produce a serious uncertainty in the predictions. As has been shown by Goldberger and Treiman,[9] in the framework of the one-neutrino universal theory of weak interactions the matrix elements of processes (1) and (2) can be expressed in first approximation in the weak-interaction constant (subject to the validity of CP invariance and the $|\Delta I| = 1$ rule[10]) in terms of four form-factors: $F_{1V}(q^2)$, $F_{2V}(q^2)$, $F_A(q^2)$, and $F_P(q^2)$, which are introduced to correspond to the general expression for the matrix element [reaction (1)]

$$\bar{u}_p \left[F_{1V} \gamma_\alpha + \frac{\mu}{2M} F_{2V} (p-n)_\beta \sigma_{\alpha\beta} + \lambda F_A \gamma_\alpha \gamma_5 + ibF_P (p-n)_\alpha \gamma_5 \right] u_n \bar{u}_e \gamma_\alpha (1+\gamma_5) u_\nu, \tag{3}$$

where the quantity $q^2 = (p-n)^2 = (l-\nu)^2$ is the momentum transfer, and the other notations are the usual ones.

It is easy to see by using the Dirac equation that the contribution of the induced pseudoscalar is proportional to the mass of the lepton.

If the vector current is conserved in weak interactions F_{1V} and F_{2V} are the isovector parts of the Dirac and Pauli form-factors of the nucleon.[1)] Concerning the axial form-factor λF_A the only thing known beyond general indications from dispersion relations is its value at $q^2 = 0$. There is a well known estimate[9,11] of the pole contribution to the form-factor bF_P of the induced pseudoscalar interaction. For the most part estimates are made on the assumption that all form-factors have the same dependence on q^2. It is not hard to see that then at neutrino energies of about 1 BeV (which is close to the conditions of the Brookhaven experiment) the contribution to the cross section proportional to F_{1V}, F_{2V}, and F_A is about the same for electrons and muons in reactions (1) and (2), and the actual value of the cross section for reaction (2) is about a third of that for reaction (1). On the other hand, inclusion of the contribution of the pseudoscalar F_P leads to a great preponderance of muon production, giving electrons in about the same proportion as for the decays $\pi \rightarrow e + \nu$ and $\pi \rightarrow \mu + \nu$.

For a comparative estimate of the contributions of the various form-factors we use the results of Yamaguchi,[7] which he derived for form-factors decreasing in the same way, of the form $F_{1V} = F_{2V} = F_A = F_P = (1 + r_0^2 q^2/12)^{-2}$ with $r_0 = 0.8 \times 10^{-13}$ cm. As a function of the neutrino energy the contribution of the pseudoscalar is a maximum at $E_\nu \approx M$. For $E_\nu = M$ the contribution of F_{1V}, F_{2V}, F_A (in units 10^{-38} cm^2) on these assumptions is about 0.85 for the cross section of process (1) and 0.33 for the cross section of process (2). For $G_P = 8G_A$ the contribution of F_P, which is the same for both processes, is 0.17. An increase of G_P by a factor three as compared with the pole estimate increases the contribution of the pseudoscalar interaction by almost a factor 10 (the interference of the pseudoscalar contribution with the axial-vector interaction is small), which leads to predominance of the number of muons over that of electrons by about a factor two and one-half for reaction (1) and six or seven for reaction (2). From this point of view experiments with muon neutrinos are to be preferred over those with antineutrinos.

Thus under the conditions of the Brookhaven experiment, with about equal numbers of neutrinos and antineutrinos in the beam, in the framework of the one-neutrino hypothesis and without contradiction with the known experimental data one can get a fivefold preponderance of muons over electrons. We emphasize that these estimates are obtained only at the price of increasing the pseudoscalar interaction constant, with the assumption that all form-factors show the same dependence.

At present we can evidently not exclude the possibility that the axial form-factor falls off more rapidly than the others as q^2 increases. Different dependences of the form-factors on q^2 can change the estimate of the ratio of the numbers of muons and electrons by a further factor of two or three without increasing the total cross section.

It is interesting to note that in experiments with electron neutrinos, for which the effectiveness of the pseudoscalar interaction is sharply diminished, the numbers of electrons and muons are equal in the framework of the one-neutrino theory.

It is clear that a five-fold increase of the number of muons on account of a large pseudoscalar interaction constant leads to a corresponding increase of the absolute value of the cross section, and improvement of the experimental data on cross sections is very desirable.

Because the experiment is made with nuclei it is probably also necessary to undertake a more detailed examination of the effects of the nucleus, for example taking into account correlations in the Fermi gas, in analogy with the way Glauber has done this for the scattering problem.[12]

It would be desirable to do experiments to determine independently the pseudoscalar contribution to the weak interaction of muons at high energies. For this purpose one can use the fact that the contribution of the pseudoscalar to the cross sections of reactions (1) and (2) decreases sharply (going to zero in the approximation $v_\mu = 1$) for forward angles of emergence of the muons. The magnitude of the pseudoscalar contribution is a maximum at about $E_\nu = M$ and falls off with increase or decrease of the energy. Because of this an experiment at larger neutrino energies would be desirable.

More accurate information about the size of the pseudoscalar in the capture of muons by nucleons at small energies would be a great help in clearing up the question.

Finally, it is obvious that from the point of view of the analysis the clearest experiments would be those with electron neutrinos, for which it will be possible to make sources only after the intensity of accelerated particles is much increased.

The two-neutrino hypothesis is attractive from the point of view of understanding the reasons that decay processes of the types $\mu \to e + \gamma, 3e$ are forbidden. We emphasize, however, that the theory of the weak interaction with symmetric neutral currents[13-16] also guarantees that such processes are forbidden, while leaving room for weak interactions to play a great part in astrophysical phenomena because of the direct neutrino-nucleon interaction.

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[1)]It is interesting to note that in principle one will have to get the asymptotic behavior of the vector form-factors, from the point of view of higher approximations, from high-energy neutrino experiments.

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INFRARED SINGULARITIES AND REGGE TRAJECTORIES IN ELECTRODYNAMICS

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IN this note we consider the implication of the previously[1] obtained dispersion relation for photon-electron scattering on the Regge trajectory for the electron-positron interaction, and also the generalization of it to the case of particles with unequal masses.

If the matrix element M_λ for photon-electron scattering, calculated by introducing a photon "mass" $\sqrt{\lambda}$ into the photon Green's function, is expressed in the form 1)

$$M_\lambda = \exp[F(t)]\,M, \qquad (1)$$

where

$$F((p'-p)^2) = \frac{i\alpha}{8\pi^3}\int\frac{dk}{k^2-\lambda}\left(\frac{2p'-k}{2p'k-k^2}-\frac{2p-k}{2pk-k^2}\right)^2, \qquad (2)$$

then, as was shown in [1], the following relation may be written for M (where m is the electron mass):

$$M = \sum_{b=s,\,u}\frac{A_b}{b-m^2}\exp\left[\beta(t)\ln\frac{m^2-b}{m^2}+\gamma(t)\right]+M_a, \qquad (3)$$

where $A_b/(b-m^2)$, $b = s, u$, are the Born terms corresponding to two second order diagrams in which the anomalous magnetic moment of the electron has been taken into account, and β and γ are power series in α; at that, in lowest order

$$\beta(t) = \frac{\alpha}{\pi}t\int_{4m^2}^{\infty}\frac{t'-2m^2}{\sqrt{t'(t'-4m^2)}}\frac{dt'}{t'(t'-t-i\varepsilon)}. \qquad (4)$$

The quantity M_a (more precisely, its invariant structure coefficients) at least in lowest (fourth) order of perturbation theory is an analytic function of s, u, t, satisfying the Mandelstam representation with cuts as singularities.

It is seen that the first term in Eq. (3) is for large s of the Regge [2] type with an exponent

$$\alpha(t) = -1+\beta(t). \qquad (5)$$

It is reasonable to suppose that the second term in Eq. (3) (M_a) can only give rise to higher order corrections to this expression. The behavior of the quantity, Eq. (5), (the Regge trajectory) is shown schematically in the figure.

The Regge equation

$$\alpha(t) = l, \quad l = 0, 1, 2, \ldots \qquad (6)$$

determines the bound states in the t channel, i.e., bound states of the electron-positron system. It has solutions only for $0 < t < 4m^2$, and at that

$$\alpha(t) = -1+\frac{\alpha}{\pi}\left[1+\frac{2t-4m^2}{\sqrt{t(4m^2-t)}}\tan^{-1}\sqrt{\frac{t}{4m^2-t}}\right]. \qquad (7)$$

In the nonrelativistic approximation ($m \to \infty$) this expression goes over into

$$\alpha((2m+E)^2) = -1+\alpha\sqrt{m/(-4E)}, \qquad (8)$$