CALCULATION OF THE ENERGY AND ANGULAR CHARACTERISTICS OF MULTIPLY CHARGED PARTICLES PRODUCED BY HIGH-ENERGY PROTONS

M. M. MAKAROV

A. F. Ioffe Physico-Technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor September 25, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 962-968 (March, 1962)

The energy spectra of multiply-charged particles emitted at a given angle and their angular correlations with the scattered protons were calculated with the aid of an electronic computer. The dependence of these characteristics on the proton beam energy, momentum distribution in the nucleus, emission angle, binding energy and mass of the multiply charged particles was investigated. The results of the calculations are compared with the experimental data and it is shown that the cascade theory presented here corresponds to the experimental picture.

INTRODUCTION

IN the interaction of high-energy particles with light and heavy nuclei, multiply charged particles, often called fragments, are produced with an appreciable cross section (of the order of several millibarns). Earlier, a hypotheses was put forward[1,2] according to which these particles are produced in quasi-elastic scattering of the incident protons and the cascade protons and neutrons on quasi-stable complexes of nucleons inside the nucleus.

The existing views on the structure of the nucleus do not contradict the production of such quasi-stable complexes. Moreover, the recently developed theories of the nucleus (the Brueckner model,[3] the cluster model,[4] and others) envisage the possibility of strong correlations between nucleons leading to the production of short-lived substructures. Owing to the existence of these correlations, the nucleus at each moment of time should be considered as consisting of a variable set of individual nucleons, pairs of nucleons, three-nucleon formations, etc. Hence in an interaction of high-energy particles with nuclei, not only the nucleon-nucleon collisions should be taken into account but also nucleon-deuteron, nucleon-Li, $d - d$, $\alpha - d$, $\alpha - \alpha$, etc. collisions. Elastic or inelastic scattering on the multiply charged particles inside the nucleus can occur in these collisions, as a result of which the particles either leave the nucleus or break up.

It should be stressed that collisions in the nucleus are basically of a nucleon-nucleon character, and hence the results of the calculation[5] by the Serber-Goldberger scheme taking into account only nucleon-nucleon interactions reflect the general features of the experimental data.

The introduction of other types of collisions and the consideration of the nucleus as a more complex structure than a Fermi gas of non-interacting nucleons permits a more accurate picture of the interaction of high-energy particles with nuclei. An indication that such an approach to high-energy nuclear reactions takes into account the more subtle effects is the small cross section for processes in which multiply charged particles are produced (about 1% of the inelastic interaction cross section for 660-MeV protons) and the fact that, on the whole, the conventional exact calculations of the intranuclear cascade are clearly not in agreement with experiment when the presence of quasi-stable complexes has to be taken into account. Here we have in mind the experimental and theoretical cross sections for the (p, pn) reactions[6] proceeding at the periphery of the nucleus, i.e., where the conditions for the production of complexes are the best.

Hence, in order to trace the more accurate effects of the interaction of high-energy particles with nuclei, we must calculate the intranuclear cascade with allowance for various groupings inside the nucleus, assuming in one way or another some probability for their production and a momentum distribution for each of the groupings in the nucleus.

Such a calculation can be carried out only if the total and differential cross sections for scattering of all particles taking part in the cascade are known for different energies. Unfortunately, the experimental data on the differential cross sections are very scanty and such a complete calculation cannot be carried out at the present time. However, with the already known pα differential cross section it has been possible to calculate[7] a nuclear cascade with allowance for a quasi-α substructure which showed considerably better agreement with experiment than the previously known calculations.

There are, however, a number of questions concerning such multiple particle interactions which can be answered without the use of the differential cross sections. This applies to the kinematic relations between the energy and the emission angle of the scattered particle, to the energy spectra of particles emitted at a given angle, etc. Comparison of these characteristics with the experimentally measured values would provide the possibility of drawing conclusions in favor of some mechanism for the production of multiply charged particles.

From the viewpoint of the cascade mechanism, there exists a number of experimental facts which require explanation. These are, first, the broad distribution of experimental points representing the dependence of the energy of the emitted particle on its emission angle as compared to the calculated curve for the elastic collision,[1] and, second, the production of multiply charged particles emitted in the backward hemisphere relative to the bombarding beam with a momentum greater than the momentum of the incident particles.[8]

In order to explain these facts and to obtain some characteristics of the quasi-elastic knock-out process for multiply charged particles we carried out kinematic calculations.

COMPUTATIONAL SCHEME

As a result of the calculations we should obtain: 1) the energy spectra of the fragments at a given angle; 2) the spectrum of the angles χ between the proton and fragment which are emitted after the collisions; 3) the dependence of the energy spectra and the correlation angles χ at a given angle on the incident particle energy, the momentum distribution of the fragments in the nucleus, emission angle, binding energy, and fragment mass.

The calculations involve the finding of the energy of the multiply charged particle E_2', the angle χ, and the momentum of the scattered nucleon p_1' (the value of p_1' is necessary for the experimental determination of the cascade-proton momentum). In the nonrelativistic approximation we can obtain the following relations for the quantities E_2', χ, p_1'

$$(m_1+m_2)^2 E_2' = \tfrac{1}{2} m_2 [\{ p_1 \cos\theta + p_2 (\cos\alpha_1 \cos\theta + \cos\alpha_2 \sin\theta)$$
$$+ \{[p_1 \cos\theta + p_2 (\cos\alpha_1 \cos\theta + \cos\alpha_2 \sin\theta)]^2$$
$$- (1 + m_1/m_2) [p_2^2 + 2m_1 \{E_\text{b}$$
$$+ E_k - p_2^2 (1/2m_2 - 1/2m_\text{t})\} + 2p_1 p_2 \cos\alpha_1]\}^{1/2}]^2,$$

$$p_1' = \{p_1^2 - p_2'^2 m_1/m_2 - 2m_1 [E_\text{b} + E_k - p_2^2 (1/2m^2 - 1/2m_\text{t})]\}^{1/2},$$
$$\cos\chi = (p_1')^{-1} [p_1 \cos\theta + p_2 (\cos\alpha_1 \cos\theta + \cos\alpha_2 \sin\theta) - p'],$$

where p_1 is the momentum of the proton beam; p_2 is the momentum of the multiply charged particle in the nucleus; p_2' is the momentum of the multiply charged particle after emission from the nucleus; θ is the angle of emission of the multiply charged particle; $\cos\alpha_i$ are the direction cosines for the momentum of the multiply charged particle in the nucleus; m_1, m_2, and m_t are the masses of the proton, fragment, and target nucleus; E_b is the binding energy of the fragment in the nucleus; and E_k is the maximum fragment energy in the nucleus.

In order to obtain the formulas we used different models for the nucleus.

1. The fragment momentum distribution in the nucleus was taken in two forms:

Gaussian distribution: $dw(p) = c_1 \exp(-p^2/p_0^2)\, dp$,
Fermi distribution: $dw(p) = c_2 p^2 dp$.

In a comparison of the experimental data on the production of protons, deuterons, and alpha particles in nuclei by high-energy particles[9] very interesting properties are observed: the mean momentum of all particles is approximately the same (210–240 MeV/c) and independent of the particle mass. This also determined the choice of the parameter p_0 in the Gaussian distribution for the given problem. The Gaussian distribution was chosen with a parameter p_0 = 193 MeV/c, which corresponds to an energy of 3 MeV for Li^7 and 2.2 MeV for Be^9.

2. For a Gaussian distribution it is necessary to determine the maximum energy of the multiply charged particle in the nucleus E_k (corresponding value of the momentum p_{max}). In the choice of p_{max} we can start from the assumption that the particle momentum in the nucleus weakly depends on the form of the particle. In a number of papers on pion production in nuclei[10] it was shown that protons in the nucleus can have momenta up to 340–390 MeV/c. Igo and Thaler[11] showed that the depth of the potential well for α particles in an average nucleus is ~35 MeV, which corresponds to a momentum up to 470 MeV/c. In this calculation we chose the value p_{max} = 415 MeV/c, which corresponds to E_k = 10 MeV for Be^9 and 13 MeV for Li^7.

Then, adding the values of the binding energies, we can obtain the value of the potential well for the fragments: 41 MeV (for Li^7 in N^{14}), 36 MeV (for Be^9 in C^{12}), which are close to the values obtained for multiply charged ions by the optical model.

The calculation was carried out for three values of the momentum p_1 – 362, 433, and 613 MeV/c—which correspond to proton energies of 70, 100, and 200 MeV—and for angles θ from 0 to 100° in 10° intervals. As fragments and targets we chose

$$Li^7\ (N^{14},\ \text{target},\ E_b = 27{,}8\ \text{MeV}),$$
$$Li^6\ (N^{14},\ \text{target},\ E_b = 16{,}2\ \text{MeV}),$$
$$Be^8\ (C^{12},\ \text{target},\ E_b = 7{,}4\ \text{MeV}),$$
$$Be^9\ (C^{12},\ \text{target},\ E_b = 26.3\ \text{MeV}).$$

The values of p_2, $\cos\alpha_1$, and $\cos\alpha_2$ were chosen by a statistical trial and error method; for each angle θ the quantities were changed 255 times.

All calculations were made on a Minsk-2 electronic computer. The program was set up in such a way that, apart from the individual values E_2', χ, and p_1' for each value of the angle θ, we calculated at once the energy spectrum and the distribution of angles χ.

RESULTS OF THE CALCULATIONS

In Fig. 1, curve 3 represents the dependence of the energy on the fragment emission angle in the case of elastic scattering of a 100-MeV proton on Li^7 and curve 4 represents the same case for scattering on a Li^7 bound in a N^{14} nucleus. The momentum distribution of the multiply charged particles in the nucleus leads to the possibility that fragments will be emitted at a given angle with energies considerably differing from the energies in the case of an elastic collision. Curves 1 and 2 (for Fermi and Gaussian fragment distributions, respectively, in the nucleus) limit the region of allowable energies of Li^7 fragments emitted from the N^{14} nucleus at various angles. If the experimental points taken from the work of Arifkhanov et al.[1] are transferred onto the curve, it turns out that they lie inside the region shown. Consequently, the existence of intranuclear momenta of multiply charged particles completely accounts for the spread in the experimental points on the $E(\theta)$ plot.

We can point to the possibility of the emission of a fragment with a considerable energy in the backward hemisphere relative to the beam. The maximum Li^7 energy at the angle $\theta = 100°$ is 20 MeV, which corresponds to a momentum of ~500 MeV/c.

Taking into account the incident proton momentum 433 MeV/c, we see that in quasi-elastic scattering the production of fragments emitted backward relative to the beam with a momentum greater than the momentum of the incident proton is possible, as has been observed experimentally.[8]

In the calculation we investigated the influence of the momentum distribution of the multiply charged particles in the nucleus on the fragment energy spectrum as a function of the binding energy and mass of the fragment (Fig. 2), on the incident particle energy (Figs. 3 and 2b), and on the emission angle. It turned out that the difference

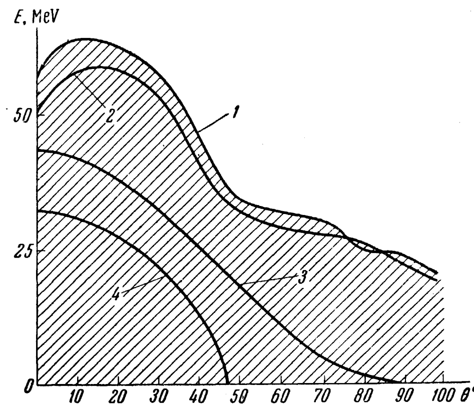

FIG. 1. Relation between the energy and the emission angle of multiply charged particles.

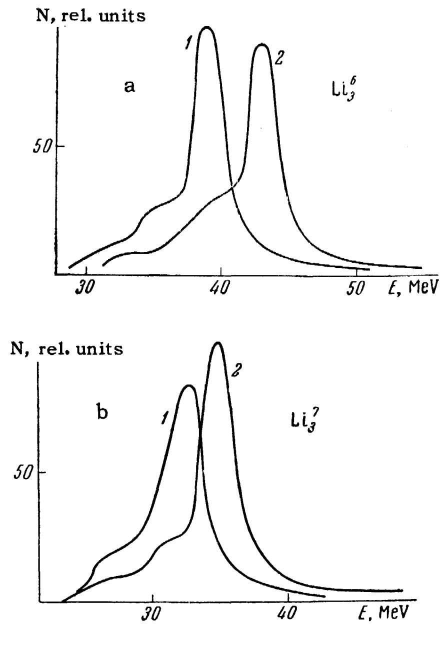

FIG. 2. Energy spectrum of Li^6 and Li^7 fragments (N^{14} target) at 0° for different momentum distributions in the nucleus: curve 1 – Gaussian distribution, curve 2 – Fermi distribution. Proton energy 100 MeV.

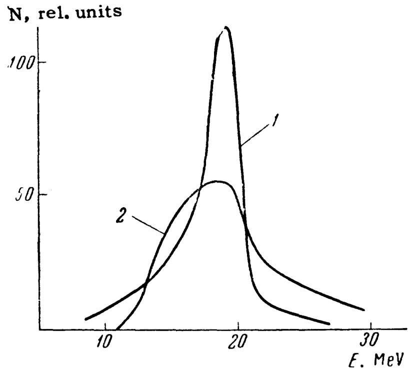

FIG. 3. Energy spectrum of Li^7 fragments at 0° for different momentum distributions in the nucleus: curve 1 — Gaussian distribution, curve 2 — Fermi distribution. Proton energy 70 MeV.

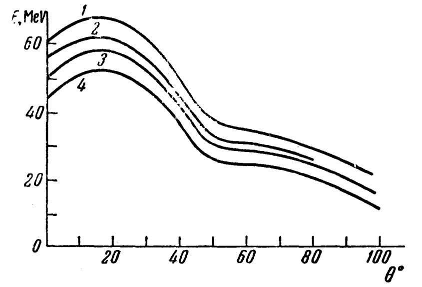

FIG. 5. Dependence of fragment energy on emission angle (limiting values): 1) Li^6 (N^{14} target), 2) Be^8 (C^{12} target), 3) Li^7 (N^{14} target), 4) Be^9 (C^{12} target).

in the fragment energy spectra due to their different momentum distribution in the nucleus increases with increasing binding energy, with increasing proton energy, for larger emission angles, i.e., for small energy transfers. It should also be mentioned that, for a Fermi momentum distribution of the fragments in the nucleus, the energy spectrum is more sensitive to the binding energy and the beam energy than for a Gaussian distribution of the intranuclear momenta.

For a comparison with the experimental data, the change in the fragment energy spectrum with a change in the emission angle at different proton energies is of interest. Figure 4 shows the Li^7 energy spectra at angles 0° and 40° for proton energies of 70, 100, and 200 MeV (the spectra were analyzed by the Ferreira-Valoschek method[12]). It follows from the figure that the greatest dependence of the energy spectra on the emission angle occurs at large proton energies. This fact obviously enables the determination of the most effective energy for the production of multiply charged particles in the knock-out case.

The dependence of the energy spectrum on the fragment mass and its binding energy in the nucleus is shown in Fig. 5. For simplicity we have shown here, instead of the energy spectra, the limiting values of the energy at a given angle. For approximately equal binding energies (Li^7 in N^{14} and Be^9 in C^{12}) the fragment spectrum softens with increasing mass (curves 3 and 4), but the influence of the binding energy sometimes becomes stronger than the influence of the fragment mass; thus the Be^9 energy spectrum from C^{12} is harder than the Li^7 fragment spectrum from N^{14} —curves 2 and 3 (owing to the small binding energy of Be^8 in the nucleus).

The distribution of the angles between fragments emitted at 0° to the beam and scattered protons is shown in Fig. 6 for E_p = 100 MeV. As is seen, there is a distinct angular correlation in the region of obtuse angles. This correlation does not depend on the shape of the fragment momentum distribution inside the nucleus (Fig. 6a), on the binding energy and mass of the fragments (Fig. 6b) at emission angles of 0°, and weakly depends on these quantities at large angles. With a decrease in the proton energy, the influence of the intranuclear momenta on the angular correlation increases.

The character of the angular correlation changes rapidly as we go to other angles of obser-

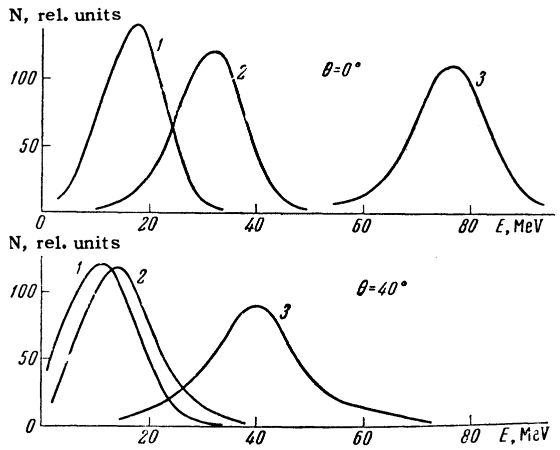

FIG. 4. Energy spectra of Li^7 fragments for different proton energies: 1) E_p = 70 MeV; 2) E_p = 100 MeV; 3) E_p = 200 MeV.

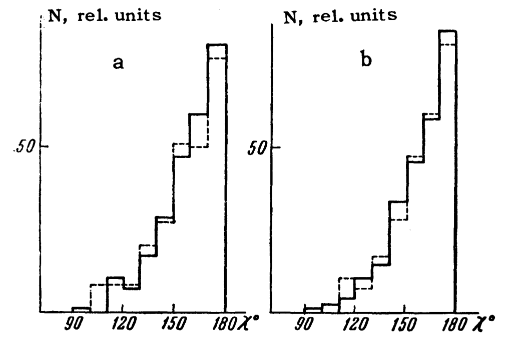

FIG. 6. Distribution of angles between fragments and protons: a) the solid line is for a Fermi distribution, the dashed line for a Gaussian distribution; E_p = 100 MeV, θ = 0°, Li^7; b) the solid line is for Li^6 and the dashed line for Li^7.

vation of fragment emission, in other words, as we go to greatly differing energy spectra. As follows from Fig. 7, the most probable angle between the fragment and the proton shifts toward the region of smaller angles. This circumstance is very important for the explanation of the mechanism of multiply-charged particle production.

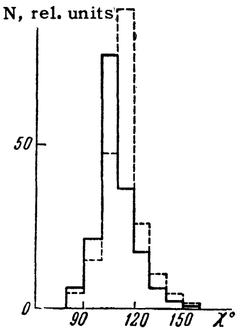

FIG. 7. Distribution of angles between fragments and protons: $E_p = 100$ MeV, $\theta = 40°$, solid line – Li^7, dashed line – Be^9

The behavior indicated above is convenient to use for a comparison with the experimental data. Since the character of the correlation weakly depends on the fragment mass, we can construct the distribution of the angles between the protons and the multiply charged particles for all fragments emitted at a given angle to the beam without separating them according to charge and mass. Figure 8 shows such a distribution for fragments emitted in the 10—30° interval from light nuclei of emulsion under bombardment by 75-MeV protons. The dashed line in the figure represents the calculated correlation for the angle 20°. A χ^2 test showed that these distributions can be considered compatible. Hence the experimentally obtained angular distribution corresponds to the model developed here.

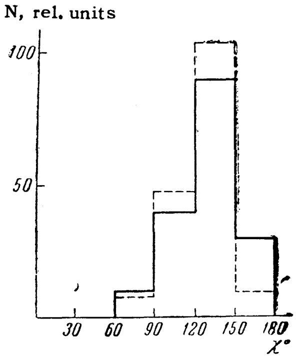

FIG. 8. Distribution of angles between fragments and protons: solid line – experimental, dashed line – calculated.

The author thanks Professor A. P. Komar, Professor N. A. Perfilov, F. G. Lepekhin, O. V. Lozhkin, V. I. Ostroumov for their interest and for discussions of the results, M. N. Ivanov for aid in setting up the computer program, and L. M. Tkach for reduction of the experimental data.

[1] Arifkhanov, Makarov, Perfilov, and Shamov, JETP **38**, 1115 (1960), Soviet Phys. JETP **11**, 806 (1960).

[2] M. M. Makarov and N. A. Perfilov, DAN SSSR **138**, 579 (1961), Soviet Phys.-Doklady **6**, 413 (1961).

[3] Brueckner, Eden, and Francis, Phys. Rev. **98**, 1445 (1955), H. A. Bethe, Phys. Rev. **103**, 1353 (1956).

[4] K. Wildermuth and Th. Kanellopoulos, CERN report 59-23, 1959.

[5] Metropolis, Bivins, Storm, Turkevich, Miller, and Friedlander, Phys. Rev. **110**, 185 (1958).

[6] Markowitz, Rowland, and Friedlander, Phys. Rev. **112**, 1295 (1958).

[7] Abate, Bellini, Fiorini, and Ratti, Nuovo cimento **22**, 1206 (1961); Zhdanov and Fedorov, JETP **41**, 1870 (1961), Soviet Phys. JETP **14**, 1330 (1961).

[8] O. V. Lozhkin, Materialy soveshchaniya po spectroskopii neĭtronodefitsitnykh yader i po primeneniyu radiokhimicheskogo i fotograficheskogo metodov pri izuchenii yadernykh reaktsiĭ (Proc. of Conf. on the Spectroscopy of Neutron-Deficient Nuclei and on the Use of Radiochemical and Photographic Techniques for the Study of Nuclear Reactions), Joint Institute of Nuclear Research, 1958, Vol. 1, p. 117.

[9] Azhgireĭ, Vzorov, Zrelov, Meshcheryakov, Neganov, and Shabudin, JETP **33**, 1185 (1957), Soviet Phys. JETP **6**, 911 (1958); P. Cüer and A. Samman, J. phys. radium **19**, 13 (1958).

[10] Azhgireĭ, Vzorov, Zrelov, Meshcheryakov, and Petrukhin, JETP **34**, 1357 (1958), Soviet Phys. JETP **7**, 939 (1958); Imhof, Easterday, and Perez-Mendez, Phys. Rev. **105**, 1859 (1957).

[11] G. Igo and R. M. Thaler, Phys. Rev. **106** (1957).

[12] E. P. Ferreira and P. Ya. Valoschek, Proc. Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1955, Report P/1076, vol. 2, p. 124.

Translated by E. Marquit
160