QUANTIZATION OF QUASI-PARTICLES WITH A PERIODIC DISPERSION LAW IN A STRONG MAGNETIC FIELD

M. Ya. AZBEL'

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The character of the spectrum of a charged quasi-particle in a crystal lattice in a very strong magnetic field **H** is explained. It is shown that the energy levels and wave functions are periodic functions of **H**.

1. Quantization of the energy levels of a quasi-particle with an arbitrary dispersion law in a sufficiently weak magnetic field, when quasi-classics holds, is well known (see [1], for example). It is interesting to comprehend how the level configuration changes with the growth of the magnetic field when the quasi-classical analysis ceases to be valid and the periodicity of the dispersion law is very essential. The present work is devoted to an elucidation of this question. The simplest case is studied, when the transitions between the bands can be neglected.

2. Let a quasi-particle have the charge e and be characterized by the dispersion law $\epsilon = \epsilon(\mathbf{p})$ (ϵ the energy, **p** the quasi-momentum), where ϵ is a periodic function: $\epsilon(\mathbf{p}+\mathbf{a}) = \epsilon(\mathbf{p})$ if $\mathbf{a} = \sum_{i=1}^{n} n_i\mathbf{a}_i$; n_i are integers such that [1]

$$\varepsilon(\hat{\mathbf{p}}) = \sum_{\mathbf{b}} \alpha_{\mathbf{b}} \exp(i\hat{\mathbf{p}}\mathbf{b}), \qquad \mathbf{b} = \sum_{i=1}^{3} m_i \mathbf{b}_i$$

(**b** the vector of the "reciprocal" lattice, m_i are integers). Then the particle Hamiltonian in a permanent magnetic field **H** has the form

$$\hat{\varepsilon} = \sum_{\mathbf{b}} \alpha_{\mathbf{b}} \exp\left\{\left(i\mathbf{P} - \frac{e\hbar}{2c}\left[\mathbf{H}\frac{\partial}{\partial\mathbf{P}}\right]\right)\mathbf{b}\right\}$$

$$= \sum_{\mathbf{b}} \alpha_{\mathbf{b}} \exp\{i\mathbf{P}\mathbf{b}\} \exp\left\{-\frac{e\hbar}{2c}\left[\mathbf{H}\frac{\partial}{\partial\mathbf{P}}\right]\mathbf{b}\right\}. \qquad (1)$$

Here the vector-potential **A** is selected in the form $\mathbf{A} = \frac{1}{2}\mathbf{H}\times\mathbf{r}$; r_i are the coordinates. Exactly as was done in the book by Landau and Lifshitz,[2] it can be shown that $\mathbf{r} = i\hbar\partial/\partial\mathbf{P}$ [the case when $\epsilon(\mathbf{p})$ is a single-valued function is considered, i.e., there is one "band"; for brevity the spin s of the particle is not considered, the associated quantization being obvious (see [3], for example) and yields $\mu_0 H\sigma$, where μ_0 is the Bohr magneton for a quasi-particle, $\sigma = -s, \ldots, s$]. In writing (1) we took into account that the terms in the exponential commute.

Let us replace **H** by $\mathbf{H}+\mathbf{H}_0$. Evidently

$$\exp\left\{-\frac{e\hbar}{2c}\left[(\mathbf{H}+\mathbf{H}_0)\frac{\partial}{\partial\mathbf{P}}\right]\mathbf{b}\right\}$$

$$= \exp\left\{-\frac{e\hbar}{2c}\left[\mathbf{H}\frac{\partial}{\partial\mathbf{P}}\right]\mathbf{b}\right\}\exp\left\{-\frac{e\hbar}{2c}\left[\mathbf{H}_0\frac{\partial}{\partial\mathbf{P}}\right]\mathbf{b}\right\}.$$

Noting that

$$\exp\{[\beta\partial/\partial\mathbf{P}]\,\gamma\}\,\psi(\mathbf{P}) = \psi(\mathbf{P} + [\gamma\beta]),$$

we easily see that

$$\hat{\Lambda} = \exp\left\{-\frac{e\hbar}{2c}\left[\mathbf{H}_0\frac{\partial}{\partial\mathbf{P}}\right]\mathbf{b}'\right\} = \exp\{\hat{\mathbf{l}}\mathbf{b}'\}$$

commutes with the Hamiltonian if $\mathbf{H}_0 = 4\pi c\mathbf{b}''/e\hbar V$ (V is the volume of the cell of the "direct" lattice **a**). This means the presence of common eigenfunctions for $\hat{\epsilon}$ and $\hat{\Lambda}$ and the possibility of considering $\hat{\Lambda}$ a number. The operator $\hat{\mathbf{l}}$ in $\hat{\Lambda}$ can here also be considered the imaginary ($i\hat{\mathbf{l}}$ is the Hermitian operator) vector $i\mathbf{l}_1$. Therefore, the replacement of **H** by $\mathbf{H}+\mathbf{H}_0$ leads to an inessential change in the origin of **P**, viz., **P** is replaced by $\mathbf{P} - \mathbf{l}_1$. This means that the energy levels and wave functions (the latter with the accuracy up to the translation $\mathbf{P}\to\mathbf{P}-\mathbf{l}_1$) are periodic functions of **H**: the fields **H** and $\mathbf{H}+\mathbf{H}_0$ lead to identical physical results.

Evidently, this denotes the periodicity of all the physical quantities with a relative amplitude on the order of unity in the formally-introduced "crystal lattice in magnetic-field space". (It is understood that the spin paramagnetism adds the evident non-periodic term $\epsilon\{H\} = \epsilon_0\{H\} + \mu_0 H\sigma$, where $\epsilon_0\{H\}$ is a periodic function.)

The differences from the customary quantum vibrations [3] are: 1) the large amplitude; 2) the periodicity in the magnetic field (and not its inverse); 3) the "universality" of the period and the absence of several harmonics.

The absence of transitions between bands (which permitted the examination of one band) and the inessentiality of the Fermi-fluid interaction are essential hypotheses of the proposed derivation.

The experimental detection of the effect is made difficult by the necessity of having a lattice with a very large period, since $H \sim 10^8$ Oe is necessary for $b_i \sim 10^{-8}$ cm. A large period can be guaranteed by a large number of atoms in the cell, by a superstructure associated with small ordering impurities, etc.

Let us describe how the change in the system of levels occurs with the growth of the magnetic field. Since the states corresponding to p and p + a are identical, by proceeding exactly as Landau and Lifshitz (see [2], Sec. 104; only periodicity of the lattice was used in the derivation in [2]), we obtain

$$\psi(\mathbf{P}) = \exp\{i\boldsymbol{\mu}\mathbf{P}\}\,\chi_{n\boldsymbol{\mu}}(\mathbf{P}),$$

where ψ is the wave function; χ is periodic in $\mathbf{P}$; $\epsilon = \epsilon_n(\mu)$; μ plays the part of a quasi-coordinate vector. The band is split into strips (n is the number of the strip); the levels in the strip are characterized by three continuous parameters (and not two as is customarily considered, see [1], for example; the presence of the third parameter, however, discrete but not continuous, could be discerned from the Zil'berman formulas [4]).

It is seen from (1) that P_H is retained and "tight" binding in weak fields corresponds to two-dimensional motion (determined by H) in a plane perpendicular to P_H and narrow, equidistant strips are manifest for given values of n and P_H ($\dot{c}b^2/ehH$ serves as a small parameter, just as in [4]). As the magnetic field grows the number of states does not change; the number of strips diminishes and their width increases (an investigation of the singularities associated with the jump changes in the number of strips by one is the subject of a separate report). For a certain "maximally effective" field the band seems to consist of several strips, between which the width and the spacing is on the order of the width of the "original" band for H = 0. As H increases further, the picture reverses and H_0 returns to a continuous spectrum for a "rational cut" as for H = 0 and is later duplicated periodically (the change in the level configuration is quasi-periodic for an "irrational cut"). It is easy to discern that the quantity $H_0 - H'$ is analogous to H' (a computation of the levels is analogous[3]).

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[1] I. M. Lifshitz and M. I. Kaganov, UFN 69, 419 (1959); Soviet Phys. Uspekhi 2 (6), 831 (1960).

[2] L. D. Landau and E. M. Lifshitz, Kvantovaya Mekhanika (Quantum Mechanics) Gostekhizdat, 1948, Sec. 104.

[3] I. M. Lifshitz and A. M. Kosevich, JETP 29, 730 (1955), Soviet Phys. JETP 2, 636 (1956).

[4] G. E. Zil'berman, JETP 30, 1092 (1956), Soviet Phys. JETP 3, 835 (1957).

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