Then μH_c is of the order

$$\mu H_c \sim T_c \sqrt{\frac{v_F}{c}\frac{e^2}{\hbar c}} \ll T_c.$$

As shown by the present author,[4] this situation occurs only in metals of the "Pippard" type for which the depth of penetration in a weak field is $\delta_p \ll \xi_0 = \hbar v_F/T_c$. Metals of "London" type transform from the normal to the superconducting state by a phase transition of the second kind in a field $H_{c2} > H_c$, where

$$H_{c2} \sim H_c\delta_L/\xi_0, \qquad \delta_L^{-1} = \sqrt{4\pi Ne^2/mc^2},$$

δ_L is the "London" depth of penetration $(\delta_L > \xi_0)$. (At the moment of the transition there is no Meissner effect. For metals of "Pippard" type the field is $H_{c2} < H_c$ and has the meaning of the supercooling field.)

Substituting Eq. (2) we find the critical field for pure "London" superconductors (among which are obviously[4] such materials as pure La, V, Nb, Ta and others, which do not obey the Rutgers formula):

$$\mu H_{c2} \sim T_c(T_c/\varepsilon_F) \ll T_c.$$

Thus also in these pure superconductors the critical field never exceeds 10^2—10^3 G.

The situation is different in superconducting alloys. The present author[5] showed that for alloys with an electron mean free path $l \ll \delta_p$ (i.e., a path which is small compared with the depth of penetration in a weak field) we have:

$$H_{c2} \sim H_c\delta_L/l.$$

The transition in a magnetic field in alloys is always of the second kind.

To make our estimate we shall use the expression for the field H_{c2} at $T = 0$ obtained by Shapoval[6]:

$$H_{c2} = 1.5\, cT_c/elv_F, \qquad \mu H_{c2} = 1.5T_c(\hbar/p_Fl). \tag{3}$$

The validity of the above formulas is limited to the region $p_Fl \gg \hbar$, i.e., to defect concentrations for which the mean free path l is large compared with the interatomic distances $a \approx \hbar/p_F$. To estimate the upper limit of the critical field in alloys we shall assume that $\hbar/p_Fl \approx 1$, whence we obtain, using Eq. (1), $H_{c\,max} \approx 10^4T^0$ G. We emphasize once more that such high fields ($\approx 10^5$ G) can in principle occur only in alloys. The less careful the preparation of these alloys the higher the critical field. The electron mean free path or the residual resistance of a sample can be used as a measure of its critical field. In particular for relatively low defect concentrations Eq. (3) can be rewritten in the form[6]

$$H_{c2} = \frac{3}{2\pi}\frac{ec\gamma T_c}{\sigma k},$$

where γ is the coefficient in the linear law for the electronic specific heat of a unit volume and σ is the conductivity. When the mean free path becomes comparable with the atomic distances we reach the upper limit of the critical field for alloys. As far as the author is aware the conductivity of such alloys has not been measured. It would be interesting to investigate in what region the law of proportionality of the critical field and the residual resistance is valid. If we had a graph of the dependence of H_c on $\rho = \sigma^{-1}$, which should be a curve with saturation, we could quite accurately predict the upper limit of the critical magnetic field for superconducting alloys.

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ABSORPTION OF SOUND IN LIQUID He II BELOW 0.6°K

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WOODRUFF[1] and Dransfeld[2] made an attempt to explain the absorption of sound in superfluid He II at temperatures below 0.6°K. At such low temperatures we are dealing with a special case. On one hand the mean free path of phonons is considerably greater than the wavelength of sound, but on the other the energy of the sound quanta $\hbar\omega$ is still small compared with the energy of thermal phonons kT. The long mean free path ensures that the indeterminacy in the energy of the acoustic phonon is small and this allows us to consider direct absorption of an acoustic phonon by thermal phonons. Woodruff and Fransfeld assumed that the acoustic phonon is absorbed by a three-phonon process. However, it is well known that the phonon spectrum in He II is stable against phonon decay and, therefore, the three-phonon process is forbidden. At low momenta the energy spectrum of phonons departs from linearity and is given by the approximate formula[3]

$$\varepsilon(p) = cp(1 - \gamma p^2), \tag{1}$$

where $\epsilon(p)$ is the phonon energy, p is its momentum, and c is the velocity of sound. With this sign of phonon dispersion, the three-phonon process is forbidden because it is not then possible to satisfy simultaneously the laws of conservation of energy and momentum (the phonon velocity is always smaller than the velocity of sound!). Consequently the calculations of Woodruff and Dransfeld, based on the three-phonon process, are not quite correct.

The acoustic phonon can be absorbed in He II only by a four-phonon process. Such processes have been considered earlier (cf. [3]). Of the possible processes the important one is that shown in Fig. 1, where q is the momentum of the acoustic phonon; p, p′, p_1 are the momenta of the thermal phonons; $|q| \ll p$. The lifetime τ of an acoustic phonon of frequency ω in such a process is given by[3]:

$$\frac{1}{\tau} = \frac{5(u+1)^4}{8\pi^3(\rho c)^2\gamma}\left(\frac{kT}{\hbar c}\right)^6 \omega. \tag{2}$$

Here $u = (\partial c/\partial \rho)(\rho/c) \approx 3$, and ρ is the density of He II. The absorption coefficient of sound is equal to

$$\alpha = \frac{1}{\tau c} = \frac{5(u+1)^4}{8\pi^3(\rho c)^2\gamma}\left(\frac{kT}{\hbar c}\right)^6 \frac{\omega}{c}, \tag{3}$$

i.e., it is proportional to T^6 and the first power of the frequency ω (Woodruff and Dransfeld obtained a different temperature dependence with α proportional to T^4).

The available data of Chase and Herlin[4] on the absorption of sound at 12 Mc do not contradict the T^6 law at T < 0.4°K, as shown in Fig. 2. A quantitative comparison of the experimental results with Eq. (3) is somewhat difficult because the quantity γ, representing the dispersion law of phonons, is not well known.[1)]

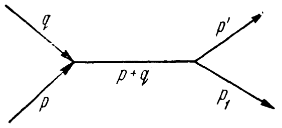

FIG. 1

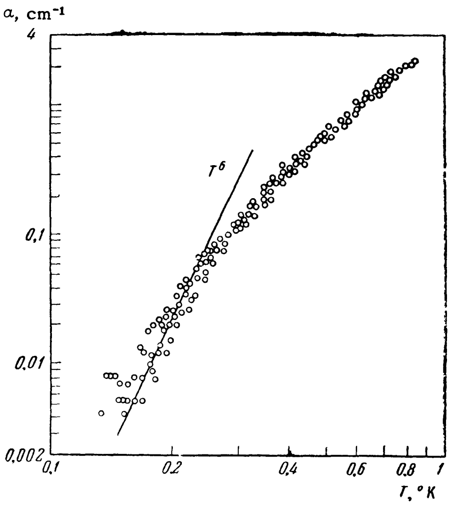

FIG. 2. Temperature dependence of the absorption coefficient of sound according to Chase and Herlin.[4]

It is difficult to obtain any information about this quantity from neutron diffraction because its magnitude is small. However, the recent results of Henshaw and Woods[5] indicate that in any case the dispersion is very slight at low phonon momenta. Quantitative agreement with the results of Chase and Herlin at T = 0.2°K is obtained by assuming that $\gamma = 1.5 \times 10^{35}$ (g.cm/sec)$^{-2}$.

The temperature variation of α obtained above is asymptotic as $T \rightarrow 0$. At temperatures of the order of 0.3°K the indeterminacy in the energy $\hbar/\tau$ becomes of the same order as the nonlinear correction in the law of energy conservation for the three-phonon process, $3\gamma p^2\hbar\omega$; in the estimate we assume that $p \approx 3kT/c$ (cf. [6]).

Consequently from this temperature (0.3°K) upwards the absorption of sound is governed not only by the four-phonon process but also by the three-phonon mechanism. The resultant complex situation needs special consideration.

The author is grateful to R. G. Arkhipov and I. E. Dzyaloshinskiĭ for valuable discussions.

[1)] An estimate obtained in[3] by interpolation of the total energy curve (including rotons) gives values of γ which are obviously too high.

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MINIMUM OF RESISTANCE IN IMPURITY-FREE CONDUCTORS

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As is well known, interelectronic collisions in sufficiently pure conductors, can become appreciable at low temperatures. This is connected with the fact that the frequency of the collisions between electrons decreases with the temperature much more slowly than the frequency of the electron-phonon collisions. However, collisions between electrons give rise by themselves to electric resistivity only if umklapp processes are taken into account (Peierls), for in the opposite case these collisions do not lead to a change in the summary momentum of the electrons. On the other hand, umklapp processes are possible only if certain conditions are satisfied. Roughly speaking, it is necessary that the Fermi surface reach the center of the Brillouin cell. In the opposite case the probability of a collision accompanied by an umklapp process is exponentially small. It is clear that at low temperatures umklapp processes are practically impossible in many metals and semiconductors. In this case the electric resistivity is obviously determined by the loss of momentum when the electrons are scattered by different inhomogeneities of the lattice (impurity atoms, phonons, dislocations, etc.) and on the boundaries of the specimen.

If l_V is the mean free path connected with the scattering by the inhomogeneities, and d is the characteristic transverse dimension of the conductor, then the resistance for $l_V \gg d$ is determined essentially by collisions between the electrons and the boundaries and, it might appear, by the effective mean free path $l_{\text{eff}} \sim d$. Actually, however, the interelectron collisions can greatly influence the process of momentum transfer to the boundaries. This is simplest to understand in the limiting case

$$d \gg l_{ee}, \quad l_V \to \infty, \tag{1}$$

where l_{ee} is the length characterizing the interelectron collisions without the umklapp processes. When $l_V, l_{ee} \gg d$, an electron situated deep in the specimen reaches the wall practically without collisions during a time $\tau \sim d/v_0$, and, accordingly, $l_{\text{eff}} \sim d$ (v_0 is the limiting velocity). Then, as in the case (1), the electron, moving like a Brownian particle, covers a distance $\sim d$ during a time $\tau \sim d^2/l_{ee}v_0$ (since $d \sim \sqrt{\tau l v_0}$), and consequently

$$l_{\text{eff}} \sim d^2/l_{ee}. \tag{2}$$

This result can also be obtained by starting from a hydrodynamic description of the electron gas, introducing the kinematic viscosity $\nu \cong v_0 l_{ee}/3$.

The hydrodynamic description is convenient because it is applicable for any ratio of l_V to d, provided $l_{ee} \ll d, l_V$; consequently, this description enables us to take into account volume collisions, too. The corresponding equation for the average electron velocity v differs from the Navier-Stokes equation for stationary processes only in the term v/τ_V, which describes the volume collisions:

$$e\mathbf{E}/m = \nu\Delta\mathbf{v} + \mathbf{v}/\tau_V.$$

Let us consider by way of an example the result of the solution of this equation for a wire of radius r in a longitudinal electric field **E**. The electric conductivity is

$$\sigma = \frac{ne^2}{p_0} l_{\text{eff}}, \qquad l_{\text{eff}} = l_V\left[1 - \frac{2}{x}\frac{I_1(x)}{I_0(x)}\right],$$

where n is the electron density, p_0 the limiting momentum, $x = r\sqrt{3}\,(l_{ee}l_V)^{-1/2}$, and $I_n(x)$ are Bessel functions of the imaginary argument. For $x \ll 1$ we have $l_{\text{eff}} \cong l_V x^2/8 = 3r^2/8l_{ee}$, in accordance with (2). The figure shows the approximate course of the temperature dependence of the resistance R(T) for the wire. In the region of the lowest temperatures, so long as $l_{ee} \gg r$ we have $l_{\text{eff}} \sim d$ and the resistance is constant. Then, starting with the temperature T_1, at which $l_{ee} \sim r$, up to a temperature T_2, for which $l_V \sim r^2/l_{ee}$, the resistance decreases quadratically with increasing temperature [according to (2); $l_{ee} \sim a(\epsilon_0/T)^2$,