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INFRARED SINGULARITIES AND REGGE TRAJECTORIES IN ELECTRODYNAMICS

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In this note we consider the implication of the previously[1] obtained dispersion relation for photon-electron scattering on the Regge trajectory for the electron-positron interaction, and also the generalization of it to the case of particles with unequal masses.

If the matrix element M_λ for photon-electron scattering, calculated by introducing a photon "mass" $\sqrt{\lambda}$ into the photon Green's function, is expressed in the form 1)

$$M_\lambda = \exp\left[F(t)\right] M, \tag{1}$$

where

$$F((p'-p)^2) = \frac{i\alpha}{8\pi^3}\int \frac{dk}{k^2-\lambda}\left(\frac{2p'-k}{2p'k-k^2} - \frac{2p-k}{2pk-k^2}\right)^2, \tag{2}$$

then, as was shown in [1], the following relation may be written for M (where m is the electron mass):

$$M = \sum_{b=s,\,u} \frac{A_b}{b-m^2}\exp\left[\beta(t)\ln\frac{m^2-b}{m^2} + \gamma(t)\right] + M_a, \tag{3}$$

where $A_b/(b-m^2)$, $b = s, u$, are the Born terms corresponding to two second order diagrams in which the anomalous magnetic moment of the electron has been taken into account, and β and γ are power series in α; at that, in lowest order

$$\beta(t) = \frac{\alpha}{\pi}\, t \int_{4m^2}^{\infty} \frac{t'-2m^2}{\sqrt{t'(t'-4m^2)}}\,\frac{dt'}{t'(t'-t-i\varepsilon)}\,. \tag{4}$$

The quantity M_a (more precisely, its invariant structure coefficients) at least in lowest (fourth) order of perturbation theory is an analytic function of s, u, t, satisfying the Mandelstam representation with cuts as singularities.

It is seen that the first term in Eq. (3) is for large s of the Regge[2] type with an exponent

$$\alpha(t) = -1 + \beta(t). \tag{5}$$

It is reasonable to suppose that the second term in Eq. (3) (M_a) can only give rise to higher order corrections to this expression. The behavior of the quantity, Eq. (5), (the Regge trajectory) is shown schematically in the figure.

The Regge equation

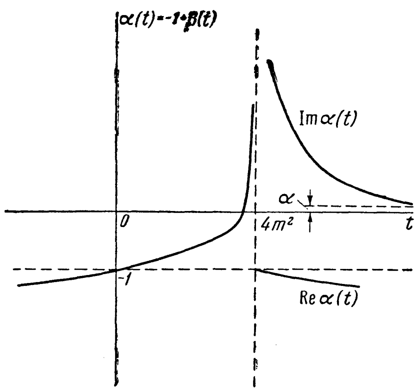
$$\alpha(t) = l, \quad l = 0, 1, 2, \ldots \tag{6}$$

determines the bound states in the t channel, i.e., bound states of the electron-positron system. It has solutions only for $0 < t < 4m^2$, and at that

$$\alpha(t) = -1 + \frac{\alpha}{\pi}\left[1 + \frac{2t-4m^2}{\sqrt{t(4m^2-t)}}\tan^{-1}\sqrt{\frac{t}{4m^2-t}}\right]. \tag{7}$$

In the nonrelativistic approximation ($m \to \infty$) this expression goes over into

$$\alpha((2m+E)^2) = -1 + \alpha\sqrt{m/(-4E)}, \tag{8}$$



which corresponds to the Coulomb levels of the electron-positron system with the radial quantum number equal to zero.

These formulas may be generalized to the case of particles with unequal masses by means of dimensional analysis.

Let us consider the scattering of a particle of charge ze, mass m, and momentum p before and p′ after the reaction, by a particle for which the corresponding quantities are Ze, M, P and P′. In order to determine the bound states of these particles one must, following Regge, find the asymptotic behavior of the matrix element M_λ for the scattering of these particles as $t \to \infty$. By dimensional analysis it will contain a term of the form $t^{-1}(t/\lambda)^{\beta(s)}$. Therefore in order to determine $\beta(s)$ it is sufficient to consider the infrared singularities of M_λ, which are given by Eq. (1) with F given by the expression (see [3])

$$-2zZ[F((p+P)^2)-F((p'-P)^2)] + z^2F((p-p')^2)+Z^2F((P-P')^2). \quad (9)$$

The exponent β is the coefficient of $\ln(1/\lambda)$ in this expression $(t \to \infty)$:

$$\beta = -zZ\frac{\alpha}{\pi}[\varphi(s)+1] + \frac{\alpha}{2\pi}\left[(z+Z)^2\left(\ln\frac{m^2}{t}+1\right)+2Z(z+Z)\ln\frac{M}{m}\right], \quad (10)$$

$$\varphi(s) = (s-m^2-M^2)\int_{(M+m)^2}^{\infty} ds'/\sqrt{k(s')}(s'-s-i\varepsilon), \quad (11)$$

$$k(s) = [s-(m-M)^2][s-(m+M)^2]. \quad (12)$$

We see that it is independent of t only in the case when the particles have opposite charges: $z = -Z$. In that case (for $z = \pm 1$) the Regge exponent is equal to

$$\alpha(s) = -1+\beta(s), \quad \beta(s) = \frac{\alpha}{\pi}[\varphi(s)+1]. \quad (13)$$

For $(m-M)^2 < s < (m+M)^2$ it is equal to

$$\alpha(s) = -1+\frac{\alpha}{\pi}\left[1+2\frac{s-m^2-M^2}{\sqrt{-k(s)}}\operatorname{arctg}\frac{s-(m-M)^2}{\sqrt{-k(s)}}\right]. \quad (14)$$

If one of the particles is at rest $(M \to \infty)$ and the other has energy E and $p^2 = m^2 - E^2$, then this formula gives for $|E| < m$

$$\alpha((M+E)^2) = -1+\frac{\alpha}{\pi}\left[1+\frac{2E}{p}\operatorname{arctg}\frac{m+E}{p}\right]. \quad (15)$$

For p close to zero this expression goes over into the Sommerfeld formula accurate to terms of order $[p/(m+E)]^2$.

The method here discussed allows one to obtain the leading Regge trajectory accurate to order α, which allows one to describe the Coulomb interaction. This conclusion agrees with that of Arbusov, Logunov et al [4] to the effect that the form of the Regge trajectory accurate to low orders in the coupling constant may be obtained in the framework of field theory. The rendition, however, of finer details of the interaction requires finer methods, as has been analyzed in [5].

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1)s, u, t denote the Mandelstam variables respectively for the direct, crossed and third channels; α is the fine structure constant, the system of units is such that $\hbar = c = 1$, the metric is such that $ab = a^0b^0 - \mathbf{a}\cdot\mathbf{b}$.

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GAMMA-QUANTUM PRODUCTION IN THE INTERACTION BETWEEN 7-BeV NEGATIVE PIONS AND NUCLEONS

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WE have previously obtained[1] the general characteristics of inelastic π^-N interactions in a 24-