"STATIC SKIN EFFECT" FOR CURRENTS IN A STRONG MAGNETIC FIELD AND THE RESISTANCE OF METALS

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It is shown that in a strong magnetic field **H** ($r \ll l$, r is the Larmor radius, l the electron mean free path) the direct current in a sample with a sufficiently good surface (whose distortions are small in comparison with r) falls off rapidly in the direction from the surface to the center of the sample ("static skin effect," Figs. 1, 3). In very strong magnetic fields [such that $r \ll l(l/d)^2$ for equal numbers of electrons n_1 and "holes" n_2, or $r \ll l(l/d)$ for $n_1 \neq n_2$, where d is the sample thickness] even the total current flows principally near the surface, in a layer of thickness l if the magnetic field is inclined to the surface or in a layer of the order of r if the field is parallel to the surface. For $n_1 = n_2$ and a polygonal cross section, the current flows near the vertices at distances of the order of l from them. Such a current distribution in the sample leads to a linear dependence of the resistance on the magnetic field (Figs. 2, 5) even for single crystals with closed Fermi surfaces (the conditions for observing this dependence and its origin are entirely different from those for the linear Kapitza law which holds for polycrystals with open Fermi surfaces). The specific results depend significantly on the conductor configuration and the orientation of the magnetic field. The directions of sharp anisotropy of the dependence $\rho(\mathbf{H})$ are determined (Fig. 4). Transformation of a "good" surface into a "bad" one results in a sharp increase in the resistance $\rho \sim H^2$ for $n_1 = n_2$, and to a sharp drop in the resistance $\rho \to$ const for $n_1 \neq n_2$ in the magnetic field mentioned.

1. PHYSICAL BASIS OF THE "STATIC SKIN EFFECT"

As is well known, there is a fundamental difference between the behavior of a conductor in a magnetic field and in its absence. In the absence of a magnetic field, all the components of the conductivity tensor σ_{ik} (which differ from zero in the anisotropic case) are infinite for infinite path length of the conduction electrons: $\sigma_{ik} \sim l \to \infty$. In a magnetic field **H** (the z axis is directed along **H**) the components σ_{xx} and σ_{yy} (and in the case when the number of "holes" equals the number of electrons n_1 also the components σ_{xy} and σ_{yx}) are equal to zero when $l = \infty$ (see[1a]):

$$\sigma_{\alpha\beta}(\mathbf{H}) \sim \sigma_{\alpha\beta}(0)\,(r/l)^2 \sim l^{-1} \to 0$$

(r is the radius of the Larmor orbit; in what follows, these components will be denoted by σ_1). This difference is especially marked in the case $n_1 = n_2$: for $H = 0$, the specimen with $l = \infty$ is an ideal conductor, $\sigma = \infty$; for $H \neq 0$, it is an ideal dielectric, $\sigma = 0$.

This difference is brought about by two causes. For $H = 0$, the electrons move in all directions without collisions at distances of the order l, while for $H \neq 0$ and $r \ll l$, the displacement in the xy-plane (for closed orbits) is always of the order of r. As a result, all the components σ_{ik} except σ_{zz} contain r instead of l, i.e., they are decreased by the factor l/r relative to the case of the absence of the magnetic field.

The additional decrease of σ_1 by the factor l/r is associated with the averaging over the equivalent random moments of the collision, which can occur at any of the l/r revolutions [see Sec. 2, the discussion of Eq. (5a)]. In this case, the randomness of the collisions, the absence of correlations between the position of the particle at a given moment and at the moment of the preceding or following collision, is a consequence only of their statistical character (in particular, thanks to this random character, the final state in statistics does not depend on the initial conditions).

In principle, collisions with a sufficiently smooth surface have a different character. They are reliably and uniquely determined by the state (coordinate and momentum) of the particle at the given instant (of course, it is assumed here that "ordinary" exchange collisions do not take place ''along the way''). In this sense, collisions with the surface are special, ''non-statistical'' collisions. [1)]

In a homogeneous field, the determinate character of surface collisions would still not lead to new results. The effective conductivity measured experimentally corresponds to a connection between the electric field and the current averaged over the specimen; the average over the depth is equivalent (everywhere, with the exception of a layer of thickness of the order r) to an average over the collision moments, since one can indicate a depth, for any given time of free motion of the electron from the surface, at which this time is realized (see Sec. 2).

However, the inhomogeneity of the electric field (normal to the surface) which arises near the surface makes collisions at different depths non-equivalent. As a result, the corresponding ''surface'' components of the conductivity for $r \ll l$ are shown to be only smaller than in the absence of the magnetic field by the factor l/r, and this means larger by a factor l/r than the ''volume'' components in the given magnetic field.

Thus, the metal would seem to consist of two parallel combined layers: a surface with a large conductivity and a volume with a small conductivity. In a sufficiently strong magnetic field, the current flows principally in the thin surface layer—the ''static skin effect'' for the current. It must be emphasized that, in contrast with the normal skin effect, 1) the current in the depth of the specimen is small but does not approach zero, and 2) the projection of the electric field in the direction of the total current is homogeneous in depth. It is curious that as $d \to \infty$ (d is the thickness of the specimen) and $r \to 0$ (physically, this corresponds to $d \gg l \gg r$) the resistance of the specimen can never approach the resistance of the unbounded metal, but can be completely determined by the near surface layer.

A sharp discontinuity in the current close to a smooth surface leads, thanks to the change in the boundary conditions, to a bending of the flow lines throughout the specimen. This also has an effect on the effective resistance (see Sec. 4). As a result, one can neglect the effect of the surface only in a sufficiently weak magnetic field, and apparently in a sufficiently strong one when $r \ll \delta$, δ being the characteristic dimension of the random distortions of the surface (in the latter case, the ''surface'' collisions cease to be determinate and are different from the ''volume'' collisions).

It is clear from the foregoing that if the character of the phenomenon in the unbounded medium depends essentially on the random character of volume collisions, [2)] then an arbitrary inhomogeneity (which makes the different collision moments non-equivalent) can be shown to be extremely significant. Consequently, the ''static skin effect'' is possible in the electrical conductivity and the thermal conductivity of metals and semiconductors of arbitrary dimensions in a sufficiently strong magnetic field both in the classical and in the quantum regions. For these same cases, one can reduce the inhomogeneity or the non-static character of the electric field to a new dependence on the magnetic field in the infinite specimen (in the non-static case, it is sufficient that the frequency $\omega \gtrsim c^2 (2\pi\sigma l^2)^{-1}$, which, for $l \sim 1$ mm and $\sigma \sim 10^{20}\ \mathrm{sec}^{-1}$, corresponds to $\omega \sim 100\ \mathrm{sec}^{-1}$).

The present work is devoted to the classical theory of galvanomagnetic phenomena in a bulk sample ($d \gg l$) for closed electron trajectories.

2. STATEMENT OF THE PROBLEM

Let us consider a plane parallel slab of thickness d (the z axis is directed along **H**, ξ is directed along the normal to the surface; $\mathbf{x} \perp \mathbf{z}, \xi$; $\eta \perp \mathbf{x}, \xi$; $0 \le \xi \le d$). Just for simplicity of exposition, we introduce the time of free path t_0 and consider the reflection from the surface to be diffuse (the consideration given below can be generalized to the case of the collision integral and an arbitrary character of reflection of the electrons from the surface).

The solution of the problem is in three parts. First, it is necessary to find the connection between the current density **j** and the electric field intensity **E**, that is, the operator (in the coordinate ξ) of the conductivity. In this case, one must take into account the discontinuity of the distribution function inside the metal (see below) brought about by the non-monotonic character of $\xi = \xi(t)$ (t is the time of revolution of the electron in its orbit), i.e., by the existence of points where $v_\xi = \dot{\xi} = 0$. This problem is solved quite simply both

[1)]Here the concrete character of the collision with the surface is immaterial; it is only important that the thickness of the layer in which the ''collision'' takes place be a very small characteristic dimension of the problem. In other words, the presence of some singularity in the mathematical behavior of the surface is necessary (where, for example, charge recombination takes place or specular reflection of particles, and so forth).

[2)]This can be made clear, for example, by comparing the results in the homogeneous and inhomogeneous cases.

by means of the kinetic equation for the distribution function and by starting out from simple physical considerations analogous to those of Chambers [2].

Second, from the electrostatic equation

$$E'_\xi(\xi) = 4\pi\rho'$$

(ρ' is found from the kinetic equation) one can express $E_\xi(\xi)$ in terms of homogeneous E_x and E_η (since curl $\mathbf{E} = 0$), and thus write $j_x(\xi)$ and $j_\eta(\xi)$ in terms of E_x and E_η. [The equation of continuity is $j'_\xi(\xi) = 0$, so that the relation j_ξ = const in a metal follows quite automatically from the kinetic equation for the electron distribution function f, leading only to the boundary condition at the surface: $j_\xi = 0$.] It is easy to understand that $\rho' \sim \sigma E/v_0$, where v_0 is the characteristic velocity of motion of the electrons and σ is the conductivity, and therefore, in metals (and generally in good conductors, for which $v_0/\sigma a \ll 1$, a being the characteristic distance over which the field changes), the equation for E_ξ reduces to $\rho' = 0$. Since $\rho' = 0$, the collision integral in the approximation of the free path time t_0 can be written in the form $(\partial f/\partial t)_{coll} = (f - f_0)/t_0$ (f_0 is the equilibrium distribution function), and the equation $\rho' = 0$ is now equivalent to the equation

$$j'_\xi(\xi) = 0, \qquad j_\xi(\xi) = \text{const} = j_\xi|_{\text{surface}} = 0, \tag{1}$$

which is obtained from the kinetic equation by integrating it over all momentum space with account of the fact that $\rho' = 0$ (the relation (1) is shown to be not a consequence of the kinetic equation, but an independent equation because our writing of the collision integral is possible only as a consequence of $\rho' = 0$).

The determination of $E_\xi(\xi)$ is the most difficult part of the problem, since Eq. (1) for E_ξ (the right hand side contains E_x, E_η) is inhomogeneous integral (since the current at the point under consideration is determined by the values of the field $E_\xi(\xi)$ at all points lying at a distance of the order of $l\varphi + r$ from the given point, where φ is the angle between z and η), and non-difference (since the current at a given point depends on the time of flight of the electron from the surface to the given point—generally speaking, a non-single-valued, function of the distance to the surface). The exact solution of this equation of course cannot be obtained. However, by taking it into account that $r \ll l$, and starting out from the form of the kernel and of the right-hand side of the equation, it is possible to deduce the character of the dependence $E_\xi(\xi)$ and, by using the physically clear uniqueness of the solution of the equation, estimate the order of magnitude of the quantities. This is sufficient to find the asymptotic values of the resistance and the Hall potential difference with accuracy up to a multiplicative factor of the order of unity. This factor is unimportant, even though it does actually depend on the concrete form of the collision integral, on the character of the reflections of the electrons from the surface, has a very complicated form and, in any event cannot be used to obtain any additional information on the electronic spectrum of metals.

Thus, because of the failure of the exact solution of the problem, it is only possible to make clear what is required for the answer to all the questions of physical interest.

Finally, in the third place, by averaging j_s ($s = x, \eta$) over ξ, we obtain the tensors of the effective conductivity $\hat{\sigma}$ and the resistivity $\hat{\rho}$:

$$\frac{1}{d}\int_0^d j_s(\xi)\,d\xi = \bar{j}_s = \sigma_{ss'}E_{s'},$$

$$\hat{\rho} = \hat{\sigma}^{-1}, \qquad E_s = \rho_{ss'}\bar{j}_{s'}, \qquad s, s' = x, \eta. \tag{2}$$

Knowledge of ρ makes it possible to determine the resistance of a plate (that is, of a specimen that is arbitrarily long but is limited in the μ direction: $0 \le \mu \le d_1$).

The presence of additional boundaries leads to an additional weak dependence of the electric field and the current on the coordinate μ which, of course, it is not necessary to take into account in the kinetic equation but which gives, in correspondence with div $\mathbf{j} = 0$ or $\int \mathbf{j}\cdot d\mathbf{S} = 0$ (where $\mathbf{S}$ is any closed surface), the boundary condition on the "new" surfaces: $j_\mu|_{\mu=0,d_1} = 0$ or the integral condition $\bar{j}_\mu = 0$, which is equivalent to the foregoing but which is more convenient for us. (This condition gives the closing of the current lines. We note that in the homogeneous case the vanishing of the component of the current normal to the surface at the surface leads to $j_\mu = j_\xi = 0$ everywhere in the specimen; there is a current only in the direction ν, $\nu \perp \mu, \xi$. In the inhomogeneous case, a "circular current" arises in the $\xi\mu$ plane.) Therefore, since $\bar{j}_\xi = \bar{j}_\mu = 0$,

$$\rho = E_\nu/\bar{j}_\nu = \mathbf{E}\cdot\bar{\mathbf{j}}/\bar{\mathbf{j}}\cdot\bar{\mathbf{j}} = \rho_{ss'}n_s n_{s'}, \qquad n_s = \bar{j}_s/\bar{j} = \bar{j}_s/\bar{j}_\nu. \tag{3}$$

We proceed to the program of solution of the problem described above. The connection between the current density $\mathbf{j}$ and the field intensity $\mathbf{E}$ in the approximation of free path time has the form [see, for example, Eqs. (2.3)—(2.5) of the work of Kaner [3]]

$$j_i = \hat{\sigma}_{ik}E_k = \frac{2e^2}{h^3}\int \gamma m^* t_0 dp_z \int_0^{2\pi} v_i(\tau)\, d\tau \times \int_{\lambda(\xi;\tau)}^{\tau} d\tau' \exp(\gamma\tau' - \gamma\tau)\, v_k(\tau')\, E_k\left(\xi + \frac{1}{\Omega}\int_{\tau}^{\tau'} v_\xi\, d\tau_1\right); \quad (4)$$

$$\xi + \Omega^{-1}\int_{\tau}^{\lambda(\xi;\tau)} v_\xi(\tau_1)\, d\tau_1 = 0, d; \qquad \lambda(\xi;\tau) \leqslant \tau. \quad (5)$$

Here $\lambda(\xi, \tau)$ is the first root of Eq. (5) preceding τ; if Eq. (5) does not have a solution, $\lambda = -\infty$; $\tau = \Omega t$, where t is the time of revolution in the orbit, and Ω is the cyclotron frequency; $\gamma = 1/\Omega t_0$.

From this formula, it is easy to obtain the result that in a homogeneous field, with accuracy up to γ inclusive at a depth ξ, $r < \xi \leq l$,

$$\sigma_{ik} \sim \langle \overline{v_i(\tau)(r_k(\tau) - r_k(\lambda))} \rangle, \qquad \mathbf{v} = \partial\varepsilon/\partial\mathbf{p}, \quad (5a)$$

where the bar indicates averaging over τ, and the angular brackets averaging over p_z. (The physical meaning of this formula is very simple: each of the electrons bears a current $e\mathbf{v}$, and the number Δn of the electrons moving in this direction is proportional to the displacement of the Fermi surface, i.e., the energy acquired in the electric field is $\Delta\epsilon = e\mathbf{E}\Delta\mathbf{r}$.)

If λ corresponds to a collision with a smooth surface, then $\lambda = \lambda(t)$; if the collision is an ordinary, random one, one must average over λ; if correlation between λ and t is absent, λ does not depend on t.

It is seen from (5a) that, first, the "surface" components σ_1 are of the order of γ, while the "volume" components are of the order γ^2, i.e., a strong inhomogeneity takes place in the current; second, averaging over the coordinate $[d\xi = v_\xi(\lambda)d\lambda]$ causes this approximation to vanish (since $\overline{v}_x = \overline{v}_y = 0$) and, consequently, in the third place, the vanishing is connected essentially with the formerly assumed homogeneity, which makes it possible, instead of determining $E_\xi(\xi)$ and then averaging j_x, j_η over ξ, to carry out the averaging over ξ at once (see Sec. 1). From (5a), it is also easy to discern the physical reason for the given order in γ of all the components σ_{ik}.

In the works of Kaner [3] and Gurevich [4] on the determination of the effective conductivity, the kinetic equation for the distribution function was solved by expansion of the distribution function in a Fourier series in the coordinate. In this case, it was not taken into account that the distribution function experiences a finite jump, not only at the surfaces of the plate ($\xi = 0$, d) but also inside it In a magnetic field parallel to the surface of the plate, the jumps correspond to depths at which many electrons can generally arrive, these collide with the surface and have the given t and p_z (i.e., they arrive at a given depth in an orbit of given radius, determined by p_z, after passing through a given section of the orbit determined by the elapsed time t). In an oblique field, the jumps correspond to depths at which the electrons which collide with the surface have an additional rotation. It is easy to understand that, for example, for cross sections where $\overline{v}_\xi = 0$ (the central cross section $p_z = 0$ is always such a cross section, owing to the central symmetry), the jumps can be absent only in a magnetic field that is strictly perpendicular to the surface of the metal, if the corresponding orbit in ordinary space is plane, which is itself a special case.

The value of this jump can easily be determined; its contribution to the conductivity is not taken into account in [3,4]. [The method of transition to equations for the Fourier components in the coordinate was used earlier for the calculation of the high frequency surface impedance of metals both in a constant magnetic field [5-7], and also in its absence. [8] However, the jump of the distribution function in the magnetic field in the depth of the metal could not be taken into account there because of the high attenuation of the high frequency field with the depth, and also owing to the insignificance of the contribution of the electrons colliding with the surface (see [7]). In the static field, the electrons colliding with the surface play a dominant role, and a correct consideration of the discontinuities arising there is necessary (see also [9]).]

We now turn to the solution of the problem under analysis. For simplicity, we assume that all the trajectories are closed. First we write

$$\hat{\sigma}_{ik} = \sigma_{ik}^{\infty} - \hat{\sigma}'_{ik}, \quad E_\xi = E_\xi^{\infty} + E'_\xi,$$

where the superscript infinity corresponds to the ordinary solution without account of the surfaces of the sample (evidently, $E_x^\infty = E_x$, $E_\eta^\infty = E_\eta$, $\sigma_{\xi k}^\infty E_k^\infty = 0$), such that $\mathbf{E}^\infty = \text{const}$, $\lambda = -\infty$, and $\hat{\sigma}'$ is determined only by the electrons intersecting the surface. We get

$$j_\alpha = \sigma_{\alpha k}^\infty E_k^\infty - \sigma'_{\alpha k}(\xi)\, E_k^\infty + \hat{\sigma}_{\alpha\xi}E'_\xi = \tilde{\sigma}_{\alpha\beta}E_\beta - \sigma'_{\alpha k}(\xi)\, E_k^\infty + \hat{\sigma}_{\alpha\xi}E'_\xi; \quad (6)$$

$$\hat{\sigma}_{\xi\xi}E'_\xi = \sigma'_{\xi k}(\xi)\, E_k^\infty; \qquad E_\xi^\infty = -\sigma_{\xi\beta}^\infty E_\beta/\sigma_{\xi\xi}^\infty \quad (7)$$

($\alpha, \beta \sim x, \eta$; i, k $\sim x, \xi, \eta$), where $\tilde{\sigma}_{\alpha\beta}$ is the ordinary tensor of "longitudinal" conductivity ($j^\infty = 0$).

Using [1a], we write out the equations for $\tilde{\sigma}$ and E_ξ^∞. We put down the final formulas only for the two cases considered in what follows: a magnetic field parallel to the surface, and an essentially oblique magnetic field (the angle between the magnetic field and the surface $\varphi \sim 1$; by γ, which is not under the integral with respect to the momenta, we mean both here and in what follows a quantity having the order of magnitude of γ). We get

a) in the case $\varphi = 0$, so that $\eta \equiv z$, $\xi \equiv y$:

$$E_y^\infty = (\delta_1\gamma^{-1}\Delta\nu + \delta_2) E_x + \delta_3\gamma^{-1}E_z;$$

$$\frac{\tilde{\sigma}}{\sigma_0} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \text{ for } n_1 \neq n_2,$$

$$\frac{\tilde{\sigma}}{\sigma_0} = \begin{pmatrix} \gamma^2\beta_{11} & \gamma\beta_{12} \\ -\gamma\beta_{12} & \beta_{22} \end{pmatrix} \text{ for } n_1 = n_2; \tag{8}$$

b) in the case $\varphi \sim 1$:

$$E_\xi^\infty = -E_\eta \operatorname{ctg}\varphi + \gamma(\delta_1 E_x + \delta_2 E_\eta);$$

$$\frac{\tilde{\sigma}}{\sigma_0} = \begin{pmatrix} \gamma^2\beta'_{11} & \gamma\delta_{12}\Delta\nu + \gamma^2\beta'_{12} \\ -\gamma\delta_{12}\Delta\nu + \gamma^2\beta'_{12} & \gamma^2\beta'_{22} \end{pmatrix};$$

$$\Delta\nu = (n_1 - n_2)/n_1; \qquad \alpha, \beta, \delta \sim 1, \qquad \sigma_0 \sim \sigma|_{H=0}. \tag{9*}$$

The solution of the second and third parts of the problem is conveniently carried out separately for $\varphi = 0$ and $\varphi \sim 1$ (in accordance with this, electrons reflected from the surface penetrate to a depth of order r or l). We shall first consider the simpler case $\varphi = 0$.

3. RESISTANCE OF THE PLATE IN A MAGNETIC FIELD PARALLEL TO THE SURFACE

We return to the determination of $E'_y(y)$. We write down Eq. (7) in the first non-vanishing approximation in γ, not making any previous assumptions on the form and order of $E'_y(y)$. In this case, it is necessary to assume that

1) $v_y = \Omega dr_y/d\tau = \dot{p}$, where $r_y \sim r$ and is periodic in τ with period 2π (a consequence of the relation $\dot{\mathbf{p}} = (e/c)\mathbf{v} \times \mathbf{H}$ and the periodicity of motion in a closed orbit);

2) by the definition (5), $r_y(\lambda) = r_y(\tau) - y$, and only those electrons collide with the surface for which this equation for λ has a solution, i.e., for which $y \le d_{larg}$ [d_{larg} is the orbit diameter $d(p_x)$ largest in p_z, $d(p_z) = r_{max}(p_z) - r_{min}(p_z)$], the max and min of r_y are taken in τ (this imposes a limit on y) $d(p_z) \ge y$ (this imposes the limit on p_z), and $r_y(\tau) \ge y + r_{min}$ (the limit on τ), so that

$$\int_\lambda^\tau = s_+ \int_{-\infty}^\tau + s_- \int_\lambda^\tau; \qquad \int_{-\infty}^\lambda = (e^{2\pi\gamma} - 1)^{-1} s_- \int_\lambda^{\lambda+2\pi};$$

$$s_- = s(r_y(\tau) - r_{min} - y), \qquad s_+ = 1 - s_-;$$

$$s(w) = 1 \text{ for } w > 0, \qquad s(w) = 0 \text{ for } w < 0$$

(s_+ guarantees the absence, and s_- the presence, of a collision with the surface);

3) $v_y(\tau') E'_y(y - r_y(\tau) + r_y(\tau'))\, d\tau' = dF(\rho - \rho')$,

$$F(x) = \int_0^x E'_y(y + \Omega^{-1}\rho')\, d\rho'.$$

For simplicity, this equation is written close to the surface $y = 0$; for $d > d_{max}$, the "interference" between these surfaces is clearly absent. $E'_y(y)$ is determined by the solution of the problem for the half-space.

As a result, one can show that Eq. (7) has the form

$$\gamma^2 \int K\left(\frac{y - y'}{r}, \frac{y'}{r}\right) E'_y\left(\frac{y'}{r}\right) \frac{dy'}{r}$$

$$= \gamma\left\{\chi_1\left(\frac{y}{r}\right) E_x + \chi_2\left(\frac{y}{r}\right) E_z\right\};$$

$$\chi_1(0) \sim \chi_2(0) \sim 1. \tag{10}$$

The kernel K and the functions χ_1 and χ_2 do not depend upon l (which is natural, since the surface is shown to exist only at distances of the order r) and are damped out (the kernel for both arguments) at distances of the order of unity. Physically, this takes place because at a given point in y there can pass without "volume" collisions only those electrons which are at distances of order r away from it.

The zeroth approximation vanishes both on the left and on the right because $\bar{v}_y = 0$. The first approximation in γ on the left is absent because the electron which is reflected from the surface collides with it again at the same depth, traveling a path of the order r; the difference between the cases of equal and unequal numbers of "holes" and electrons does not appear because for collisions with the surface not only n_1 and n_2 are essential, but also the quantity $d(p_z)$, which is different for electrons and "holes;" the fundamental effect of the surface is connected with the fact that the electrons which would have the path length l have the path length r as a result of the collision with the surface.

Equation (10) shows that

$$E'_y(y) = \gamma^{-1}\{\chi_3(y/r) E_x + \chi_4(y/r) E_z\},$$

$$\chi_3(0) \sim \chi_4(0) \sim 1, \tag{11}$$

*ctg = cot.

where the functions χ_3 and χ_4 have the same character as χ_1 and χ_2, and are determined from an equation which no longer contains any parameters. These functions could easily be represented in terms of the function $G(\xi, \xi')$:

$$\int K(\xi, \xi')\, G(\xi', \xi'')\, d\xi' = s(\xi - \xi'').$$

We note that a comparison of (11) and (8) shows a sharp inhomogeneity of the Hall field E_y for $n_1 = n_2$.

The solution of Eq. (7) close to the surface $y = d$ has of course quite a similar character. Substituting the expression for E'_y in Eq. (6), we get as a fundamental approximation in γ (the functions χ_{ik} have the same character as χ_3 and χ_4):

$$\sigma_0^{-1} j_\alpha = s_{\alpha\beta} E_\beta; \qquad \alpha, \beta = x, z;$$

$$s_{\alpha\beta} = \begin{pmatrix} \alpha_{11} + \chi_{11} & \alpha_{12} + \chi_{12} \\ \alpha_{12} + \chi_{21} & \alpha_{22} + \chi_{22} \end{pmatrix} \text{ for } n_1 \neq n_2. \tag{12}$$

$$s_{\alpha\beta} = \begin{pmatrix} \gamma^2\beta_{11} + \chi_{11} & \gamma\beta_{12} + \chi_{12} \\ -\gamma\beta_{12} + \chi_{21} & \beta_{22} + \chi_{22} \end{pmatrix} \text{ for } n_1 = n_2. \tag{13}$$

The value of E_μ in the plate is determined, as was shown above, from the condition $\bar{j}_\mu = 0$. Here it is easy to see that the current density changes appreciably over distances of the order r, and for $n_1 = n_2$ and $\nu \neq z$, it falls off appreciably at this distance (Fig. 1)—the "skin effect" for the static current. However, it must be kept in mind that, in contrast with the skin effect in a variable field, 1) the current at infinity does not tend to zero but to a small finite value; 2) the tangential field does not depend on the depth.

By averaging the relation (13) over y, in accord with (2), we can convince ourselves that for $n_1 \neq n_2$ the resistivity tensor has in the first approximation in γ the same form as for an infinitely thick plate: the resistance approaches saturation in strong fields ($r \ll l$).

For $n_1 = n_2$, the resistivity tensor [which is obtained from (13)] has the form

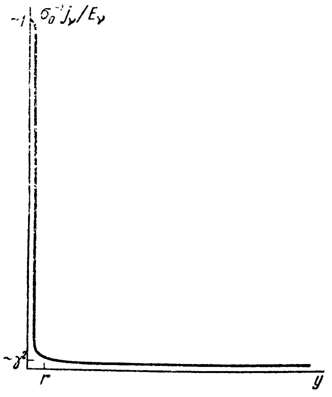


FIG. 1. Damping of a constant current in a parallel magnetic field.

$$\hat{\rho} \sim \sigma_0^{-1} (s_1 \gamma^2 + s_2 \gamma l/d)^{-1} \begin{pmatrix} \beta_3 & \beta_2\gamma \\ -\beta_2\gamma & \beta_1\gamma^2 + \delta_1\gamma l/d \end{pmatrix};$$

$$s_i, \beta_i, \delta_i \sim 1. \tag{14}$$

For $\gamma \gg l/d$, i.e., for $r \gg l^2/d$, which corresponds to the "intermediate" region of magnetic fields $H_0 \ll H \ll H_0 l/d$, $H_0 \sim cp_0/el$, the tensor $\hat{\rho}$ has the usual form. For $r \ll l^2/d$, i.e., $H \gg H_0 l/d$, in the case of a specimen with good surface ($\delta \ll l^2/d$, δ is the characteristic dimension of the random deformations of the surface)

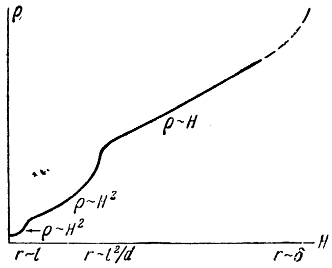
$$\hat{\rho} = \sigma_0^{-1} \begin{pmatrix} \beta'_3 \gamma^{-1} d/l & \beta'_2 d/l \\ \beta'_2 d/l & \beta'_1 \end{pmatrix}, \qquad \beta'_i \sim 1. \tag{15}$$

This signifies [see (3)] that in almost all directions of the magnetic field in the plane of the plate relative to the current, for $H \gtrsim H_0 d/l$ and apparently for $H \lesssim H_0 l/\delta$ (see Sec. 1), a transition takes place from the quadratic increase of resistance with the magnetic field to the linear increase: from $\rho \sim \rho_0 (H/H_0)^2$ to $\rho \sim \rho_0 (H/H_0) d/l \sim \rho_0 d/r$ (see Fig. 2). As was emphasized above, this takes place only in specimens with good surface. In this connection, the transformation of a bad surface to a good one leads for $H \gg H_0 d/l$ to a sharp decrease in the resistance and the transformation from $\rho \sim H^2$ to $\rho \sim H$. The case $\mathbf{H} \parallel \mathbf{j}$ is an exception, in which the resistance tends to saturation for $H \gg H_0$ in any case.

As is seen from the foregoing, the transition to the linear law $\rho \sim H$ is associated with the unusual "static skin effect" for a current in a strong magnetic field. We note that the linear law assumed here differs in principle from Kapitza's law: the former takes place both in single crystals and in polycrystals with closed electron orbits, but only for a good surface on the specimen, i.e., for $r \gg \delta$; the latter is possible only in polycrystals with a definite type of open trajectory [1b] and does not depend on the state of the surface of the specimen.

To conclude this section, we note that for all discussions we have essentially assumed only $d > 2r_{max}$, i.e., the absence of "interference" be-

FIG. 2. Dependence of $\rho(H)$ for $n_1 = n_2$.



tween the surfaces of the plate. Therefore the formulas that have been obtained are valid both for $l < d$ and also for $l > d > 2r_{max}$.

4. RESISTANCE OF THE PLATE IN AN OBLIQUE MAGNETIC FIELD

The solution for the oblique magnetic field is more complicated than for the parallel case. This is associated with the fact that

$$r_\xi(\tau) = \frac{1}{\Omega}\int^{\tau} v_\xi \, d\tau_1,$$

which entered into the argument E'_ξ at the right hand side of Eq. (4) is not a periodic function of τ in contrast with $r_y(\tau)$, which determines the argument of E'_y in the parallel field. Therefore, it is not possible to write down the first approximation in γ of (7) without making assumptions about the form of the function $E'_\xi(\xi)$.

Inasmuch as $d \gg l$, we limit ourselves, as was done in Sec. 3, to the determination of the shape of $E'_\xi(\xi)$ near the surface $\xi = 0$ (that is, for $\xi \lesssim l$). By computing the first approximation in γ of the right side of Eq. (7), we establish the fact that it is the sum of terms of two types: these fall off and change 1) at a distance of the order of l, and 2) at distances of the order of r. The "interference" terms, which fall off at a distance l and oscillate over the distance $\Delta\xi$, $l \gg \Delta\xi \gtrsim r$, are absent [the latter is not accidental; it can be shown that the presence of similar terms would have led, generally speaking, to the absence of a solution of Eq. (7)]. This suggests that perhaps $E'_\xi(\xi)$ also consists of a similar sum. The origin of the terms which are damped over a distance l is quite evident: the electrons reflected from the surface and passing into the depth of the metal carry with them information about the surface and about the field near it; they are damped as a result of volume collisions, i.e., exactly over a distance of the order of l.

In addition, there are electrons [always, with exception of the magnetic field normal to the surface and of $r_\xi(\tau)$ which is monotonic for all orbits] which, upon being reflected from the surface, return to it and again collide (such, for example, are the electrons of the central cross section which generally do not reach the depth of the metal, having $\bar{v}_\xi = 0$ as a result of the central symmetry of the Fermi surface). Such a repeated collision is possible for electrons which "in the mean" are flying both to the surface and away from it ($\bar{v}_\xi < 0$ or $\bar{v}_\xi > 0$), but only at distances of the order of r. Consequently, there is a change in the distribution function and in the current at distances of the order of r. (In a magnetic field parallel to the surface, all collisions of the electrons with the surface are of this sort, and both the current and E_ξ change only over a distance r.)

In this connection, the electrons must be divided into three groups for all computations: 1) $\bar{v}_\xi > 0$, with $\lambda \sim -\xi/|\bar{v}_\xi T|$; $\varphi \sim 1$ was taken precisely to make this formula valid (for $\varphi = 0$, $\bar{v}_\xi = 0$, $\xi \sim r$, $\lambda \sim \pi$ is possible); 2) $\bar{v}_\xi < 0$, ξ and p such that the electrons do not collide with the surface, $\lambda = -\infty$; 3) $\bar{v}_\xi < 0$, ξ and p such that the electrons having $\lambda \sim \pi$ collide with the surface. It is convenient to introduce the discontinuity function, as was done in Sec. 3.

Thus we make the following assumptions. We set

$$E'_\xi(\xi) = \chi(\xi) + \psi(\xi), \tag{16}$$

where the function $\psi(\xi)$ falls off and decreases over a distance l, while $\chi(\xi)$ does the same over the distance r. Comparing similarly changing terms separately[3] we get the same equation for χ, ψ. Transforming to dimensionless variables, and setting $a(\xi) = \tilde{a}(l\xi_1)$, $b(\xi) = \tilde{b}(r\xi_1)$, where a and b are functions which change over the distances l and r, respectively, we finally get

$$\gamma^2 \int L(\xi_1, \xi_1')\,\tilde{\chi}(\xi_1')\,d\xi_1' = \gamma\chi^{(\alpha)}(\xi_1)E_\alpha + \gamma h(\xi_1)\int_0^\infty \tilde{\chi}(\xi_1')\,d\xi_1' + \int_0^\infty K_1(\xi_1, \xi_1')\,\tilde{\psi}(\xi_1')\,d\xi_1', \tag{17}$$

$$\int_0^\infty K(|\xi_1 - \xi_1'|)\,\tilde{\psi}(\xi_1')\,d\xi_1' = \gamma\psi^{(\alpha)}(\ _1)E_\alpha + \gamma p(\xi_1)\int_0^\infty \tilde{\chi}(\xi_1')\,d\xi_1' + \gamma^2\Lambda(\xi_1)\int_0^\infty K_2(\xi_1')\,\tilde{\chi}(\xi_1')\,d\xi_1', \tag{18}$$

where all the known functions (i.e., all functions except $\tilde{\chi}$ and $\tilde{\psi}$) are of the order of unity at zero and change over distances of the order unity.

The Wiener-Hopf equation (18) makes it possible to express $\tilde{\psi}$ in terms of $\tilde{\chi}$ and, by substituting it in (17), to find $\tilde{\chi}$. The physically evident uniqueness of the solution of the initial integral equation makes it possible to conclude that the correct formula for $E'_\xi(\xi)$ is obtained in the same way, and that, consequently, the substitution justifies the initial assumption (16).

It is not difficult to understand from (17) and (18) that

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[3]In other words, from $f_1(\xi/r) + f_2(\xi/l) = 0$, where $f_1(x)$, $f_2(x)$ are damped out at $x \sim 1$ when $r \to 0$, we get $f_2 = 0$, whence $f_1 = 0$ also.

$$\tilde{\chi}(0) \sim \gamma^{-1}, \qquad \int_0^\infty \tilde{\chi}(\xi_1')\, d\xi_1' \sim 1, \qquad \tilde{\psi}(0) \sim \gamma; \tag{19}$$

$\tilde{\chi}(\xi_1)$ and $\tilde{\psi}(\xi_1)$ change and are damped out over distances of the order of unity. As was to have been expected, the form of $E_\xi^{\xi}(\xi)$ is the same for $n_1 = n_2$ and $n_1 \neq n_2$ (for the same reasons as in Sec. 3).

Now, substituting (16) and (19) in Eq. (6), we obtain the connection between $j_\alpha(\xi)$ and $E_\alpha(\alpha = x, \eta)$, and from $\bar{j}_\mu = 0$ we find E_μ and the relation of j_ν with E_ν. Here χ leads to a sharp, almost antisymmetric (since $\chi^2 \sim \gamma^{-1}$, and $\int_0^\infty \tilde{\chi} d\xi_1' \sim 1$) change in the current density over distances of the order of r, and ψ leads to a sharp change over distances of the order (Fig. 3) l—the "static skin effect" for the current.[4)]

Averaging the current density over the depth, we get the effective conductivity tensor $\hat{\sigma}$:

$$\frac{\hat{\sigma}}{\sigma_0} = \begin{pmatrix} \gamma^2\beta_{11}' & \gamma\delta_{12}\Delta\nu + \gamma^2\beta_{12}' \\ -\gamma\delta_{12}\Delta\nu + \gamma^2\beta_{12}' + \gamma(l/d)\,\delta_1\cos\varphi & \gamma^2\beta_{22}' + \gamma(l/d)\,\delta_2\cos\varphi \end{pmatrix}, \qquad \delta_{ik} \sim \delta_i \sim 1. \tag{20}$$

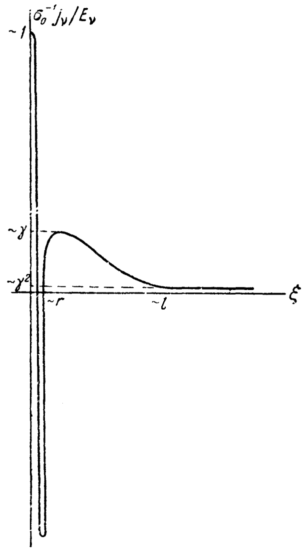

FIG. 3. Damping of a constant current in an oblique magnetic field; $\sigma_0 = \sigma|_{H=0}$.

It is seen from (20) that, as also for $\varphi = 0$, the ordinary formulas are valid in the field $H_0 \ll H \ll H_0(d/l)$.[1] For $\varphi = \pi/2$, that is, in a field normal to the surface, the ordinary formulas [1] are valid for $H \gg H_0$. Physically this is easy to understand. All the effects brought about by the surface, as was mentioned in Sec. 1, are connected with the fact that the inhomogeneity of the field, which arises because of the surface, changes the asymptote (averaged over the depth) of the "surface" conductivity tensor. If now $\xi \equiv z$, then only E_z is inhomogeneous (E_x, E_y = const), and the asymptotic values of σ_{iz} and σ_{zk} do not depend on the randomness of the volume collisions and therefore do not change.

The absence of corrections to $\sigma_{x\alpha}'$ is the result of $\bar{v}_x = 0$; this is also easy to obtain by considering the order of E_ξ.

We return to the resistivity in the case of a very strong field $H \gg H_0 l\, (r\cos\varphi)^{-1}$, when "screening" of the current by a layer of the order of l takes place. For $n_1 = n_2$, we have

$$\frac{\hat{\rho}}{\rho_0} = \begin{pmatrix} \gamma^{-2}\beta_1 & \gamma^{-2}\beta_2 \\ \gamma^{-1}(d/l)\,\beta_3 & \gamma^{-1}(d/l)\,\beta_4 \end{pmatrix}, \qquad \beta_i \sim 1. \tag{21}$$

Consequently, a difference from the ordinary formulas arises here when the direction of the resulting current, the magnetic field and the normal to the surface lie in a single plane. In this case, there is a linear law (Fig. 2)

$$\rho \sim \rho_0 (d/l)(H/H_0) \sim \rho_0 d/r, \qquad \rho \ll \rho_0 (H/H_0)^2, \tag{22}$$

and all the conclusions are analogous to the conclusions of Sec. 3, only the current density falls off over the distance l by a factor of γ^{-1} and not over r by a factor of γ^{-2}, as for the case $\varphi = 0$. Inasmuch as the $\rho_{\eta\eta} \sim \rho_0$ when $\varphi = 0$, the dependence of ρ on the angle φ between the magnetic field and the plane of the sample (that is, between z and η) has the form shown in Fig. 4a, i.e., the resistance changes quickly over a narrow range of angles close to $\varphi = 0$, and also close to $\varphi = \pi/2$ if $\psi = 0$ (ψ is the angle between ν and η). Upon rotation of the magnetic field in the plane of the plate ($\varphi = 0$), just as in the ordinary case, a sharp anisotropy takes place close to the direction $\mathbf{j} \parallel \mathbf{H}$ (only the transition takes place from $\rho \sim \rho_0 \sim \rho_0 d/r$, and not to $\rho \sim \gamma^{-2}\rho_0$, as in the ordinary case). The essential difference from the ordinary case lies in the presence, at $\varphi \neq 0$ or $\pi/2$, of a strong anisotropy close to the **H** direction, which lies in the $\nu\xi$ plane, upon rotation of the magnetic field which makes the given angle with the surface (Fig. 4b).

4)For $\varphi = 0$, the "skin effect" does not vanish at $n_1 = n_2$, owing to the sharp rise in E_ξ' as $\varphi \to 0$.

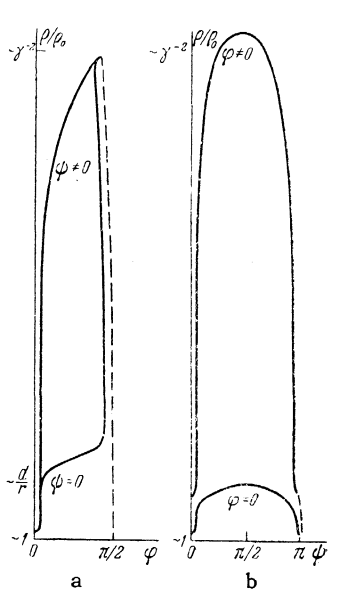

FIG. 4. Anisotropy in the dependence $\rho(\mathbf{H})$; $\rho_0 = \rho|_{H=0}$.

For $n_1 \neq n_2$,

$$\frac{\hat{\rho}}{\rho_0} = \begin{pmatrix} \gamma^{-1}(l/d)\beta_1' & \gamma^{-1}\beta_2' \\ -\gamma^{-1}\beta_2' & \beta_3' \end{pmatrix}, \qquad \beta_i' \sim 1. \tag{23}$$

Here only $\rho_{\eta\eta}$ always goes to saturation in strong fields $H \gg H_0$, as in the ordinary case.[1a] For all other directions of the magnetic field, the resistance is appreciably larger than in the ordinary case and increases linearly with the magnetic field and nowhere tends toward saturation:

$$\rho \sim \rho_0 (l/d)(H/H_0) \sim \rho_0 l^2/rd \gg \rho_0 \tag{24}$$

(see Fig. 5). For $r \gtrsim \delta$, a transition evidently takes place to the ordinary formulas; for the existence of a linear portion on the curve $\rho(H)$, it is necessary that $\delta \ll l^2/d$. The form of the polar diagrams of ρ/ρ_0 in φ for $\psi \neq 0$ and in ψ for $\varphi \neq 0, \pi/2$, in accord with Eqs. (21) and (23), are similar to $\rho(\psi)$ for $\varphi = 0$ in Fig. 4b (only $0 \leq \varphi \leq \pi/2$); for $\varphi = 0$ there is no strong anisotropy in ψ and for $\psi = 0$ there is no strong anisotropy in φ, as in the ordinary case.

At first glance, it appears that the increase of resistance (24) contradicts the fact that, in comparison with the ordinary case, the conductivity tensor of the deep layers of the metal does not change, the surface conductivity $\sigma_{\eta\eta}$ increases [see (20)] and the conductivity of the metal is determined by the surface layer (of thickness of the order of l). The reason for the increase in the resistance lies in the following. As is seen from (9), in an unbounded metal, where the current flows in the same direction everywhere, the larger conductivity ($\rho \sim \rho_0$) is associated with the appearance of the Hall field, γ^{-1} times larger than the field along the direction of the current. The increase of σ' in the case under our consideration leads to the result that there arises an anomalously large surface current. The condition for the continuity $\bar{j}_\eta = 0$ is guaranteed in the first approximation only in the surface layer, which gives (in the whole specimen) a Hall field of the order of the longitudinal field E_ν. As a result, the current in the ν direction in the interior of the specimen falls sharply off in comparison with the case of the unbounded metal, showing significantly less "surface" current which, in turn, is much less than the "bulk" current in the case of a bulk conductor. (In addition, in contrast to the unbounded sample, a rotational current arises in the metal.) Thus the increase in the resistance is associated with the sharp fall-off in the anisotropy of the "surface" conductivity tensor, which leads everywhere to a Hall field that is much less than the Hall field in the unbounded sample, and to $j_\mu \neq 0$ in the depth of the specimen. This is a consequence of the change of the "effective" boundary conditions, of which we spoke in Sec. 1.

5. RESISTANCE OF A WIRE IN A STRONG MAGNETIC FIELD

A knowledge of the solution for the plate makes it possible, for $d \gg l$, to find the resistance of a thin wire with $d \gg l$. We isolate a layer of thickness of the order of l close to the surface of the wire. Outside this layer, in the depth of the wire, there is no surface, $\mathbf{E} = \text{const} = \mathbf{E}^\infty$, and $j_i \sim \sigma_{ik}^\infty E_k^\infty$. In the layer isolated by us, any section of length d_1 with $l \ll d_1 \ll d$ can be regarded as a plate. Since all the quantities, and in particular the distribution function, vary appreciably only along the direction of the local normal $\boldsymbol{\xi}$, to the surface, one can repeat the discussions of Sec. 2. Inasmuch as the surface layer is decisive in a sufficiently strong magnetic field, a knowledge of the effective conductivity of the plate $\sigma = 1/\rho$ for $n_1 \neq n_2$ immediately yields $j_\nu = \sigma(\beta, \mathbf{H}) E_\nu$, and the resistance of the wire is equal to

$$\rho = 2\pi \Big/ \int_0^{2\pi} \sigma(\beta, \mathbf{H})\, d\beta;$$

β determines the direction of ξ_1. If $\mathbf{H}$ is not directed along the axis of the wire ν, then for $r \ll l^2/d$ the distance $\rho \sim \rho_0 l^2/rd$ (if $\mathbf{H} \parallel \nu$, $\rho \sim \rho_0$ everywhere); the current flows in a layer of thickness of the order of l.

For $n_1 = n_2$, the conditions for the appearance of the "skin effect" become somewhat more stringent. The fact is that, in accord with Secs. 3, 1, account of the dependence of the Hall effect on a single coordinate only leads to a "skin effect" for selected values of β. The boundedness of the dimensions of the specimen in the two directions gives a dependence of the Hall fields on two coordinates (in the kernel of the integral equation, λ depends on two coordinates). As a result, carrying out calculations similar to the one-dimensional case, and determining F ($E = -\operatorname{grad} F$) from the condition $\rho' = 0$, we get in all components of the effective conductivity tensor contributions of the order $\gamma l^2/d_1 d_2$ (d_1, d_2 are the characteristic diameters of the wire), and for $r \ll l\,(l^2/d_1 d_2)$ the value $\rho \sim \rho_0 (l/r)(d_1 d_2/l^2)$. The cases $\mathbf{H} \perp \nu$, where $\rho \sim \rho_0 (l/r)^2$, and $\mathbf{H} \parallel \nu$, where $\rho \sim \rho_0$, are exceptions.

It is curious that if the transverse cross section of the wire is polygonal (with a sufficient degree of accuracy) and $\mathbf{H}$ is not parallel to one of the planes that bound the wire and is not perpendicular to its axis, then the "skin effect" takes on an unusual character: for $\delta \ll r \ll l\,(l^2/d_1 d_2)$ the total current also (and not only the current density) flows fundamentally only in the angles of the polygon at distances from the vertices of the order of l. The difference between the case of a smooth curve which bounds the transverse cross section and the polygon lies in the fact that in the first case there is everywhere a weak dependence on two coordinates, and the current is fundamentally uniformly distributed in a layer of the order of l; in the second case, there is a region of essential dependence on both coordinates—in the angles of the polygon, where the same current is centered; in the remaining places, close to the surface, the dependence on the second coordinate is exponentially weak.

It is clear from the foregoing that the presence of hills and valleys on the surface can have a striking effect on the resistance of the wire, because of the strong anisotropy of the effective conductivity. The parts which are oriented in a special manner relative to the magnetic field (for example, for the case $n_1 = n_2$, the parts of the surface parallel to the magnetic field, where the conductivity has a sharp maximum) give an anomalously high (and in a sufficiently strong magnetic field, the definitive) contribution to the conductivity. In particular, the character of the anisotropy ρ of a polygonal wire differs essentially from the character of the anisotropy of ρ for a cylindrical wire.

As a result, the resistance is seen to be very sensitive to the form of the configuration of the wire surface. For $\mathbf{H}$ not parallel and not perpendicular to ν, and for cylindrical wire, $\rho(\mathbf{H})$ is the same as in Fig. 5 for $n_1 \neq n_2$; for $n_1 = n_2$, the form of $\rho(\mathbf{H})$ differs from that shown in Fig. 2 only in that the linear portion begins at $r \sim l^3/d_1 d_2$.

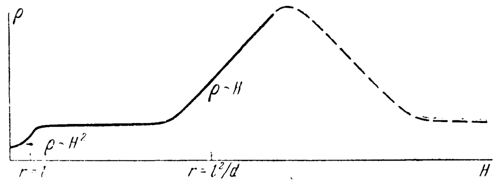

FIG. 5. Dependence of $\rho(H)$ for $n_1 \neq n_2$.

The anisotropy of the resistance (as also the values of the Hall field in all cases) can easily be found from the formulas that have been given (in particular, for a cylindrical wire and $n_1 = n_2$, the dependence of ρ on the angle (z, ν) is the same as $\rho(\varphi)$ for $\psi = 0$ in Fig. 4a).

We note that the temperature dependence of the resistance has a similar character; this can be obtained directly from the given formulas upon the substitution $l^{-1} = l_i^{-1} + l_{ee}^{-1} + l_{eph}^{-1}$, where $l_i \sim \text{const}$, $l_e \sim T^{-2}$, $l_{eph} \sim T^{-5}$ are the lengths of the paths determined by collisions with impurities, electrons, and phonons respectively.

6. CONCLUSION

1. In a sufficiently strong magnetic field, the resistance of a wire with a "good" surface ($\delta \ll r$) is generally proportional to the magnetic field. For $n_1 \neq n_2$ and $\mathbf{H}$ not parallel to ν, the resistance $\rho \sim \rho_0 l^2/rd_1$ for $r \ll l^3/d_1 d_2$ (for $\mathbf{H} \parallel \nu$, we have $\rho \sim \rho_0$); for $n_1 = n_2$ and $\mathbf{H} \cdot \nu \neq 0$, the value of ρ is $\rho \sim \rho_0 d_1 d_2/rl$ for $r \ll l^3/d_1 d_2$ (for $\mathbf{H} \cdot \nu = 0$, we have $\rho \sim \rho_0 \gamma^{-2}$); $d_2 > d_1$. In the first case ($n_1 \neq n_2$) the temperature dependence of ρ is determined by the relation $\rho \sim l$, in the second ($n_1 = n_2$), by the relation $\rho \sim l^{-2}$.

The transformation of a "bad" surface into a "good" surface leads in a sufficiently strong magnetic field, for the case $n_1 = n_2$, to a decrease in the resistance by a factor of $l^3/rd_1 d_2$, while for $n_1 \neq n_2$ it leads to an increase in field by a factor of l^2/rd_1.

2. The resistance of the plate (the length of which $d_1 \gg l^2/r$) for $n_1 \neq n_2$ has the following form in the general case: $\rho \sim \rho_0 l^2/rd$. We have the following exceptions when $\rho \sim \rho_0$, as in the "ordinary" case: a) the magnetic field parallel or perpendicular to the surface of the plate; b) the magnetic field lying in a plane with the normal ξ to the surface of the plate and to the direction of the total current ν.

The dependence of the resistance on the angle φ (between **H** and ξ) and ψ (between the projection of **H** on the surface of the plate and ν) is shown in Fig. 4. For $\psi = 0$ and $\varphi = 0$ and $\pi/2$, the strong anisotropy is absent.

3. The resistance of the plate (the length of which $d_1 \gg l^2/r$) for $n_1 = n_2$ is the same in the general case as ordinarily: $\rho \sim H^2$. An exception is the case of the magnetic field parallel to the surface ($\varphi = 0$) but not parallel to the total current, or lying in a single plane with ξ and ν ($\psi = 0$), but not perpendicular to the surface of the plate: in both cases $\rho \sim \rho_0 d/r$. The anisotropy of the resistance is shown in Fig. 4.

4. The transformation of a ''bad'' surface (with deformations of the order r) of the plate into a ''good'' surface (with deformations much smaller than r) leads to cases which differ from the ordinary (see points 2 and 3 in the conclusions) to an increase ($n_1 \neq n_2$) or decrease ($n_1 = n_2$) in the resistance by a factor l^2/rd.

5. All the new effects noted above are associated with the ''static skin effect'' for a current in a strong magnetic field. In a magnetic field parallel to the surface, the current flows primarily in a layer of thickness of the order of r, in an oblique field, in a layer of the order of l in the plate and in a cylindrical wire, and at angles (at distances of the order l from the vertices of the angles) of a polygon, if the transverse cross section of the wire takes such a form. In a multi-connected sample, both the external and internal surfaces are important.

6. The experimental investigation of the dependence ρ(**H**) makes it possible to determine the length of path l, the radius of the orbit r and the dimensions of random deformations of the surface (which characterize it qualitatively) from a large number of characteristic points in a single experiment on a single specimen.

The determination of l and r from different points for a single specimen makes it possible in addition to make clear to what degree the values of l and r are characteristic for the material and do not depend on the concrete experiment.

Note added in proof (February 12, 1963). It is easy to note that the form of the statistics did not play a role in the conclusions; therefore upon satisfaction of the conditions given in the work for r and l, these results will be valid for semiconductors also.

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