SOME ISOTOPIC RELATIONS FOR REACTIONS INVOLVING FOUR PARTICLES IN THE FINAL STATE

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Assuming charge invariance, relations between the cross sections for reactions involving four particles in the final state are derived under the condition of a resonance interaction in the three particle system. Resonances in the $K\overline{K}\pi$, $K\pi\pi$ (or $\overline{K}\pi\pi$) and $K\overline{K}\overline{K}$ systems are considered.

It is well-known that the hypothesis of charge invariance of the strong interactions leads to a whole series of relations between the cross sections for reactions going through various isotopic spin channels. Many such relations have recently been obtained in a series of articles, and those in particular which appear under the condition of a resonance interaction of the particles in the final state were investigated in detail.[1-3] The comparison of such relations with experiment enables us to determine, for example, the isotopic spin of the resonant state.

In this connection, the isotopic spin analysis in the articles specified was carried out only under the assumption of a resonance interaction in a two particle system. However, as is known, resonances in the three π meson system were recently experimentally discovered, and it is expected that experimental investigations aimed at observing possible three-particle resonances in other elementary particle systems (for example, K mesons and hyperons) will be undertaken in the next year. These resonant states may be observed in reactions involving four particles in the final state as certain "peaks" in the distribution of the effective mass Q of the three particles.

At the same time, it is of interest to perform an analysis of such reactions, starting from considerations of charge invariance of the strong interactions. The presence of a resonance interaction in a state with a definite value of the isotopic spin T of the system of particles leads to certain relations between the cross sections which, when compared with experiment, would enable us to determine the isotopic spin of a given three-particle resonant state.

In the present article, we obtain a set of relations between reaction cross sections under the condition of a resonance interaction in the systems $K\overline{K}\pi$, $K\pi\pi$ (or $\overline{K}\pi\pi$) and $K\overline{K}\overline{K}$. The investigation of the possibility of resonances in such systems is of great interest, since it would enable us to test [4,5] certain theoretical conjectures.

In certain special cases, additional relations appear which correspond to the case when the interaction of two particles in a definite isotopic spin state dominates in the three-particle resonant state. A comparison of such relations with experimental data may turn out to be useful in connection with certain theoretical considerations about three-particle resonances.[6]

We consider reactions of the type

$$\pi^- + p \to N + K + \overline{K} + \pi, \quad (1)$$

$$K^- + p \to Y + K + \overline{K} + \pi, \quad (2)$$

$$\pi^- + p \to Y + K + \pi + \pi, \quad (3)$$

$$K^{\pm} + p \to N + K\,(\overline{K}) + \pi + \pi, \quad (4)$$

$$\overline{p} + p \to K + \overline{K} + \pi + \pi. \quad (5)$$

Here Y denotes a Λ or Σ hyperon.

Selecting $K\overline{K}\pi$ as the subsystem for reactions (1) and (2), and $K\pi\pi$ as the subsystem for reactions (3)—(5), in the first case we obtain for the amplitudes of the processes

$$M = \sum_{T\tau t} C\left(\lambda, \tfrac{1}{2}, T;\ \lambda_3, \tfrac{1}{2}\right) C\left(1, \tfrac{1}{2}, t;\ \alpha, \beta\right) \times C\left(t, \tfrac{1}{2}, \tau; \alpha + \beta, \gamma\right) C\left(\tau\eta T; \alpha + \beta + \gamma, \eta_3\right) A_t^{\tau T}. \quad (6)$$

Here the C (jj'J; mm') are Clebsch-Gordan coefficients; λ and λ_3 are the isospin and the third-component of the isospin of the π (K) meson in the initial state; η and η_3 are the same quantities for the nucleon (hyperon) in the final state; α, β, γ are the third-components of the isospin of the π, $\overline{K}$, and K mesons, respectively; t is the total isospin of the π and $\overline{K}$ mesons, τ is the same for the $K\overline{K}\pi$ system, and T is the total isospin of the final or initial state.

A similar expression also occurs for the amplitudes of reactions (3)—(5):

$$M=\sum_{T\tau t} C\left(\lambda, \tfrac{1}{2}, T;\ \lambda_3, \tfrac{1}{2}\right) C\,(1, 1, t; \alpha, \beta)$$

$$\times C\left(t, \tfrac{1}{2}, \tau; \alpha+\beta, \gamma\right) C\,(\tau\eta T; \alpha+\beta+\gamma, \eta_3)\, B_t^{\tau T}. \quad (7)$$

Here the notation is similar to that of the preceding case and does not require special explanation, with only this difference: in the final state the K meson is replaced by a π meson.

The amplitudes $A_t^{\tau T}$ and $B_t^{\tau T}$ are functions of the momenta (and other variables) of the corresponding particles, and furthermore $B_t^{\tau T}$ is multiplied by $(-1)^t$ upon inversion of the momenta of the π mesons. Using (6) and (7), one can obtain the following relations for the case in which there is a resonance with isotopic spin $\tau = 2$ in the $K\bar{K}\pi$ system:

$$\sigma\,(\pi^-p \to pK^+K^-\pi^-) = \sigma\,(\pi^-p \to pK^0\bar{K}^0\pi^-),$$
$$\sigma\,(\pi^-p \to nK^0K^-\pi^+) = \sigma\,(\pi^-p \to nK^+\bar{K}^0\pi^-),$$
$$\sigma\,(\pi^-p \to pK^+K^-\pi^-) : \sigma\,(\pi^-p \to pK^0K^-\pi^0)$$
$$: \sigma\,(\pi^-p \to nK^0K^-\pi^+) = 9:18:4, \quad (8)$$

$$2\sigma\,(K^-p \to \Sigma^+K^+K^-\pi^-) = 2\sigma\,(K^-p \to \Sigma^+K^0\bar{K}^0\pi^-)$$
$$= 2\sigma\,(K^-p \to \Sigma^-K^+K^-\pi^+) = 2\sigma\,(K^-p \to \Sigma^-K^0\bar{K}^0\pi^+)$$
$$= \sigma\,(K^-p \to \Sigma^+K^0K^-\pi^0) = \sigma\,(K^-p \to \Sigma^-K^+\bar{K}^0\pi^0). \quad (9)$$

The presence in the $K\bar{K}\pi$ system of resonances with isotopic spin $\tau = 1$ or $\tau = 0$ would lead to the second of relations (8); however, in case of a resonance with $\tau = 0$ reactions in which the charge of the $K\bar{K}\pi$ system is not equal to zero would be strongly suppressed. As far as reactions (2) are concerned, the presence of resonances with $\tau = 1$ and $\tau = 0$ would lead to the following relations:

for $\tau = 1$

$$\sigma\,(K^-p \to \Lambda K^+\bar{K}^0\pi^-) = \sigma\,(K^-p \to \Lambda K^0K^-\pi^+),$$
$$\sigma\,(K^-p \to \Sigma^0K^+\bar{K}^0\pi^-) = \sigma\,(K^-p \to \Sigma^0K^0K^-\pi^+); \quad (10)$$

for $\tau = 0$, besides (10)

$$2\sigma\,(K^-p \to \Lambda K^+K^-\pi^0) = \sigma\,(K^-p \to \Lambda K^0K^-\pi^+). \quad (11)$$

We remark that relation (11) will also be fulfilled in that case when the reaction goes through a resonant state of the $K\pi$ system with isospin $T = 1/2$.

If there is a resonance in the $K\pi\pi$ system with isospin $\tau = 5/2$, then

$$\sigma\,(\pi^-p \to \Sigma^+K^+\pi^-\pi^-) : \sigma\,(\pi^-p \to \Sigma^+K^0\pi^0\pi^-) :$$
$$\sigma\,(\pi^-p \to \Sigma^-K^+\pi^-\pi^+):$$
$$\sigma\,(\pi^-p \to \Sigma^-K^0\pi^0\pi^+) = 2:8:1:2. \quad (12)$$

In the event of resonances in the $K\pi\pi$ (or $\bar{K}\pi\pi$) system with $\tau = 3/2$, we have

$$\sigma\,(K^-p \to pK^-\pi^+\pi^-) = \sigma\,(K^-p \to n\bar{K}^0\pi^+\pi^-),$$
$$\sigma\,(\bar{p}p \to K^+K^-\pi^+\pi^-) = \sigma\,(\bar{p}p \to K^0\bar{K}^0\pi^+\pi^-),$$
$$18\sigma\,(K^+p \to pK^0\pi^0\pi^+) = 9\sigma\,(K^+p \to pK^+\pi^-\pi^+)$$
$$+ 4\sigma\,(K^+p \to nK^0\pi^+\pi^+). \quad (13)$$

If it turns out that the isospin of the resonant state of the $K\bar{K}\pi$ system is $\tau = 1$ or else $\tau = 3/2$ or $\tau = 1/2$ for the $K\pi\pi$ system, then, besides relations (8)—(13), additional equalities between the reaction cross sections emerge in certain limiting cases. It may turn out that not all resonant amplitudes will have the same order of magnitude, depending on different values of the lower index (for example, A_t^{1T} for different possible values of t).

Thus, if $|A_{1/2}^{1T}| \gg |A_{3/2}^{1T}|$, we have in this limiting case

$$2\sigma(\pi^-p \to pK^0K^-\pi^0) = \sigma(\pi^-p \to pK^0\bar{K}^0\pi^-),$$
$$2\sigma(K^-p \to \Lambda K^+K^-\pi^0) = \sigma(K^-p \to \Lambda K^+\bar{K}^0\pi^-),$$
$$2\sigma(K^-p \to \Sigma^-K^+\bar{K}^0\pi^0) = \sigma(K^-p \to \Sigma^-K^+K^-\pi^+),$$
$$2\sigma(K^-p \to \Sigma^+K^0K^-\pi^0) = \sigma(K^-p \to \Sigma^+\bar{K}^0K^0\pi^-). \quad (14)$$

In the opposite case, for $|A_{3/2}^{1T}| \gg |A_{1/2}^{1T}|$, we have

$$18\sigma\,(\pi^-p \to pK^0\bar{K}^0\pi^-) = 2\sigma\,(\pi^-p \to pK^+K^-\pi^-)$$
$$= 9\sigma\,(\pi^-p \to pK^0K^-\pi^0),$$
$$2\sigma\,(K^-p \to \Sigma^+K^0\bar{K}^0\pi^-) = \sigma\,(K^-p \to \Sigma^+K^0K^-\pi^0),$$
$$2\sigma\,(K^-p \to \Lambda K^+\bar{K}^0\pi^-) = \sigma\,(K^-p \to \Lambda K^+K^-\pi^0),$$
$$18\sigma\,(K^-p \to \Sigma^-K^+K^-\pi^+) = 2\sigma\,(K^-p \to \Sigma^-K^0\bar{K}^0\pi^+)$$
$$= 9\sigma\,(K^-p \to \Sigma^-K^+\bar{K}^0\pi^0). \quad (15)$$

If $|B_1^{\tau T}| \gg |B_0^{\tau T}|, |B_2^{\tau T}|$, then for a resonant state of the $K\pi\pi$ (or $\bar{K}\pi\pi$) system with isospin $\tau = 3/2$, we have

$$2\sigma(K^+p \to pK^0\pi^0\pi^+) = \sigma(K^+p \to pK^+\pi^+\pi^-),$$
$$2\sigma(\pi^-p \to \Lambda K^+\pi^0\pi^-) = \sigma(\pi^-p \to \Lambda K^0\pi^-\pi^+),$$
$$2\sigma(\pi^-p \to \Sigma^0K^+\pi^0\pi^-) = \sigma(\pi^-p \to \Sigma^0K^0\pi^-\pi^+),$$
$$2\sigma\,(\pi^-p \to \Sigma^-K^0\pi^0\pi^+) = \sigma\,(\pi^-p \to \Sigma^-K^+\pi^-\pi^+),$$
$$2\sigma\,(K^-p \to p\bar{K}^0\pi^0\pi^-) = \sigma\,(K^-p \to pK^-\pi^+\pi^-),$$
$$2\sigma\,(\bar{p}p \to K^+\bar{K}^0\pi^0\pi^-) = 2\sigma\,(\bar{p}p \to K^0K^-\pi^0\pi^+)$$
$$= \sigma\,(\bar{p}p \to K^+K^-\pi^-\pi^+). \quad (16)$$

In the case of a resonance interaction with $\tau = 1/2$

$$2\sigma\,(K^+p \to pK^+\pi^-\pi^+) = \sigma\,(K^+p \to pK^0\pi^0\pi^+),$$
$$2\sigma\,(\pi^-p \to \Lambda K^0\pi^-\pi^+) = \sigma\,(\pi^-p \to \Lambda K^+\pi^0\pi^-),$$
$$2\sigma\,(\pi^-p \to \Sigma^0K^0\pi^-\pi^+) = \sigma\,(\pi^-p \to \Sigma^0K^+\pi^0\pi^-),$$
$$2\sigma\,(\pi^-p \to \Sigma^-K^+\pi^-\pi^+) = \sigma\,(\pi^-p \to \Sigma^-K^0\pi^0\pi^+),$$
$$2\sigma\,(K^-p \to pK^-\pi^-\pi^+) = \sigma\,(K^-p \to p\bar{K}^0\pi^0\pi^-),$$
$$2\sigma\,(\bar{p}p \to K^+K^-\pi^-\pi^+) = \sigma\,(\bar{p}p \to K^+\bar{K}^0\pi^0\pi^-),$$
$$2\sigma\,(\bar{p}p \to K^0\bar{K}^0\pi^-\pi^+) = \sigma\,(\bar{p}p \to K^0K^-\pi^0\pi^+). \quad (17)$$

In conclusion, we quote the relations which emerge for the case of a $K\bar{K}\bar{K}$ resonance with isospin $\tau = 3/2$:

$$2\sigma(K^-p \rightarrow pK^+K^-K^-)$$
$$= \sigma(K^-p \rightarrow pK^0\bar{K}^0K^-) = \sigma(K^-p \rightarrow nK^+\bar{K}^0K^-). \quad (18)$$

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