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# *ELECTRON SCATTERING IN AN EXTERNAL FIELD WITH LARGE MOMENTUM TRANSFER*

S. Ya. GUZENKO

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

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An asymptotic expression is obtained in the "one-logarithm" approximation for the scattering cross section of relativistic electrons in an external field, for the case of large momentum transfer to the field.

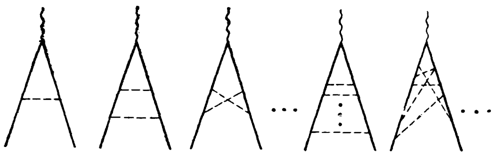
1. Abrikosov[1] has found the cross section for scattering of an electron by an external field for the case of large momentum transfer in the "doubly-logarithmic" approximation, in which the main terms in the matrix element in the n-th order of perturbation theory are of the type $e^{2n}L^{2n}$ (L is the logarithm of a large energy).[2]

For the example of a vertex part, the writer and Fomin[3] have shown that it is possible to calculate the terms that come next after the main ones, of the type $e^{2n}L^{2n-1}$ ("one-logarithm" approximation). In the present paper a calculation is made in the one-logarithm approximation for the cross section for electron scattering in an external field in the high-energy region: $\epsilon \gg m$, $pq \gg m^2$ (p and q are the four-momenta of the incident and scattered electron, and $\epsilon$ is its energy).

2. Let us first find the contribution to the matrix element from the main diagrams, in which all of the photon lines run across the vertex at which the external field acts (see figure).

To begin with we consider the contribution from diagrams with one photon line (diagram a):

$$M_0^{(1)} = \frac{e^2}{\pi i}\int \frac{d^4k}{(2\pi)^2} \frac{\overline{u}_q \gamma_\mu (\hat{q} - \hat{k} + m) \hat{A} (\hat{p} - \hat{k} + m) \gamma_\mu u_p}{(-2pk + k^2 + p^2 - m^2 + i\varepsilon)(-2qk + k^2 + q^2 - m^2 + i\varepsilon)(k^2 + i\varepsilon)}, \tag{1}$$



a b c

where $u_p$ and $u_q$ are the spinor amplitudes for the incident and scattered electron and $A_\mu$ is the potential of the external field.

For $p^2 = m^2$, $q^2 = m^2$ the integration over the region of small momenta leads to divergences; therefore we shall regard $p^2 - m^2$ and $q^2 - m^2$ as differing from zero by small quantities, and set $p^2 = m^2$, $q^2 = m^2$ only after the removal of the infrared divergences (Article 5).

As in [1,2] we break up the vector k into components in the plane of p and q and perpendicular to this plane:

$$k = u(p - \gamma q) + v(q - \gamma p) + k_\perp, \qquad k_\perp^2 = -2pqz,$$

where $k_\perp p = k_\perp q = 0$, $\gamma = m^2/2pq$. The new variables u and v vary between infinite limits, and z ranges from zero to infinity ($k_\perp$ is a spacelike vector); the fourth variable is the angle in the plane of $k_\perp$.

The terms in the numerator of the integrand in Eq. (1) which are quadratic in k make a contribution in the high-momentum region and will be considered separately (Article 3). The terms linear in $k_\perp$ do not lead to logarithmic terms. When we further use the fact that $\hat{p}u_p = mu_p$ and transfer the factors $\hat{p}$ and $\hat{q}$ across the $\gamma_\mu$, we get, on dropping terms of the order $m^2/2pq$

$$M_0^{(1)} = \frac{e^2}{\pi i} \overline{u}_q \hat{A} u_p \frac{1}{4\pi} \times \iiint \frac{(1 - u - v)\, du\, dv\, dz}{[u(1-v) - \alpha + \gamma v + z - i\varepsilon][v(1-u) - \beta + \gamma u + z - i\varepsilon][uv - z + i\varepsilon]} + M_1^{(1)} \equiv \frac{e^2}{4\pi^2} \overline{u}_q \hat{A} u_p J\{1 - u - v\} + M_1^{(1)}, \tag{2}$$

where $\alpha = (p^2 - m^2)/2pq$, $\beta = (q^2 - m^2)/2pq$; and $M_1^{(1)}$ is the contribution to $M_0^{(1)}$ which comes from large values of k.

Let us first find Re $J\{1\}$. The logarithmic terms will be correctly taken into account if the integration over u, v is taken between the limits (−1, 1) (cf. [3]), and then the only contribution to the integral over z is that from the residue at the point $z = uv$:

$$\operatorname{Re} J\{1\} = -\pi \int_{-1}^{1} du \int_{-1}^{1} dv \frac{\theta(uv)}{(u-\alpha+\gamma v)(v-\beta+\gamma u)}$$

$$= -\pi\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right) + O(1).$$

There are contributions to $J\{u\}$ from the residues from the second and third denominators

$$J\{u\} = J\{v\} = 4\pi\ln\gamma + O(1).$$

For $M_0^{(1)}$ we finally get

$$\operatorname{Re} M_0^{(1)} = -\frac{e^2}{4\pi}\bar{u}_q\hat{A}u_p\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta} + 4\ln\gamma\right) + M_1^{(1)}. \quad (3)$$

Im $M_0^{(1)}$ also contains one-logarithm terms, but their contribution to the cross section is of the order $e^4L^2$, which is not retained in our present approximation.

Let us now consider the main diagrams with n photon lines. Combining all of the diagrams in $M_0^{(n)}$ (diagram c) and symmetrizing the integrand in the variables $k_1, \ldots, k_n$ (see Appendix), we get to one-logarithm accuracy

$$M_0^{(n)} = \left(\frac{e}{\pi i}\right)^n \bar{u}_q\hat{A}u_p\left(\frac{1}{4\pi}\right)^n\frac{1}{n!}\int\ldots\int\frac{(1-u_1-\ldots-u_n-v_1-\ldots-v_n)\,\Pi du_i dv_i dz_i}{\Pi[u_i(1-v_i)-\alpha+\gamma v_i+z_i-i\varepsilon][v_i(1-u_i)-\beta+\gamma u_i+z_i-i\varepsilon][u_iv_i-z_i+i\varepsilon]} + M_1^{(n)}, \quad (4)$$

where $M_1^{(n)}$ is the contribution to $M_0^{(n)}$ from the region of large momenta of the virtual photons (more exactly, from regions where at least one of the photon momenta is large).

In the expression written the integrations over the variables $(u_i, v_i, z_i)$ relating to each photon can be carried out independently. The calculations are analogous to those for $M_0^{(1)}$ and give the following result:

$$\operatorname{Re} M_0^{(n)} = \left(-\frac{e^2}{4\pi}\right)^n \bar{u}_q\hat{A}u_p\left[\frac{1}{n!}\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^n + \frac{1}{(n-1)!}\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}\cdot 4\ln\gamma\right] + M_1^{(n)}. \quad (5)$$

In the square of the absolute value of the matrix element the one-logarithm terms in Im $M_0^{(n)}$ lead to terms of the type $e^{2k}L^{2k-2}$. Therefore in the one-logarithm approximation Im $M_0^{(n)}$ does not contribute to the cross section.

3. Let us now find the contribution from the main diagrams which comes from the region of high momenta of the virtual photons. One-logarithm terms arise if the photon line carrying a large momentum is located higher than all the other photon lines. The integration over this large momentum is carried out independently, and those over the others, as in Article 1. The result is the following expression for the contribution to $M^{(n)}$ which comes from the region of large momenta:

$$M_1^{(n)} = \left(\frac{e^2}{\pi i}\right)^n\left(\frac{1}{4\pi}\right)^{n-1}\frac{(-\pi i)^{n-1}}{(n-1)!}\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}$$

$$\times\int\frac{d^4k}{(2\pi)^2}\frac{\bar{u}_q\gamma_\mu\hat{k}\hat{A}\hat{k}\gamma_\mu u_p}{(k^2-2pk)(k^2-2qk)k^2} = \frac{e^2}{\pi}\bar{u}_q\hat{A}u_p\cdot\frac{1}{4}\ln\frac{\Lambda^2}{2pq}$$

$$\times\left(-\frac{e^2}{4\pi}\right)^{n-1}\frac{1}{(n-1)!}\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}. \quad (6)$$

The one-logarithm contributions from diagrams in which there are photon lines that do not run across the main vertex (electron proper-energy insertions and side vertices) cancel each other, with the exception of proper-energy parts that lie below all the photon lines. Their contribution is given by (cf. [4])

$$M_2^{(n)} = \frac{e^2}{\pi}\bar{u}_q\hat{A}u_p\left(-\frac{1}{4}\ln\frac{\Lambda^2}{m^2} - \frac{1}{2}\ln\frac{p^2-m^2}{m^2} - \frac{1}{2}\ln\frac{q^2-m^2}{m^2}\right)$$

$$\times\left(-\frac{e^2}{4\pi}\right)^{n-1}\frac{1}{(n-1)!}\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}. \quad (7)$$

Here charge renormalization and wave-function renormalization have been performed.

The sum of the contributions (6) and (7) is independent of the cut-off momentum Λ and is given by

$$M_1^{(n)} + M_2^{(n)} = \frac{e^2}{\pi}\bar{u}_q\hat{A}u_p\left(\frac{1}{4}\ln\gamma - \frac{1}{2}\ln\frac{\alpha\beta}{\gamma^2}\right)\left(-\frac{e^2}{4\pi}\right)^{n-1}\frac{1}{(n-1)!}$$

$$\times\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}. \quad (8)$$

4. Diagrams obtained from the main diagrams (see figure) by the insertion of proper-energy parts in photon lines also give a contribution in the one-logarithm approximation. It can be shown that the only contribution is that from the imaginary part of the proper-energy function. For example, an insertion in $M_0^{(1)}$ gives

$$M_3^{(1)} = -\bar{u}_q\hat{A}u_p\frac{e^4}{12\pi^2}$$

$$\times\int\frac{\theta(uv-z-4\gamma)\,du\,dv\,dz}{[u(1-v)-\alpha+\gamma v+z][v(1-u)-\beta+\gamma u+z][uv-z]}$$

$$= \bar{u}_q\hat{A}u_p\frac{e^4}{36\pi^2}\ln^3\gamma.$$

By inserting a proper-energy part in each photon line of $M_0^{(n)}$ in turn and symmetrizing the integrand in $k_1, \ldots, k_n$, we get

$$M_3^{(n)} = -\bar{u}_q \hat{A} u_p \frac{e^4}{12\pi^2}\left(-\frac{e^2}{\pi i}\right)^{n-1} \frac{1}{n!} \int \ldots \int \frac{[\theta(u_1 v_1 - z_1 - 4\gamma) + \ldots + \theta(u_n v_n - z_n - 4\gamma)]\, \Pi du_i dv_i dz_i}{\Pi [v_i(1-u_i) - \alpha + \gamma u_i + z_i - i\varepsilon][u_i(1-v_i) - \beta + \gamma v_i + z_i - i\varepsilon][u_i v_i - z_i + i\varepsilon]}$$

$$= \bar{u}_q \hat{A} u_p \frac{e^4}{36\pi^2} \ln^3\gamma \cdot \left(-\frac{e^2}{4\pi}\right)^{n-1} \frac{1}{(n-1)!}\left(2\ln\gamma \ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}. \quad (9)$$

The contribution from the photon proper-energy insertions corresponding to the external potential is of the form

$$M_4^{(n)} = -\bar{u}_q \hat{A} u_p \frac{e^2}{3\pi} \ln\gamma \cdot \left(-\frac{e^2}{4\pi}\right)^{n-1} \frac{1}{(n-1)!} \times \left(2\ln\gamma \ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)^{n-1}. \quad (10)$$

Combining the expressions (5) and (8)–(10), we find the final expression for the matrix element for electron scattering in an external field:

$$M = \sum_{n=0}^{\infty} M^{(n)} = \bar{u}_q \hat{A} u_p \left[1 - \frac{4}{3}\frac{e^2}{\pi}\ln\gamma + \frac{e^2}{\pi}\left(\frac{1}{4}\ln\gamma - \frac{1}{2}\ln\frac{\alpha\beta}{\gamma^2}\right) + \frac{e^4}{36\pi^2}\ln^3\gamma\right] \exp\left\{-\frac{e^2}{4\pi}\left(2\ln\gamma\ln\frac{\alpha\beta}{\gamma} - \ln^2\frac{\alpha}{\beta}\right)\right\}. \quad (11)$$

5. Let us now consider the scattering of an electron in an external field with the emission of photons whose total energy does not exceed $\Delta\epsilon \ll \epsilon$ ($\epsilon$ is the energy of the electron).

The matrix element for scattering with the emission of one such photon is of the well known form (cf. [5])

$${}_1M = \frac{\sqrt{4\pi}e}{\sqrt{2|\mathbf{k}|}}\left(\frac{2qe}{2qk + q^2 - m^2} - \frac{2pe}{2pk - p^2 + m^2}\right) M,$$

where $k_\mu$ is the momentum of the photon and $e_\mu$ is its polarization.

Summing the square of the absolute value of ${}_1M$ over the polarizations and integrating over photon energies from 0 to $\Delta\epsilon$, we get the following expression for the cross section for scattering with the emission of a photon of energy not exceeding $\Delta\epsilon$:

$$d\sigma^{(1)} = d\sigma^{(0)}\,[J(p, q) + J(-p) + J(q)],$$

$$J(p, q) = -2\pi e^2 \int_0^{\Delta\varepsilon} \frac{d^3\mathbf{k}}{(2\pi)^3 |\mathbf{k}|} \frac{8pq}{(2qk + q^2 - m^2)(2pk - p^2 + m^2)}$$

$$= -\frac{e^2}{\pi}\left(2\ln\gamma\ln\frac{\Delta\varepsilon}{\varepsilon} - \ln\gamma\ln\frac{\alpha\beta}{\gamma} + \frac{1}{2}\ln^2\frac{\alpha}{\beta}\right) + O(1),$$

$$J(-p) + J(q) = 2\pi e^2 \int_0^{\Delta\varepsilon} \frac{d^3\mathbf{k}}{(2\pi)^3|\mathbf{k}|}\left[\frac{4m^2}{(2qk + q^2 - m^2)^2} + \frac{4m^2}{(2pk - p^2 + m^2)^2}\right]$$

$$= \frac{e^2}{\pi}\left(-2\ln\frac{\Delta\varepsilon}{\varepsilon} + \ln\frac{\alpha\beta}{\gamma^2}\right) + O(1); \quad (12)$$

$d\sigma^{(0)}$ is the cross section for scattering without emission of a photon.

It can be shown that to one-logarithm accuracy the cross section with the emission of r photons is of the form

$$d\sigma^{(r)} = d\sigma^{(0)} \int_0^{\Delta\varepsilon} \frac{d^3\mathbf{k}_1}{(2\pi)^3} \int_0^{\Delta\varepsilon - |\mathbf{k}_1|} \frac{d^3\mathbf{k}_2}{(2\pi)^3} \cdots \int_0^{\Delta\varepsilon - |\mathbf{k}_1| - \ldots - |\mathbf{k}_{r-1}|} \frac{d^3\mathbf{k}_r}{(2\pi)^3} \frac{1}{r!} \prod_1^r \frac{2\pi e^2}{|\mathbf{k}_i|}$$
$$\times \left\{\prod_1^r \frac{-8pq}{(2qk_i + q^2 - m^2)(2pk_i - p^2 + m^2)} + \sum_1^r \left[\frac{4m^2}{(2qk_i + q^2 - m^2)^2} + \frac{4m^2}{(2pk_i - p^2 + m^2)^2}\right] \prod_{s \neq i} \frac{-8pq}{(2qk_s + q^2 - m^2)(2pk_s - p^2 + m^2)}\right\}. \quad (13)$$

Carrying out the integration over $k_r$, we get an expression of the type (12) with $\Delta\epsilon$ replaced by $\Delta\epsilon - |\mathbf{k}_1| - \ldots - |\mathbf{k}_{r-1}|$. Expanding this expression in powers of $\mathbf{k}$, we can show that the added terms give contributions of the order of $\Delta\epsilon/\epsilon$. Therefore we can replace all upper limits in Eq. (13) by $\Delta\epsilon$. The result obtained is

$$d\sigma^{(r)} = d\sigma^{(0)}\left[\left(-\frac{e^2}{\pi}\right)^r \frac{1}{r!}\left(2\ln\gamma\ln\frac{\Delta\varepsilon}{\varepsilon} - \ln\gamma\ln\frac{\alpha\beta}{\gamma} + \frac{1}{2}\ln^2\frac{\alpha}{\beta}\right)^r + \frac{e^2}{\pi}\left(-2\ln\frac{\Delta\varepsilon}{\varepsilon} + \ln\frac{\alpha\beta}{\gamma^2}\right)\left(-\frac{e^2}{\pi}\right)^{r-1}\frac{1}{(r-1)!}\left(2\ln\gamma\ln\frac{\Delta\varepsilon}{\varepsilon} - \ln\gamma\ln\frac{\alpha\beta}{\gamma} + \frac{1}{2}\ln^2\frac{\alpha}{\beta}\right)^{r-1}\right]. \quad (14)$$

Summing $d\sigma^{(r)}$ over r, we get the cross section for scattering with the emission of photons of energy not exceeding $\Delta\epsilon$:

$$d\sigma = \sum_{r=0}^{\infty} d\sigma^{(r)} = d\sigma^{(0)}\left[1 + \frac{e^2}{\pi}\left(-2\ln\frac{\Delta\varepsilon}{\varepsilon} + \ln\frac{\alpha\beta}{\gamma^2}\right)\right] \times \exp\left\{-\frac{e^2}{\pi}\left(2\ln\gamma\ln\frac{\Delta\varepsilon}{\varepsilon} - \ln\gamma\ln\frac{\alpha\beta}{\gamma} + \frac{1}{2}\ln^2\frac{\alpha}{\beta}\right)\right\}. \quad (15)$$

Substituting the value of $d\sigma^{(0)}$ according to Eq. (11), we get

$$d\sigma = d\sigma_0\left(1 - \frac{13}{6}\frac{e^2}{\pi}\ln\frac{m^2}{2pq} - 2\frac{e^2}{\pi}\ln\frac{\Delta\varepsilon}{\varepsilon} + \frac{e^4}{18\pi^2}\ln^3\frac{m^2}{2pq}\right) \times \exp\left\{-2\frac{e^2}{\pi}\ln\frac{m^2}{2pq}\ln\frac{\Delta\varepsilon}{\varepsilon}\right\}, \quad (16)$$

where $d\sigma_0$ is the cross section for electron scattering in an external field in first-order perturbation theory.

The logarithmic terms in the parentheses in Eq. (16) are the desired corrections to the doubly-logarithmic approximation of [1]. We note that part of these corrections have been taken into account by Yennie, Frautschi, and Suura[6] by another method in a treatment of the infrared divergence.

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## APPENDIX

We shall explain the symmetrization procedure for the simplest example of $M_0^{(2)}$ (diagram b):

$$M_0^{(2)} = \left(\frac{e^2}{\pi i}\right)^2 \bar{u}_q \hat{A} u_p \left(\frac{1}{4\pi}\right)^2 \int \frac{(1-u_1-u_2-v_1-v_2)(1-u_1)du_1du_2dv_1dv_2dz_1dz_2}{[u_1v_1-z_1][u_2v_2-z_2][v_1(1-u_1)-\alpha+\gamma u_1+z_1]} \frac{1}{[(v_1+v_2)(1-u_1-u_2)-\alpha+\gamma(u_1+u_2)+z_1+z_2]}$$
$$\times \frac{1}{[(u_1+u_2)(1-v_1-v_2)-\beta+\gamma(v_1+v_2)+z_1+z_2]}\left[\frac{1-v_1}{u_1(1-v_1)-\beta+\gamma v_1+z_1}\right.$$
$$\left.+\frac{1-v_2}{u_2(1-v_2)-\beta+\gamma v_2+z_2}\right] + M_1^{(2)} \equiv \left(\frac{e^2}{\pi i}\right)^2 \bar{u}_q \hat{A} u_p \left(\frac{1}{4\pi}\right)^2 J + M_1^{(2)}. \quad \text{(A.1)}$$

We have denoted by $M_1^{(2)}$ the contribution from large momenta (the corresponding terms have been omitted in the numerator of the integrand in Eq. (A.1). In the denominators we have dropped terms of the type of $(z_1z_2)^{1/2}\cos(\varphi_1-\varphi_2)$, as was also done in [3].

Reducing the expression in square brackets in Eq. (A.1) to a common denominator, we get

$$J = \int \frac{(1-u_1-u_2-v_1-v_2)\, du_1\, du_2\, dv_1\, dv_2\, dz_1\, dz_2}{(u_1v_1-z_1)(u_2v_2-z_2)[u_1(1-v_1)-\alpha+\gamma v_1+z_1][u_2(1-v_2)-\alpha+\gamma v_2+z_2]}$$
$$\times \frac{(1-u_1)}{[v_1(1-u_1)-\beta+\gamma u_1+z_1][(v_1+v_2)(1-u_1-u_2)-\beta+\gamma(u_1+u_2)+z_1+z_2]}$$
$$\times\left[1-\frac{(u_1+u_2)v_1v_2+\beta(1-v_1-v_2)+2\gamma v_1v_2+z_1v_2+z_2v_1}{(u_1+u_2)(1-v_1-v_2)-\alpha+\gamma(v_1+v_2)+z_1+z_2}\right]. \quad \text{(A.2)}$$

Estimates show that the second term in the square brackets in Eq. (A.2) gives a contribution of the order $L^2$, which is not of "one-logarithm" type. Writing the remaining integral in the form

$$J = \frac{1}{2}\int \frac{(1-u_1-u_2-v_1-v_2)\, du_1\, du_2\, dv_1\, dv_2\, dz_1\, dz_2}{(u_1v_1-z_1)(u_2v_2-z_2)[u_1(1-v_1)-\alpha+\gamma v_1+z_1][u_2(1-v_2)-\alpha+\gamma v_2+z_2]}$$
$$\times\left[\frac{1-u_1}{v_1(1-u_1)-\beta+\gamma u_1+z_1}+\frac{1-u_2}{v_2(1-u_2)-\beta+\gamma u_2+z_2}\right] \quad \text{(A.3)}$$

and then reducing to a common denominator in Eq. (A.3), we again omit terms of order $L^2$. The result is then

$$J = \frac{1}{2}\int \frac{(1-u_1-u_2-v_1-v_2)\, du_1dv_1dz_1}{[u_1(1-v_1)-\alpha+\gamma v_1+z_1][v_1(1-u_1)-\beta+\gamma u_1+z_1][u_1v_1-z_1]}$$
$$\times \frac{du_2dv_2dz_2}{[u_2(1-v_2)-\alpha+\gamma v_2+z_2][v_2(1-u_2)-\beta+\gamma u_2+z_2][u_2v_2-z_2]}. \quad \text{(A.4)}$$

Here the integrations over $k_1$ and $k_2$ can now be carried out independently.

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173