ON THE CROSS SECTIONS FOR ELEMENTARY PROCESSES WITH LARGE ABSORPTION

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Collisions between nonrelativistic particles accompanied by a large absorption have been considered. It is shown that $\sigma_s < \sigma_a$, irrespective of the spatial distribution of the absorption. For absorption which decreases with the distance like $\exp(-r^2/b^2)$ and $\exp(-r/c)$ the magnitude of the cross sections weakly depends on the real part of the potential and especially on its behavior at distances which are much smaller than the pion Compton wavelength.

INTRODUCTION

THE interpretation of the cross sections for the interaction of elementary particles in the nonrelativistic region can be made in a number of cases without taking into account relativistic effects. When there is a large probability for the absorption of the particles in a collision, this permits the use of classical optics. Indeed, for waves with small l a large part of the absorption takes place at considerable distances, since only a very small fraction of the particles passes at small distances (large attenuation of the wave function as a result of absorption). For large l the collision parameter is generally large and therefore the details of the behavior of the particles at small r are not important.

The possibility of describing such processes in the language of the optical model is of great importance, since in collision of nucleons with antinucleons and (perhaps) with kaons a large amount of absorption is observed. An attempt to explain the nucleon-antinucleon cross sections in classical language was made by Ball and Chew.[1] Their results, however, differ from the experimental results during the last few years at low energies.[2,3] We shall show that the crude theory of Ball and Chew cannot claim to describe the experimental results in a satisfactory way.

The problem on nucleon-antinucleon collisions will be solved below by means of the ordinary Schrödinger equation with a complex potential. The calculations were made on an M-20 computer which permitted the investigation of a broad class of complex potentials. The Coulomb field was not taken into account in the calculations. For c.m.s. energies $E_{cms} = 20$ MeV and 40 MeV the Coulomb field plays an important role at angles $\theta < 0.2$ rad and $\theta < 0.1$ rad, respectively. At energies < 20 MeV, we are dealing with collisions between an antineutron and a proton or an antiproton and a neutron.

1. CHOICE OF THE POTENTIAL

We have assumed that the imaginary part of the potential is independent of the orbital angular momenta and the spins of the incident particles. Although such an assumption is indeed a very crude one, the dependence of the annihilation on the angular momentum of the incident particles should be very weak, in view of the multiple nature of pion production.

In the quasi-classical model of Ball and Chew it is assumed that the absorption occurs only for $r < 4-5 \times 10^{14}$ cm, i.e., $2\lambda_c$ for a nucleon. We approximated such a model by taking the imaginary part of the potential in the form

$$W = -W_0 \exp\{-r^4/a^4\}, \qquad (1)$$

which decreases sufficiently strongly with increasing distance. Use of rectangular potentials was undesirable, as has been shown in work on nuclear physics.

We also used potentials with an imaginary part of the Gaussian type:

$$W = -W_0 \exp\{-r^2/b^2\}. \qquad (2)$$

Finally, we made the most detailed study for potentials of the type

$$W = -W_0 \exp\{-r/c\}. \qquad (3)$$

Such potentials are more convenient for calculation than a potential with a Yukawa-type singularity, while the difference between (3) and

$$W = -W_0 c r^{-1} \exp\{-r/c\} \qquad (4)$$

for problems with strong absorption is small, since the region of small r does not have to be considered in the case of large W_0. On the other hand, it is clear that any elementary processes decrease at large distances in accordance with the Yukawa law (4) and that a similar behavior should be expected also from the absorption potentials.

The real part of the potential was not specified in detail. Since we considered it necessary to investigate the general features of the problem, we omitted such details as the presence of tensor forces and spin-orbital interactions. The tensor force makes it necessary to solve a system of radial Schrödinger equations, which slows down the machine calculations and does not permit the study of a broad class of imaginary and real potentials. The spin-orbital interaction only lowers the centrifugal barrier slightly in some cases and increases it in others, which can be important for the angular distribution and the polarization, but not for the total cross sections.

Hence the real potential was taken in the form of the sum of three terms:

$$V = U_1 + U_2 + U_3. \tag{5}$$

Here we assumed that U_1 is the force of longest range:

$$U_1 = \pm f^2 e^{-\mu r}/\mu r, \tag{6}$$

where μ is the pion mass ($\hbar$ and c have been set equal to unity).

We investigated both positive and negative U_1. For antinucleons four s states are possible, namely: ${}^1S_0^1$, ${}^1S_0^3$, ${}^3S_1^1$, and ${}^3S_1^3$ (for $l \neq 0$ there are eight states). On the upper left is the ordinary spin multiplicity, on the upper right is the isospin multiplicity. Of these states, U_1 is positive (from charge conjugation) for ${}^1S_0^3$ and ${}^3S_1^1$ and is negative in the other two states. The picture is similar for $l \neq 0$. However, since we have not specified the potentials in detail for the individual l and J, the calculation with U_1 of a given signature was carried out for all waves. The potential U_2, which includes the interaction at medium distances, was taken negative. The two pion terms of the interaction always have a minus sign, but other contributions to the potential for intermediate r are not excluded. If U_2 is a large positive quantity, then the cross section could be greatly diminished, in sharp contradiction to experiment.

We took the potential U_2 in the form

$$U_2 = -A_1 e^{-2\mu r} r^{-2} (1 + A_2 r)(1 - e^{-\beta^2 r^2}). \tag{7}$$

The last factor in (7) eliminates the pole. (It is important only for very small distances and does not play any role in our problem.)

The short-range potential U_3 was taken in the form

$$U_3 = B e^{-6\mu r}. \tag{8}$$

Such a potential with a positive sign reproduces well the repulsion at small distances in the nucleon-nucleon problem. On the other hand, in the presence of an interaction through vector ρ mesons with a mass of the order 5.3μ, the basic term of the potential can vary like $\exp(-5.3\mu r)/r$ and give repulsion for identical particles, while for an interaction of antiparticles with particles it can give attraction in some states and repulsion in others (if the isospin of the field is unity).

The constant B was taken in most cases as 2000 MeV, and for all values of l considered we used the same signature, either positive or negative. For absorption subject to the law (1), we also considered the case B = 4000 MeV. The constant f^2 was taken in agreement with the present data as 0.09μ. The constant A_1 was chosen so that it more or less satisfactorily reflected the behavior of the pp interaction in the ${}^1S_0^3$ state at medium distances; its value was taken equal to 100 MeV. We also used the values $A_2 = 0.7\ F^{-1}$ and $\beta^2 = 2.0\ F^{-2}$.

Since in formulas (6) and (8) we investigated both signatures, the class of investigated potentials was quite broad.

2. RESULTS OF THE CALCULATION

We calculated the absorption and scattering cross sections for E_{cms} = 20, 40, 70, and 120 MeV. We first considered the short-range potential (1). In this potential, W_0 was taken equal to 2000 and 4000 MeV and a was taken as 0.56 F and 0.54 F. Quasi-classically, such a large absorption ensures a sticking coefficient T $= \sigma_a/(2l+1)\pi\lambda\!\!\!^-{}^2 \cong 1$. However, accurate calculations show that at medium energies, even if the antinucleon energy is sufficient to overcome the centrifugal barrier, T is much less than unity. According to [1], for all waves for which the centrifugal barrier is below the kinetic energy, we have T = 1.

It is seen from Table I that even for a large negative U_3 the value of T for s waves is 10—15% below unity. This is especially so for particles with $l = 1$, for which T is half that in the case of Ball and Chew. For T = 1 we have $\sigma_s = \sigma_a$. For smaller T, however, the ratio of the scattering and the absorption depends on the phase-shift of the scattered wave. If we take $T = 1 - |\eta|^2$ and $\sigma_s/(2l+1)\pi\lambda\!\!\!^-{}^2 = \Sigma = |1 - \eta^2|$, then Σ is minimal

Table I. Partial scattering and absorption cross sections for $W = -W_0 \exp(-r^4/a^4)$

Potential parameters	$B=0$, $U_1>0$, $W_0=+2000$, $a^4=0{,}1$				$B=+2000$, $U_1<0$, $W_0=+2000$, $a^4=0{,}1$		$B=-2000$, $U_1<0$, $W_0=+2000$, $a^4=0{,}085$			
E, MeV	40		70		40		40		70	
l	T	Σ	T	Σ	T	Σ	T	Σ	T	Σ
0	0.611	0.411	0.679	0.680	0.524	0.228	0.838	0.838	0.357	0.415
1	0.163	0.0121	0.296	0.033	0.141	0.089	0.377	0.267	0.546	0.350
2	0.0035	$2.45\cdot10^{-4}$	0.0136	$1\cdot10^{-3}$	$3.4\cdot10^{-3}$	$1.4\cdot10^{-2}$	$7.6\cdot10^{-3}$	$1.9\cdot10^{-2}$	0.0294	0.0593
3			$1.9\cdot10^{-4}$	$9.2\cdot10^{-5}$	$2.8\cdot10^{-5}$	$1.5\cdot10^{-3}$	$4.5\cdot10^{-5}$	$1.5\cdot10^{-3}$	$3.2\cdot10^{-4}$	$6.2\cdot10^{-3}$

Potential parameters	$B=-2000$, $U_1<0$, $W_0=+4000$, $a^4=0{,}085$				$B=-4000$, $U_1<0$, $W_0=+4000$, $a^4=0{,}085$			
E, MeV	40		70		40		70	
l	T	Σ	T	Σ	T	Σ	T	Σ
0	0.766	0.267	0.783	0.361	0.887	0.465	0.881	0.435
1	0.310	0.235	0.460	0.292	0.454	0.360	0.615	0.478
2	$8.1\cdot10^{-3}$	$1.9\cdot10^{-2}$	$3.06\cdot10^{-2}$	$5.7\cdot10^{-3}$	$1.17\cdot10^{-2}$	$2.20\cdot10^{-2}$	$4.53\cdot10^{-2}$	$7.44\cdot10^{-2}$
3	$5.7\cdot10^{-5}$	$1.6\cdot10^{-3}$	$3.98\cdot10^{-4}$	$6.2\cdot10^{-3}$	$7.4\cdot10^{-5}$	$1.6\cdot10^{-3}$	$5.2\cdot10^{-4}$	$6.6\cdot10^{-3}$

Table II. Total cross sections for $W = -W_0 \exp(-r^4/a^4)$

Potential parameters	$B=0$, $U_1>0$, $W_0=2000$, $a^4=0{,}1$		$B=+2000$, $U_1<0$, $W_0=2000$, $a^4=0{,}1$	$B=-2000$, $U_1<0$, $W_0=+2000$, $a^4=0{,}1$		$B=-4000$; $U_1<0$, $W_0=+4000$, $a^4=0{,}085$	
E, MeV	40	70	40	40	70	40	70
σ_a, mb	35	29	28	62	48	72	53
σ_s, mb	14	14	15.5	39	30	52	41.5
σ_s/σ_a	0.4	0.5	0.55	0.63	0.63	0.72	0.78

for real η. The quantity Σ is equal to unity for $\arg\eta = \pi/2$ and is maximal for $\arg\eta = \pi$. For small energies, $\arg\eta$, as a rule, is small, and the scattering cross section is close to a minimum for a given T. Consideration of Table I shows that for $l = 0$ the quantity Σ is close to a minimum and hence Σ is considerably less than T.

The total scattering cross sections (Table II) are much less than the absorption cross sections. Ball and Chew[1] assumed that, apart from waves with T = 1, for which $\Sigma = T$, waves which do not pass through the centrifugal barrier are still scattered, and hence $\sigma_s > \sigma_a$. We have shown however, that also when the potential U_3 has a positive sign there is still considerable absorption of s waves and for 70 MeV, also p waves. The scattering is small, and hence the fact that the potentials U_1 and U_3 are of different sign for different l changes the ratio of the scattering and absorption very little, and in the general case $\sigma_s < \sigma_a$, contrary to the conclusions of Ball and Chew.

This result, as we see, agrees with the current experimental data, but the absolute value of the absorption cross sections (for U_3 negative) is approximately $^2/_3$ of those in [1] and $^1/_3$ to $^2/_3$ of the experimental value.[3] This led us to consider potentials (2) and (3), which decrease more slowly with increasing r.

Table III lists T and Σ for the potential (2), which occupies an intermediate position between (1) and (3) as regards absorption. The scattering for $l = 0$ proves to be large (close to maximum), owing to the combination of the phase-shifts from the real and imaginary potentials. On the whole, however, as is seen from Table IV, we have $\sigma_s < \sigma_a$. The experimental cross sections can be explained only for $b^2 = 0.6$. Here the absorption is extended over a region of the pion radius or somewhat larger.

The potential (3) has been investigated in greatest detail. In this case, for those l for which the centrifugal barrier is inside the pion radius, the coefficient T is close to unity (Tables V and VI). An important feature of the potential (3) is the weak sensitivity of the cross sections to the behavior of the real part of the potential at small distances

Table III. Partial cross sections for $W = -W_0 \exp(-r^2/b^2)$

Potential parameters	$W_0 = +2000,\ U_1 > 0,\ B = 0,\ b^2 = 0.5$				$U_1 < 0,\ B = 2000,\ W_0 = +2000,\ b^2 = 0.5$		$U_1 < 0,\ B = -2000,\ W_0 = +2000,\ b^2 = 0.8$	
E, MeV	40		70		40		40	
l	T	Σ	T	Σ	T	Σ	T	Σ
0	0.758	1.534	0.846	1.758	0.7836	1.286	0.8232	1.821
1	0.484	0.155	0.674	0.397	0.525	0.1073	0.6636	0.3741
2	0.078	$2.1\cdot10^{-3}$	0.227	0.0163	0.09	0.0110	0.245	0.0175
3	$3.4\cdot10^{-3}$	$2.2\cdot10^{-3}$	0.0200	$2.39\cdot10^{-3}$	$4\cdot10^{-3}$	$1.5\cdot10^{-3}$	$2.3\cdot10^{-2}$	$1.4\cdot10^{-2}$

Table IV. Total cross sections for a Gaussian potential

Potential parameters	$U_1 > 0,\ B = 0,\ W_0 = 2000,\ b^2 = 0.5$		$U_1 < 0,\ B = 2000,\ W_0 = 2000,\ b^2 = 0.5$	$U_1 < 0,\ B = -2000,\ W_0 = 2000,\ b^2 = 0.8$		
E, MeV	40	70	40	40	70	120
σ_a, mb	82	84	88	133	120	110
σ_s/σ_a*	0.76	0.74	0.60	0.74	0.75	0.7

*The ratio σ_s/σ_a proves to be quite stable relative to the choice of the real part of the potential; only for $U_3 > 0$ does it decrease somewhat.

Table V. Coefficients T and Σ for $W - W_0 \exp(-r/c)$

Potential parameters	$B = 0,\ U_1 > 0$; $W_0 = +2000,\ c = 0.35$		$B = 0,\ U_1 > 0$; $W_0 = +2000,\ c = 0.5$		$B = 0,\ U_1 > 0$; $W_0 = +2000,\ c = 0.42$				$U_1 > 0,\ W_0 = +2000,\ B = 2000,\ c = 0.42$		$U_1 > 0,\ W_0 = +2000,\ c = 0.42,\ B = -2000$	
E, MeV	40		40		40		70		40		40	
l	T	Σ	T	Σ	T	Σ	T	Σ	T	Σ	T	Σ
0	0.9185	1.237	0.957	1.456	0.937	1.445	0.975	1.339	0.937	1.428	0.937	1.441
1	0.676	0.222	0.884	0.793	0.799	0.465	0.916	0.742	0.789	0.446	0.805	0.482
2	0.1544	$9\cdot10^{-3}$	0.570	0.144	0.336	0.038	0.597	0.146	0.330	0.037	0.343	0.039
3	0.0179	0.0034	0.1778	$9.5\cdot10^{-3}$	0.0625	0.0013	0.197	0.0114	0.0622	$1.3\cdot10^{-3}$	0.0628	$1.3\cdot10^{-3}$

Potential parameters	$U_1 < 0,\ B = +2000$; $W_0 = +2000,\ c = 0.42$		$U_1 < 0,\ B = +2000$; $W_0 = +2000,\ c = 0.385$		$U_1 < 0,\ B = -2000,\ W_0 = +2000,\ c = 0.385$			
E, MeV	40		40		40		70	
l	T	Σ	T	Σ	T	Σ	T	Σ
0	0.955	1.310	0.949	1.19	0.953	1.234	0.9815	1.239
1	0.833	0.422	0.788	0.303	0.812	0.356	0.922	0.613
2	0.367	0.044	0.263	0.0257	0.282	0.0300	0.542	0.111
3	0.0675	0.0024	0.0381	0.00175	0.0386	0.00182	0.136	0.0096
4	$2.67\cdot10^{-3}$	$2.03\cdot10^{-4}$			$4.6\cdot10^{-3}$	$2.1\cdot10^{-4}$	$2.6\cdot10^{-2}$	$1.13\cdot10^{-3}$

Table VI. Total cross sections for $W = -W_0 \exp(-r/c)$

Potential parameters	$B = 0,\ U_1 > 0,\ W_0 = +2000,\ c = 0.42$			$W_0 = +2000,\ U_1 > 0\ c = 0.42$; $B = +2000$	$W_0 = +2000,\ U_1 > 0\ c = 0.42$; $B = -2000$	$U_1 < 0;\ W_0 = +2000,\ c = 0.385,\ B = 2000$			$U_1 < 0,\ W_0 = +2000,\ B = -2000,\ c = 0.385$				$U_1 < 0,\ W_0 = +2\cdot10^4,\ c = 0.21,\ B = -2000,$	
E, MeV	20	40	70	40	40	40	70	120	20	40	70	120	40	70
σ_s, mb	120	95	78	93	97	71	60	50	91	84	66	56	73	65
σ_s/σ_a	0.6	0.55	0.51	0.54	0.555	0.45	0.45	0.43	0.47	0.48	0.48	0.48	0.53	0.57

Table VII. Experimental values of σ_a and σ_s

E, MeV (c.m.s.)	22	45	71	118
σ_a, mb	175	101	99	66
σ_s, mb	107	85	64	52
σ_s/σ_a	0.61	0,84	0.65	0.79

(U_3). The sign of U_1 also does not have a strong effect on the absorption and has a still weaker effect on the scattering. Such a stability relative to the real potential makes it possible to investigate the dependence of the cross sections on the two parameters W_0 and c, which determine the absorption.

3. COMPARISON WITH EXPERIMENT

In this way we have shown that for all investigated potentials the scattering cross section is less than the absorption cross section.

The experimental data on the absorption and elastic scattering cross sections[3] (by elastic scattering we understand the sum of the cross sections for charge exchange and true elastic scattering) are shown in Table VII.

Comparison with the data given in Table VI shows that the calculated scattering cross sections are in good agreement with experiment for $W_0 = 2000$ MeV and $c \cong 0.38$ F. Here the absorption cross sections proved to be somewhat greater than experimental, but the difference is not so great if we take into account the crudeness of the model.

With an increase in W_0, the corresponding value of c slowly decreases, and only for $W_0 = 20$ M, where M is the nucleon mass, do we have $c \sim 1/M$. It should be noted that for the potential (3) the quantity σ_a is equal, with good accuracy to $\pi a c^2$, where a depends on E, W_0, and the real potential. For $W_0 = 2000$ MeV, $U_1 < 0$, $U_3 < 0$, and $E = 40$ MeV, we have $a \sim 35$, and the effective radius is of the order 6c; with an increase in energy to 120 MeV, we have $a \sim 25$ and the radius drops to 5c. For $W_0 = 20$ GeV, we have $a \sim 100$.

The decrease in the cross sections with increasing energy takes place more slowly than in experiment, but the qualitative agreement between the model with an exponential potential W and experiment is indisputable.

It is possible that similar considerations can be applied to kaons.

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