A DIFFRACTION MECHANISM FOR THE PROCESS INVERSE TO THE $\pi \to \mu + \nu$ DECAY IN THE FIELD OF A NUCLEUS

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The process $\nu + A \to \pi + \mu + A$ is studied for high-energy neutrinos. It is shown that the main contribution to the cross section for this process is determined by the strong interaction of the pion with the nucleus, which is taken into account phenomenologically by using the formulae describing diffraction on a "black sphere."

In connection with the program of experimental investigations of the interactions of neutrinos with matter sketched by Pontecorvo [1], there have appeared several theoretical papers [2] devoted to this problem. However, until now, only the Coulomb mechanism for inelastic processes appearing in collisions of high energy neutrinos with matter has been considered (cf. e.g. [2,3]). In the present paper a peculiar mechanism, will be considered, allowing to take into account phenomenologically the strong interactions; the method is related to the wave properties of the particles participating in reactions of the type

$$\nu + A \to \pi + \mu(e) + A, \quad (1)$$

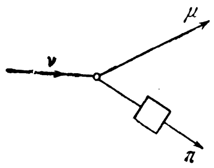
$$\nu + A \to K + \mu(e) + A, \quad (2)$$

$$\nu + A \to n + p + \mu(e) + A \quad (3)$$

etc.

For neutrinos of sufficiently high energy, there will be formed in the processes (1)—(3) fast particles (π, K, n, p) capable of strong interactions with nuclei. If processes (1)—(3) occur in the region of space outside the nucleus, they can be realized by means of the diffraction mechanism proposed by Landau and Pomeranchuk [4]. Indeed, under such circumstances the nucleus can be considered as a "black" (totally absorbing) body with respect to strongly interacting particles (the details of the strong interaction turn out to be inessential), and the inelastic processes (1)—(3) occur via the strong absorption of pions, nucleons, and strange particles by the nuclear particles (by absorption we mean here scattering of the strongly interacting particles by the nuclei).

The wave character of the diffraction mechanism of inelastic processes has been discussed in detail in the papers of Akhiezer and Pomeranchuk [5] and of Pomeranchuk and Feĭnberg [6]. The processes (1)—(3) can be computed on the basis of the "exact" wave function of the particle that is subject to diffraction in the final state [5]. However, the computations become simpler and more intuitive if the diagram technique proposed by Zhizhin and the author [7] for the treatment of inelastic diffraction processes is used.



The process which is inverse to $\pi \to \mu + \nu$ is described by the diagram in the figure. The box in the figure corresponds to elastic diffraction scattering of the pion on the "black" nucleus. (It is not important to have a specific model for the "black" nucleus, the pion-nucleus scattering amplitude may be taken from experiment.) The lines corresponding to the nucleus have been omitted from the figure, since the nucleus acquires only a small recoil in these processes and its motion is negligible [7]

The Coulomb interaction of the muons and pions with the nucleons can be taken into account in a trivial manner as has been done in the paper of Shabalin and the author [3]. As will be seen below, the Coulomb mechanism may turn out to be of secondary importance as compared to the diffraction mechanism, at least in the region of very high neutrino energies ($E_\nu \gtrsim 2 - 3$ BeV).

The neutrinos and muons do not interact strongly with nuclei and therefore their diffraction can be neglected. In order that the process (1) be realized outside the nucleus, in which case one may neglect the contribution of heavier intermediate states and the dominant role is played by the diagram represented in the figure, it is necessary that the momentum transfer to the nucleus along the direction of motion of the neutrino be sufficiently small [6]:

$$q_{\parallel} \lesssim 1/R, \tag{4}$$

Here $R = A^{1/3}/m_\pi$, R is the radius of the nucleus, A is the atomic weight and m_π is the pion mass ($\hbar = c = 1$).

Besides condition (4) it is necessary that the nucleus act as a whole with respect to the pion and the diffuse character of the nuclear boundary be negligible. This condition is fulfilled if the momentum transfer component perpendicular to the direction of motion of the neutrino satisfies the inequality

$$q_{\perp} \lesssim \mu. \tag{5}$$

The conservation laws imply that in order that the inequality (4) be satisfied, it is necessary that the neutrino energies in the laboratory system (l.s.) be of the order

$$E_{\nu L} \gtrsim (m_\pi + m_\mu)^2 R/2, \tag{6}$$

where m_μ is the muon mass.

For an iron nucleus $E_{\nu L}^{\min} \approx 830$ MeV and for a lead nucleus $E_{\nu L}^{\min} \approx 1280$ MeV. Thus one can expect that the diffraction mechanism will become dominant for neutrino energies $E_{\nu L}$ larger than several BeV. According to the rules formulated in [7] the matrix element for the process inverse to the decay $\pi \to \mu + \nu$ in the field of a nucleus will have the form [1])

$$M = \frac{Gf}{\sqrt{E_\pi}} \frac{|\mathbf{p}_{\pi L}|}{(p_\nu - p_\mu)^2 - m_\pi^2} \frac{2\pi R}{q_\perp} J_1(q_\perp R)$$
$$\times \bar{u}_\mu (\hat{p}_\nu - \hat{p}_\mu)(1 + \gamma_5) u_\nu 2\pi\delta(E_{\nu L} - E_{\mu L} - E_{\pi L}), \tag{7}$$

where $G = 10^{-5}/m_N^2$ (m_N is the nucleon mass) is the universal weak-interaction constant, $E_{\pi L}$ and $p_{\pi L}$ are the energy and momentum of the pion in the l.s., p_ν is the 4-momentum of the neutrino, p_μ — the 4-momentum of the muon, u_μ and u_ν are the four-component spinors describing the spin states of the muon and neutrino, respectively, $E_{\nu L}$ and $E_{\mu L}$ are the energies of the muon and neutrino in the l.s., $J_1(x)$ is the Bessel function of the first kind, and $f((p_\nu - p_\mu)^2)$ is the form-factor for the $\pi \to \mu + \nu$ decay, equal to $f^2 \approx m_\pi^2$ [8] for real pions. If the l.s. angle of emission of the muon is $\theta \sim m_\mu/E_{\mu L}$ and $E_{\mu L} \sim E_{\pi L}$, the variable $(p_\nu - p_\mu)^2$ will deviate from its mass shell value by a quantity of the order of $2 - 3m_\pi^2$.

[1])Here we use a metric in which the scalar product of 4-vectors is $ab = a_0 b_0 - \mathbf{a}\cdot\mathbf{b}$.

Thus, by comparing the experimental results with the theoretical differential cross section of the process (1), computed from the matrix element (7), one can obtain information about the behavior of the form-factor f as a function of the square of the 4-momentum $(p_\nu - p_\mu)^2$ of the virtual pion.

In the following computations, in order to estimate the cross section for the process (1), it will be assumed that $f \approx \text{const} \approx m_\pi$. The differential cross section for the process (1) computed on the basis of the matrix element (7) has the following form:

$$d\sigma = \frac{G^2 f^2 m_\mu^2 [J_1(q_\perp R)]^2 R^2}{8(2\pi)^3 q_\perp^2 E_{\nu L}^2} ds d\mathbf{q}_\perp \int \frac{[(Pp_\pi)^2/M^2 - m_\pi^2](p_\mu p_\nu)}{[p_\mu p_\nu + \Delta/2]^2}$$
$$\times \frac{d^3 p_\pi}{E_\pi} \frac{d^3 p_\mu}{E_\mu} \delta^4(g - p_\mu - p_\pi), \tag{8}$$

where $g^2 = (p_\mu + p_\pi)^2 = s$, in the l.s.; $g_0 = E_{\nu L}$, P is the 4-momentum of the nucleus before the collision, M is the mass of the nucleus and $\Delta = m_\pi^2 - m_\mu^2$. For computational convenience we separate in Eq. (8) an explicitly relativistic invariant integral, which becomes a function of the variable s if the condition [2]) $s \gg -t$ (t is the square of the 4-momentum transferred to the nucleus) is satisfied.

In the laboratory system one obtains the following expression for the angular distribution, derived from (8) in the region of small angles and large energies, $E_\mu \gg m_\mu$, $E_\pi \gg m_\pi$:

$$d\sigma = \frac{G^2 R^2 f^2 m_\mu^2 [J_1(q_\perp R)]^2 dq_\perp (E_{\nu L} - E_{\mu L}) dE_{\mu L}}{8\pi q_\perp E_{\nu L}^2}$$
$$\times \left[\frac{1}{m_\mu^2 + E_{\mu L}\Delta/E_{\nu L} + k_\mu^2} - \frac{\Delta E_{\mu L}/E_{\nu L}}{(m_\mu^2 + E_{\mu L}\Delta/E_{\nu L} + k_\mu^2)^2}\right] dk_\mu^2, \tag{8a}$$

where $k_\mu^2 = E_{\mu L}^2 \theta_\mu^2$ and θ_μ is the angle of emission of the muon with respect to the direction of motion of the neutrino.

Equation (8) implies that in the extreme relativistic case the produced muons will be emitted inside a narrow cone with $\theta_\mu \lesssim m_\mu/E_\mu$. It can be shown that the pions will be emitted preferentially within angles $\theta_\pi \lesssim m_\pi/E_\pi$. However, Eq. (8a) is not convenient for the estimate of the total cross section of the process (1) and for the derivation of the differential distribution in terms of the square of total mass of the generated particles.

These calculations are conveniently carried out in the center-of-mass system of the pion and muon. In the c.m.s. the integral in Eq. (8) is

[2])The condition $s \gg (-t)$ is satisfied even when $s = s_{\min} = (m_\pi + m_\mu)^2$, since, according to the inequalities (4) and (5), $t_{\max} \lesssim m_\pi^2$.

easily computed. Carrying out the integrations over $\mathbf{p}_\mu$, $\mathbf{p}_\pi$, and the azimuthal angles of the vector $\mathbf{q}_\perp$, and neglecting inessential corrections, we obtain the following expression for the differential cross section for the process inverse to the decay $\pi \rightarrow \mu + \nu$:

$$d\sigma = \frac{R^2G^2f^2m_\mu^2\,[J_1(q_\perp R)]^2\,dq_\perp}{16\pi q_\perp}\,\frac{(s-\Delta)\sqrt{(s-s_{min})(s-\delta)}\,ds}{s^3}, \tag{9}$$

where $\delta = (m_\pi - m_\mu)^2$. The integration over $q_\perp$ in (9) is to be carried out over the interval $0 \le q_\perp \lesssim \mu_\pi$ [cf. Eq. (5)]. For heavy nuclei $m_\pi R \gg 1$ and therefore

$$\int_0^{m_\pi} [J_1(q_\perp R)]^2\, dq_\perp/q_\perp \approx 1/2.$$

Thus the following simple expression is obtained for the differential distribution of the particles generated in reaction (1) with respect to the square of their total mass, s,

$$d\sigma = \frac{R^2G^2f^2m_\mu^2}{32\pi}\,\frac{(s-\Delta)\sqrt{(s-s_{min})(s-\delta)}}{s^3}\,ds. \tag{10}$$

A kinematic analysis of the reaction (1) shows that condition (4) is satisfied if $s \lesssim 2E_{\nu L}/R$, i.e., that Eq. (10) is valid in the interval

$$s_{min} \leqslant s \lesssim 2E_{\nu L}/R = s_{max}. \tag{11}$$

The emission angles of muons in the laboratory system are limited by the condition $\theta_\mu \lesssim 1/A^{1/3}$ in order for (11) to be true ($\theta_{\mu\,max} \approx 10°$ for a lead nucleus).

Integrating Eq. (10) with respect to s in the interval (11) and taking into account that $\delta \ll s_{min}$ we obtain an estimate for the total cross section for the process (1) valid when it is dominated by the diffraction scattering of the pion on the nucleus for $s_{min} \ll s_{max}$:

$$\sigma \approx \frac{R^2G^2f^2m_\mu^2}{32\pi}\left[\ln\frac{\sqrt{s_{max}}+\sqrt{s_{max}-s_{min}}}{\sqrt{s_{max}}-\sqrt{s_{max}-s_{min}}} - 2\sqrt{1-\frac{s_{min}}{s_{max}}} - \frac{2\Delta}{3s_{min}}\left(1-\frac{s_{min}}{s_{max}}\right)^{3/2}\right] \approx \frac{R^2G^2f^2m_\mu^2}{32\pi}\ln\frac{8E_{\nu L}m_\pi}{e^2 s_{min}A^{1/3}}. \tag{12}$$

It is evident from Eq. (12) that the cross section for the diffractive production of pions and muons by fast neutrinos increases slowly (logarithmically) with increase of the incident energy $E_{\nu L}$. If one restricts the l.s. angles to $\theta_\mu \lesssim m_\mu/E_{\mu L}$, Eq. (8a) implies that the total diffraction cross section is energy independent[4] and has the order of magnitude $\sigma \approx R^2G^2f^2m_\mu^2/32\pi$. For an energy $E_{\nu L}$ = 5 BeV the cross section (12) reaches an appreciable value for intermediate and heavy nuclei: $\sigma_{Pb} \approx 1.6 \times 10^{-40}$ cm^2 for a lead nucleus, $\sigma_{Fe} \approx 1 \times 10^{-40}$ cm^2 for an iron nucleus. Under the same conditions the Coulomb mechanism leads to cross sections $\sigma_{Pb} \approx 7 \times 10^{-42}$ cm^2, $\sigma_{Fe} \approx 1 \times 10^{-42}$ cm^2[3] i.e., by two orders of magnitude less than in the case of the diffractive mechanism.

It is evident that the formula (10) is valid also for the process inverse to the $K \rightarrow \mu + \nu$ decay in the field of a nucleus with only slight modifications, to take into account the fact that the numerical value of the form factor f^2 is by approximately an order of magnitude smaller for the K-meson than for the pion[8] and the replacement $m_\pi \rightarrow m_K$. The cross section for the process inverse to the $\pi(K) \rightarrow e + \nu$ decay in the field of a nucleus will be smaller by a factor $(m_e/m_\mu)^2$ as compared to the process inverse to the $\pi(K) \rightarrow \mu + \nu$ decay, as was also found in the case of the Coulomb mechanism[3], and will therefore be naturally depressed.

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