THE COALESCENCE OF PHOTONS IN THE COULOMB FIELD OF A NUCLEUS

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Expressions are obtained for the probability of the coalescence of photons in the Coulomb field of a nucleus, in the limiting cases of photons with frequencies large and small compared with the electron mass. For high frequencies the coalescence of photons is investigated by means of the dispersion-relation method. The low-frequency case is treated by means of the radiative corrections to the Lagrangian for the electromagnetic field.

1. One of the nonlinear effects of quantum electrodynamics which has not been studied so far is the coalescence of photons in the Coulomb field of a nucleus. The coalescence of photons can occur not only in an external field, but also in interaction with particles, for example electrons (this process is obviously the inverse of the double Compton effect). For the case of electrons this process has been treated by Fried [1] for small frequencies. We shall be principally interested in the coalescence of photons at large frequencies $\omega_i \gg m$ (where m is the mass of the electron and ω_i is the frequency of the photons). As will be shown below, in the high-frequency region the main mechanism for the coalescence of photons is the process in the Coulomb field of a nucleus. In the low-frequency region, on the other hand, the main effect is the coalescence in interaction with electrons.

For the high-frequency region we shall study the process of coalescence in the Coulomb field of a nucleus by means of the method of dispersion relations. We find the expression for the probability of the coalescence of photons in this range of frequencies. For comparison, the contribution from the coalescence on electrons is also estimated for the high-frequency case. Besides this we treat the case of coalescence of photons in the Coulomb field of a nucleus in the low-frequency region also. To do this we use the expression for the radiative corrections to the Lagrangian for the electromagnetic field.

2. In studying the coalescence of photons in the Coulomb field of a nucleus by the method of dispersion relations, as in the treatment of photon-photon scattering, [2] it is convenient to deal with the total amplitude A, which is connected with the matrix element M for the coalescence by the relation

$$M\delta(\omega_1 + \omega_2 - \omega_3) = -\frac{\pi}{2} Ze^5 (2\omega_1\omega_2\omega_3)^{-1/2} \int \frac{d^3q}{q^2} A\delta^4(k_1 + k_2 - k_3 - \mathbf{q}), \quad (1)$$

where k_1, k_2, and k_3 are the four-momenta of the photons before and after the coalescence (ω_1, ω_2, and ω_3 are the frequencies of these photons), Ze is the charge of the nucleus, and **q** is the momentum transferred to the nucleus.

The amplitude A can be written in the form

$$A = A_1 + A_2 + A_3 + A_{1e} + A_{2e} + A_{3e},$$

where the partial amplitudes A_1, A_2, A_3 correspond to the following processes:

$$(k_1, e_1) + (k_2, e_2) \to (k_3, e_3), \quad (2.I)$$
$$(-k_3, e_3) + (k_2, e_2) \to (-k_1, e_1), \quad (2.II)$$
$$(k_1, e_1) + (-k_3, e_3) \to (-k_2, e_2) \quad (2.III)$$

(e_i are the polarization vectors of the photons). The partial amplitudes A_{1e}, A_{2e}, A_{3e} are those for the exchange processes with respect to the variables (k_1, e_1) and (k_2, e_2), and are obtained from A_1, A_2, A_3 by the interchange (k_1, e_1) $\rightleftarrows$ (k_2, e_2).

The amplitudes A_1, ... A_{1e}, ... depend both on the scalar products k_ie_j and on the scalar invariants $s = -(k_1 + k_2)^2$, $t = (k_1 - k_3)^2$, $u = (k_2 - k_3)^2$, which because of the conservation law are connected by the relation $-s + t + u = q^2$. Owing to the crossing symmetry of the processes (2), the expressions for all of these amplitudes can be obtained from the expression for any one partial amplitude, for example A_1, by means of interchanges:

$$A_1 \to A_2 \text{ for } (k_1, e_1) \leftrightarrow (-k_3, e_3), \quad s \leftrightarrow -u, \quad t \to t;$$
$$A_1 \to A_3 \text{ for } (k_2, e_2) \leftrightarrow (-k_3, e_3), \quad s \leftrightarrow t, \quad u \to u;$$
$$A_1 \to A_{1e} \text{ for } (k_1, e_1) \leftrightarrow (k_2, e_2), \quad t \leftrightarrow u, \quad s \to s;$$
$$A_1 \to A_{2e} \text{ for } (k_1, e_1) \to (-k_3, e_3) \to (k_2, e_2) \to (k_1, e_1), \quad s \to -t \to -u \to s;$$

$$A_1 \to A_{3e} \text{ for } (k_2, e_2) \to (-k_3, e_3) \to (k_1, e_1) \to (k_2, e_2),$$
$$s \to -u \to -t \to s. \tag{3}$$

Therefore it suffices to consider the amplitude in the first channel.

For definite real values of the variables s, t, and u the amplitudes A_1, ... have both real and imaginary parts. The imaginary part of the amplitude A_1 is connected with an element of the scattering matrix $S = 1 + iT$ by the relation

$$-\frac{i}{2}\langle k_3, e_3 | T - T^+ | k_1, e_1; k_2, e_2\rangle$$
$$= -\frac{\pi}{2} Ze^5 (2\omega_1\omega_2\omega_3)^{-1/2} \int \frac{d^3q}{q^2} \operatorname{Im} A_1 \delta^4 (k_1 + k_2 - k_3 - q),$$

where T^+ is the operator which is the Hermitian adjoint of the operator T.

It follows from the unitarity condition on the operator T

$$T - T^+ = iT^+T \tag{4}$$

that in the first nonvanishing order of perturbation theory (in the order Ze^5) $\operatorname{Im} A_1$ is represented by the Feynman diagrams shown in Fig. 1 (electron lines with cross marks denote free particles).

As we shall see, the imaginary parts of the amplitudes A_1, A_{1e} are different from zero for $s \geq 4m^2$, $\operatorname{Im} A_2$ and $\operatorname{Im} A_{3e}$ for $u \leq -4m^2$, and $\operatorname{Im} A_3$ and $\operatorname{Im} A_{2e}$ for $t \leq -4m^2$.

In what follows we confine ourselves to the treatment of the coalescence of photons under the condition that the momenta of the photons in the initial state are parallel to each other (this is the most interesting case, since then the momentum transferred to the nucleus can be a very small quantity, which leads to a large value of the probability of coalescence). To this case there corresponds the value of the invariant $s = 0$.

When now for fixed s we try to find from the imaginary part of an amplitude an expression for the real part, it is convenient to use the dispersion relations in the form of Eq. (2.6) of a paper by Mandelstam [3] [see the expression (14) of the present paper]. For these dispersion relations it is necessary to know the imaginary parts of the amplitudes A_2, A_3, A_{2e}, A_{3e}. Since it is more customary to deal with channel (2.I), we shall first calculate the imaginary part of the amplitude A_1

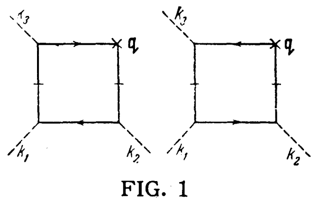

FIG. 1

for the values of s, t, u which correspond to $s = 0$ in channels II, III, IIe, IIIe. We then get the expressions for $\operatorname{Im} A_2$, $\operatorname{Im} A_3$, $\operatorname{Im} A_{2e}$, $\operatorname{Im} A_{3e}$ from $\operatorname{Im} A_1$ by using the crossing-symmetry property (3).

For this purpose we write down the explicit expression for $\operatorname{Im} A_1$, corresponding to the diagrams of Fig. 1,

$$\operatorname{Im} A_1 = \frac{1}{4\pi^2} \int d^4v\,\delta(kv)\,\delta(v^2 - s + 4m^2) \frac{S}{(vp - s)(vp' - s - q^2)}, \tag{5}$$

$$S = \operatorname{Sp}\left(\frac{i}{2}(\hat{k} + \hat{v}) - m\right)$$
$$\times \gamma_\mu \left(\frac{i}{2}(\hat{v} - \hat{p}) - m\right) \gamma_\nu \left(\frac{i}{2}(\hat{k} - \hat{v}) + m\right)$$
$$\times \gamma_4 \left(\frac{i}{2}(\hat{v} - \hat{p}') - m\right) \gamma_\sigma e_{1\mu} e_{2\nu} e_{3\sigma}, \tag{5'}$$

where $v = p_1 - p_2$, $k = k_1 + k_2$, $p = k_1 - k_2$, $p' = k_3 - q = 2k_3 - k_1 - k_2$ [p_1 and p_2 are the four-momenta of the free electron and positron in the unitarity condition (4)], and $\hat{k} = k_\mu \gamma_\mu$ (γ_μ are the Dirac matrices).

3. Let us now get the expression for the coalescence amplitude A in the case of large photon frequencies $\omega_i \gg m$. In the variables s, t, and u (when we keep in mind that the momenta of the photons in the initial state are parallel to each other) the condition $\omega_i \gg m$ is equivalent to $s = 0$, $t = 2\omega_1\omega_3 (1 - \cos\theta) \gg 4m^2$, $u = 2\omega_2\omega_3 (1 - \cos\theta) \gg 4m^2$, where θ is the angle at which the photon formed by the coalescence emerges. θ is consequently subject to the restriction $\theta \gg \max[2m(\omega_1\omega_3)^{-1/2}, 2m(\omega_2\omega_3)^{-1/2}]$.

Let us begin with channels III and IIIe. To get the expressions for $\operatorname{Im} A_3$ and $\operatorname{Im} A_{3e}$ for $s = 0$, according to crossing symmetry, Eq. (3), we must calculate the imaginary part of the amplitude A_1 for $t = 0$. If $t = 0$ the amplitude A_1 can be represented in the form

$$A_1 = (2\pi)^{-2} [a_1 \delta_{\mu\nu} k_{2\sigma} + b_1 \delta_{\nu\sigma} k_{2\mu} + c_1 \delta_{\mu\sigma} k_{1\nu} + d_1 \delta_{\mu\sigma} k_{3\nu}$$
$$+ f_1 k_{2\mu} k_{1\nu} k_{2\sigma} + g_1 k_{2\mu} k_{3\nu} k_{2\sigma}] e_{1\mu} e_{2\nu} e_{3\sigma}. \tag{6}$$

From the condition that the expression (6) be gauge-invariant one has the following relations between the coefficients a_1, b_1, ... :

$$sc_1 + ud_1 = 0, \quad a_1 + b_1 - \tfrac{1}{2} sf_1 - \tfrac{1}{2} ug_1 = 0,$$

and the contributions of the third and fourth terms in Eq. (6) cancel. Therefore we need to know only the expressions for, say, the coefficients a_1, b_1, f_1.

Calculating the trace (5′) and carrying out the integration over v in Eq. (5) (for $t = 0$), we get the following expressions for the imaginary parts of the coefficients a_1, b_1, ... (see Appendix)

$$\operatorname{Im} a_1 \approx \frac{J_1(s)}{s+q^2}(s\omega_3 - q^2\omega_1),$$

$$\operatorname{Im} b_1 \approx 2\frac{J_1(s)}{s+q^2}(s(\omega_2-\omega_1) - q^2\omega_1),$$

$$\operatorname{Im} f_1 \approx 4J_1(s)\,\omega_3/(s+q^2),$$

$$\operatorname{Im} g_1 \approx -4J_1(s)\,\omega_1/(s+q^2), \tag{7}$$

where

$$J_1(s) \approx -(\pi/s)\ln(s/m^2)\,\theta(s-4m^2) \quad (\text{for } s \gg 4m^2).$$

(Here we have kept only the terms which lead to the largest contributions to the real part of the amplitude.)

When we now use the crossing symmetry (3), we get from Eqs. (6) and (7) the expressions for the amplitudes A_3 and A_{3e}

$$A_3 = (2\pi)^{-2}[a_3\delta_{\mu\sigma}k_{3\nu} + b_3\delta_{\nu\sigma}k_{3\mu} + f_3k_{1\sigma}k_{3\nu}k_{3\mu} + g_3k_{2\sigma}k_{3\nu}k_{3\mu}]\,e_{1\mu}e_{2\nu}e_{3\sigma}, \tag{8}$$

where

$$\operatorname{Im} a_3 \approx -\frac{J_1(-t)}{q^2-t}(t\omega_2 - q^2\omega_1),$$

$$\operatorname{Im} b_3 \approx -2\frac{J_1(-t)}{q^2-t}(t(\omega_1+\omega_3) - q^2\omega_1)$$

$$\operatorname{Im} f_3 \approx -4J_1(-t)\,\omega_2/(q^2-t),$$

$$\operatorname{Im} g_3 \approx 4J_1(-t)\,\omega_1/(q^2-t) \tag{8'}$$

and

$$A_{3e} = (2\pi)^{-2}[a_{3e}\delta_{\nu\sigma}k_{3\mu} + b_{3e}\delta_{\mu\sigma}k_{3\nu} + f_{3e}k_{3\nu}k_{3\mu}k_{2\sigma} + g_{3e}k_{3\mu}k_{3\nu}k_{1\sigma}]\,e_{1\mu}e_{2\nu}e_{3\sigma}, \tag{9}$$

where

$$\operatorname{Im} a_{3e} \approx -\frac{J_1(-u)}{q^2-u}(u\omega_1 - q^2\omega_2),$$

$$\operatorname{Im} b_{3e} \approx -2\frac{J_1(-u)}{q^2-u}(u(\omega_2+\omega_3) - q^2\omega_2),$$

$$\operatorname{Im} f_{3e} \approx -4J_1(-u)\,\omega_1/(q^2-u),$$

$$\operatorname{Im} g_{3e} \approx 4J_1(-u)\,\omega_2/(q^2-u). \tag{9'}$$

Next, to get the expressions for the imaginary parts of the amplitudes A_2 and A_{2e} for $s = 0$, we must calculate the imaginary part of the amplitude A_1 for $u = 0$. Using the condition of gauge invariance, we have for A_1

$$A_1 = (2\pi)^{-2}[a_1\delta_{\mu\nu}k_{1\sigma} + b_1\delta_{\mu\sigma}k_{1\nu} + f_1k_{2\mu}k_{1\nu}k_{1\sigma} + g_1k_{3\mu}k_{1\nu}k_{1\sigma}]\,e_{1\mu}e_{2\nu}e_{3\sigma}. \tag{10}$$

Calculation of the trace (5′) and integration over v in Eq. (5) for $u = 0$ give the following expressions for $\operatorname{Im} a_1$, $\operatorname{Im} b_1$, ... (see Appendix):

$$\operatorname{Im} a_1 \approx 0, \quad \operatorname{Im} b_1 \approx 2J_1(s)(-s\omega_1 + q^2\omega_2)/(s+q^2),$$

$$\operatorname{Im} f_1 \approx -4J_1(s)\,\omega_3/(s+q^2), \operatorname{Im} g_1 \approx 4J_1(s)\,\omega_2/(s+q^2). \tag{11}$$

Using the crossing symmetry (3), we get for A_2 and A_{2e}

$$A_2 = (2\pi)^{-2}[a_2\delta_{\nu\sigma}k_{3\mu} + b_2\delta_{\mu\sigma}k_{3\nu} + f_2k_{2\sigma}k_{3\nu}k_{3\mu} + g_2k_{1\sigma}k_{3\nu}k_{3\mu}]\,e_{1\mu}e_{2\nu}e_{3\sigma}, \tag{12}$$

where

$$\operatorname{Im} a_2 \approx 0, \quad \operatorname{Im} b_2 \approx -2J_1(-u)(-u\omega_3 + q^2\omega_2)/(q^2-u),$$

$$\operatorname{Im} f_2 \approx 4J_1(-u)\,\omega_1/(q^2-u), \operatorname{Im} g_2 \approx 4J_1(-u)\omega_2/(q^2-u) \tag{12'}$$

and

$$A_{2e} = (2\pi)^{-2}[a_{2e}\delta_{\mu\sigma}k_{3\nu} + b_{2e}\delta_{\nu\sigma}k_{3\mu} + f_{2e}k_{1\sigma}k_{3\mu}k_{3\nu} + g_{2e}k_{2\sigma}k_{3\mu}k_{3\nu}]\,e_{1\mu}e_{2\nu}e_{3\sigma}, \tag{13}$$

where

$$\operatorname{Im} a_{2e} \approx 0, \quad \operatorname{Im} b_{2e} \approx -2J_1(-t)(-t\omega_3 + q^2\omega_1)/(q^2-t),$$

$$\operatorname{Im} f_{2e} \approx 4J_1(-t)\,\omega_2/(q^2-t), \operatorname{Im} g_{2e} \approx 4J_1(-t)\,\omega_1/(q^2-t). \tag{13'}$$

We have found the expressions for the imaginary parts of the amplitudes in which we are interested. Knowing them, we can find by means of the dispersion relations the expression for the real part of the total amplitude A. For fixed s ($s = 0$) it is convenient to use the dispersion relations in the form of Eq. (2.6) of [3]

$$A = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{\operatorname{Im} A_3(t') + \operatorname{Im} A_{2e}(t')}{t'-t}dt' + \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{\operatorname{Im} A_2(u') + \operatorname{Im} A_{3e}(u')}{u'-u}du', \tag{14}$$

which contain the imaginary parts of the amplitudes A_2, A_{2e}, A_3, A_{3e}, which we have calculated [in Eq. (14) the momentum **q** transferred to the nucleus is fixed].

First, starting from the expressions (8), (9), (12), (13) for the partial amplitudes A_2, A_{2e}, A_3, A_{3e}, we can write for the total amplitude A the expression

$$A = (2\pi)^{-2}[a\delta_{\mu\sigma}k_{3\nu} + b\delta_{\nu\sigma}k_{3\mu} + fk_{3\mu}k_{3\nu}k_{1\sigma} + gk_{3\mu}k_{3\nu}k_{2\sigma}]\,e_{1\mu}e_{2\nu}e_{3\sigma}, \tag{15}$$

where the coefficients a, b, ... are expressed in terms of the coefficients of the partial amplitudes in channels III, IIIe, II, IIe in the following way:

$$a = a_3 + b_{3e} + b_2 + a_{2e}, \quad b = b_3 + a_{3e} + a_2 + b_{2e},$$

$$f = f_3 + g_{3e} + g_2 + f_{2e}, \quad g = g_3 + f_{3e} + f_2 + g_{2e}. \tag{15'}$$

Now, substituting in Eq. (14) the expressions for $\operatorname{Im} a_3$, $\operatorname{Im} a_{3e}$, ... given in Eqs. (8′), (9′), (12′), (13′), we get for Re a, Re b, ...

$$\mathrm{Re}\, a \approx (\omega_1/2t)\ln^2(t/m^2), \qquad \mathrm{Re}\, b \approx (\omega_2/2u)\ln^2(u/m^2),$$
$$\mathrm{Re}\, f \approx \mathrm{Re}\, g \approx 0. \tag{16}$$

(Here we have kept only the terms that give the largest contribution to the probability of coalescence.)

It must be noted that for coalescence $t > 0$ and $u > 0$ ($s = 0$). Furthermore the imaginary part of the amplitude A is equal to zero, and consequently only the real part of the amplitude contributes to the probability of coalescence. Keeping this in mind, we finally get for the matrix element M of Eq. (1) the expression

$$M \approx -\frac{1}{16\pi}\frac{Ze^5}{(2\omega_1\omega_2\omega_3)^{1/2}}\frac{1}{t+u}$$
$$\times\left[(e_1e_3)(e_2k_3)\frac{\omega_1}{t}\ln^2\frac{t}{m^2} + (e_2e_2)(e_1k_3)\frac{\omega_2}{u}\ln^2\frac{u}{m^2}\right]. \tag{17}$$

4. The probability for two photons to coalesce into one is defined as the number of such events per unit time referred to unit flux of incident photons, and is connected with the matrix element M by the relation

$$dR = (2\pi)^{-4}\omega_3^2|M|^2 do. \tag{18}$$

Substituting in Eq. (18) the expression (17) for M, we get

$$dR \approx \frac{Z^2\alpha^5}{2^7\pi\omega_1\omega_2\omega_3^3}\frac{do}{(1-\cos\theta)^4}\Big[(\mathbf{e}_1\mathbf{e}_3)(\mathbf{e}_2\mathbf{n}_3)\ln^2\frac{2\omega_1\omega_3(1-\cos\theta)}{m^2}$$
$$+(\mathbf{e}_2\mathbf{e}_3)(\mathbf{e}_1\mathbf{n}_3)\ln^2\frac{2\omega_2\omega_3(1-\cos\theta)}{m^2}\Big]^2, \tag{19}$$

where $\mathbf{n}_3$ is the unit vector in the direction of the momentum $\mathbf{k}_3$.

Averaged over the polarizations of the photons, the probability of coalescence is given by the formula

$$dR \approx \frac{Z^2\alpha^5}{2^9\pi\omega_1\omega_2\omega_3^3}\frac{\sin^2\theta\, do}{(1-\cos\theta)^4}$$
$$\times\Big[\ln^4\frac{2\omega_1\omega_3(1-\cos\theta)}{m^2} + \ln^4\frac{2\omega_2\omega_3(1-\cos\theta)}{m^2}$$
$$+\left(\ln^2\frac{2\omega_1\omega_3(1-\cos\theta)}{m^2} + \ln^2\frac{2\omega_2\omega_3(1-\cos\theta)}{m^2}\right)^2\cos^2\theta\Big]. \tag{20}$$

this formula is valid for $\omega_i \gg m$ and $\theta \gg \max[2m(\omega_1\omega_3)^{-1/2}, 2m(\omega_2\omega_3)^{-1/2}]$.

In the small angle region $\theta \ll 1$ the expression (20) takes the form

$$dR \approx \frac{Z^2\alpha^5}{2^4\pi\omega_1\omega_2\omega_3^3}\frac{do}{\theta^6}$$
$$\times\left\{\ln^4\frac{\omega_1\omega_3\theta^2}{m^2} + \ln^4\frac{\omega_2\omega_3\theta^2}{m^2} + \ln^2\frac{\omega_1\omega_3\theta^2}{m^2}\ln^2\frac{\omega_2\omega_3\theta^2}{m^2}\right\}. \tag{21}$$

This expression does not hold for very small angles $\theta \ll m/\omega_i$. Starting from Eq. (5), one can show that $(dR/do)_{\theta=0} = 0$. In addition, as we see

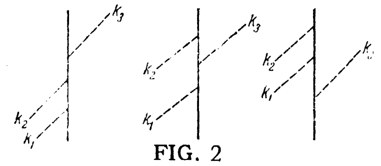

FIG. 2

from Eq. (20), $(dR/do)_{\theta=\pi} = 0$. The probability of coalescence takes its maximum value for $\theta \sim m/\omega_i$.

To estimate the total probability of coalescence, we integrate Eq. (21) over the angle variables with the lower limit taken as $\theta = \max[2m(\omega_1\omega_3)^{-1/2}, 2m(\omega_2\omega_3)^{-1/2}]$. For $\omega_1 = \omega_2 = \omega_3/2 = \omega$ we have for the total probability

$$R \approx 0.05\, Z^2 r_0^5 m/\omega, \tag{22}$$

where r_0 is the classical electron radius. If $\omega_1 \ll \omega_2$, $\omega_3 \sim \omega$, we have

$$R \approx 2\cdot 10^{-3} Z^2 r_0^5 \frac{m}{\omega}\frac{\omega_1}{\omega}\ln^4\frac{\omega}{\omega_1}. \tag{23}$$

We note that for $\omega_1 = 0$ the expression (23) goes to zero. This is a simple consequence of Furry's theorem and Ward's identity.

5. For comparison let us estimate the probability of coalescence of photons on electrons at high frequencies. This process is shown graphically in Fig. 2 [the remaining three diagrams are obtained from those shown by the interchange $(k_1, e_1) \rightleftarrows (k_2, e_2)$]. A simple estimate of these graphs leads to the following expression for dR (for $\omega_1 = \omega_2 = \omega_3/2 = \omega$):

$$dR \approx C\frac{\alpha^3}{\omega^5}\frac{do}{\theta^4}, \tag{24}$$

where C is a numerical constant. Integrating this expression over the angles with a lower limit of about $\theta \sim m/\omega$, we get

$$R \approx C' r_0^3 m/\omega^3 \tag{25}$$

It can be seen from a comparison of the expressions (22) and (25) that at high frequencies coalescence of photons occurs mainly in the Coulomb fields of nuclei.

6. Let us now consider the coalescence of photons in the Coulomb field of a nucleus at low frequencies $\omega_i \ll m$. In this case we can obtain the probability of coalescence very simply if we use the expression for the radiative corrections to the Lagrangian for the electromagnetic field.

The matrix element for the process considered is now represented by the expression

$$M = \frac{i}{8\pi}\frac{Ze^5}{(2\omega_1\omega_2\omega_3)^{1/2}}\int\frac{d^3q}{q^2}I_{\mu\nu\sigma4}$$
$$\times(k_1, k_2, -k_3, -\mathbf{q})\, e_{1\mu}e_{2\nu}e_{3\sigma}\delta^4(k_1+k_2-k_3-\mathbf{q}), \tag{26}$$