PHASE SHIFT ANALYSIS OF ELASTIC p-p SCATTERING AT 660 MeV

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A phase-shift analysis of elastic p-p scattering at an energy of 660 MeV is performed under the assumption that the imaginary part of the scattering phase shifts for the $^3F_{2,3,4}$ and $^1S_0$ states can be neglected. A single set of phase shifts was obtained in the interval $\overline{\chi^2} \le \chi^2 \le 2\overline{\chi^2}$ and four sets in the interval $2\overline{\chi^2} \le \chi^2 \le 3\overline{\chi^2}$. Curves, calculated on the basis of the most probable phase shifts, for the angular dependence of the quantities $\sigma(\vartheta)$, $P(\vartheta)$, $C_{nn}(\vartheta)$, $C_{kp}(\vartheta)$, $D(\vartheta)$, $R(\vartheta)$, and $A(\vartheta)$ are presented.

THE phase-shift analysis of the N-N interaction at energies above the threshold for pion production is of great interest. In this energy region, p-p scattering has been studied in most detail at an energy of approximately 660 MeV, the energy at which a large program of experimental research on the elastic and inelastic p-p interactions was carried out on the Dubna synchrocyclotron.

Many features of the analysis of the inelastic interaction at an energy of 660 MeV were found to be reflected in the Mandelstam resonance model,[1] according to which pion production takes place in a small number of states. Consequently, in performing the phase-shift analysis it is expedient to take advantage of the indications of this model and to take account of the production of pions only in the $^1D_2$ and $^3P_{0,1,2}$ states. Under this assumption, the number of experiments pertaining to elastic p-p scattering which have been performed turns out to be sufficient to perform a phase-shift analysis.

Below are presented the results of the completed analysis that enables us, on the one hand, to see the picture of the elastic p-p interaction in different spin states far from threshold for the production of pions, and, on the other hand, to obtain indications for planning further experiments with regard to p-p scattering at 660 MeV.

The present analysis follows the work of Kazarinov and Silin[2,3] in many respects. A calculation of the one-meson contribution to the scattering amplitude was performed according to formulas available in these articles and in the work of Cziffra et al,[4] with the pion-nucleon coupling constant equal to 0.08. The value of the orbital momentum l_{max}, above which it is possible to describe the scattering by the one-meson Feynman diagram, was determined according to Kazarinov.[5] In this connection, it was found that $l_{max} = 4$.

Data from a series of articles with regard to the measurement of the differential cross section $\sigma(\vartheta)$[6,7] (12 points), polarization $P(\vartheta)$ (14 points),[8] the parameters $D(\vartheta)$[9] and $R(\vartheta)$[10] (10 points), and also $C_{nn}(\vartheta)$[11,12] (3 points), $C_{kp}(\pi/2)$[13] and the value of the total p-p scattering cross section for an energy of 660 MeV[14] were used in the phase analysis.

The parametrization of Stapp et al,[15] in which for the inelastic region it is necessary to regard the phase shifts and coefficients of mixing as complex, was adopted. In this connection, it was assumed:

For singlet transitions

$$\overline{\delta}_l \equiv \overline{\delta}_l^R + i\overline{\delta}_l^I;$$

for triplet transitions with $l = j$

$$\overline{\delta}_{l,j} \equiv \overline{\delta}_{l,j}^R + i\overline{\delta}_{l,j}^I;$$

for transitions with $l = j \pm 1$

$$\overline{\delta}_{j\pm1,j} \equiv \overline{\delta}_{j\pm1,j}^R + i\overline{\delta}_{j\pm1,j}^I, \qquad \varepsilon_j \equiv \varepsilon_j^R + i\varepsilon_j^I,$$

where $\overline{\delta}^I \ge 0$ in virtue of the unitarity of the S matrix.

In accordance with the Mandelstam model, the phases of the $^1D_2$, $^3P_{0,1,2}$ states were assumed to be complex, while the imaginary parts of the phase shifts of $^1S_0$, $^3F_{2,3}$ states and the parameter ϵ_2 were assumed equal to zero. A value found earlier by Soroko,[16] equal to 18.24°, was used for $\overline{\delta}^I(^1D_2)$.

The phase shifts were determined by the method of least squares. The search for minima of the functional χ^2 was accomplished on the Joint Institute electronic computer by the method of lineari-

Phase shifts	Values of the phase shifts (in degrees)				
	Solution 1, $\chi^2 = 47$	Solution 2, $\chi^2 = 62{,}1$	Solution 3, $\chi^2 = 67.2$	Solution 4, $\chi^2 = 82.7$	Solution 5, $\chi^2 = 83{,}1$
$\bar{\delta}^R\,(^1S_0)$	—21.2±8.5	12.8±6.8	— 8.5±6.9	18.1±5.4	—17.2±4.5
$\bar{\delta}^R\,(^3P_0)$	—37.7±7.4	—51.5±6.6	—23.6±4.1	—88.2±5,6	—39.5±10,1
$\bar{\delta}^R\,(^3P_1)$	—15.2±5.0	—5.7±6.6	— 2.7±3.6	35.4±3.9	—39.1±4.4
$\bar{\delta}^R\,(^3P_2)$	56.8±9.3	45.7±2.2	—59.0±3.2	9.3±1.1	13.0±3.1
$\bar{\delta}^R\,(^1D_2)$	4.2±2.7	2,2±6.3	— 0,5±3.8	— 1.5±4.8	— 4,9±3.3
ε_2^R	— 0,1±4.2	—3.1±3,0	— 0.1±2.7	— 0,9±1.4	— 9,3±4.6
$\bar{\delta}^R\,(^3F_2)$	— 6.3±1.1	—7.4±1.8	— 3.8±1.0	—14.8±1.5	5,9±2.2
$\bar{\delta}^R\,(^3F_3)$	3.0±1.6	—5.5±3.1	9.8±1.0	—13.5±2,0	3.5±2,3
$\bar{\delta}^R\,(^3F_4)$	— 3.8±1.0	—6.5±0.7	6.2±1.4	— 1.3±0.5	7.8±0.8
$\bar{\delta}^R\,(^1G_4)$	7.8±1.0	0.8±1.7	— 4.6±1.1	2.0±1.2	— 5.5±0.8
$\bar{\delta}^I\,(^1S_0)$	—	—	—	—	—
$\bar{\delta}^I\,(^3P_0)$	1.9±10.1	7.6±10.2	— 3.2±9.0	5.6±6.7	— 2.6±10.9
$\bar{\delta}^I\,(^3P_1)$	— 2.0±3.4	8,5±5.2	2,1±4.5	3.9±2.7	— 1,4±3.5
$\bar{\delta}^I\,(^3P_2)$	+29.1±6.3	10,6±2,0	20,1±3.2	13.2±1.8	23,8±2.4
$\bar{\delta}^I\,(^1D_2)$	18.2	18.2	18.2	18.2	18.2

zation.[17] After more than 100 searches (for $l_{max} = 4$ and $\overline{\chi^2} = 28$) one solution, hereafter referred to as solution 1, was found in the interval $\overline{\chi^2} \leq \chi^2 \leq 2\overline{\chi^2}$ ($\chi^2 = 47$), and four solutions with χ^2 equal to 62.1, 67.2, 82.7, and 83.1 (solutions 2, 3, 4, and 5, respectively) were found in the interval $2\overline{\chi^2} \leq \chi^2 \leq 3\overline{\chi^2}$. The sets of phase shifts thus obtained are presented in the table. Figures 1—3 illustrate the angular dependences of the experimentally measured quantities, calculated with the aid of solution 1.

In order to clarify the stability of solution 1 and verify the correctness of the assumptions made during the analysis, solution 1 was made more precise by additional variation of the following pairs of parameters: $\bar{\delta}^I(^1S_0)$ and $\bar{\delta}^I(^1D_2)$; $\bar{\delta}^I(^3F_2)$ and ϵ_2^I; $\bar{\delta}^I(^3F_2)$ and $\bar{\delta}^I(^3F_3)$ with $\epsilon_2^I = 0$. As a result, for a practically unchanged value of the goodness-of-fit criterion $\chi^2/\overline{\chi^2} \cong 1.5$, good confirmation of the assumed conditions was obtained, and $\bar{\delta}^I(^1D_2)$ proved to be equal to 14.3 ± 4.3°. Also a strong change in $\chi^2/\overline{\chi^2}$ was not observed upon increasing l_{max} from 4 to 5. The results of these investigations indicate, on the one hand, that whatever underestimate occurs in the introduced number of parameters it is negligible. On the other hand, the fact that the criterion $\chi^2/\overline{\chi^2}$ constantly remains somewhat larger than unity points, possibly, to an overstated accuracy of certain of the experimentally determined quantities.

It is of interest to note that with the already existing accuracy of the experimental data, the number of solutions turned out to be small, and solution 1 goes smoothly over (as will be shown in a more detailed report) into the analogue of Stapp's first solution. In all solutions, apart from the errors, $\bar{\delta}^I(^1D_2)$ and $\bar{\delta}^I(^3P_2)$ differ significantly from zero with $\epsilon_2 \sim 0$. The values of $\bar{\delta}^R(^1S_0)$ and $\bar{\delta}^R(^1D_2)$, found previously as solution – a in [12] for p-p scattering at 650 MeV, as well as the value of $A(\pi/2)$ predicted by Golovin et al,[11] are in agreement with the corresponding values given by solution 1 of the present article.

A more assured choice of the most reliable solution of phase shifts will be possible after additional measurements of a series of scattering parameters at specific points. At present the analysis is being made more precise with account of relativistic effects.

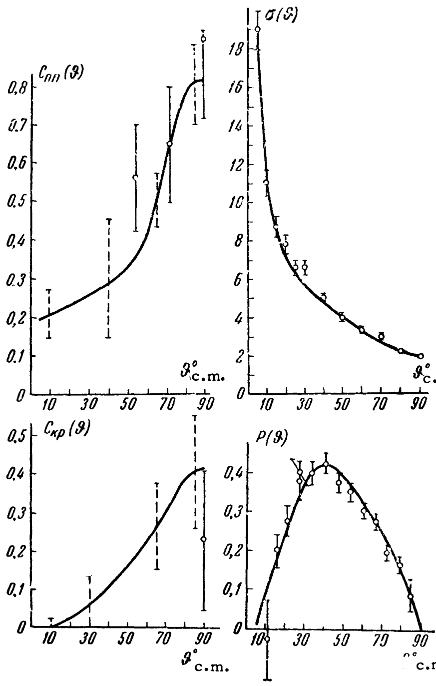

FIG. 1. Angular dependence of $\sigma(\vartheta)$, $P(\vartheta)$, $C_{nn}(\vartheta)$, $C_{kp}(\vartheta)$ according to solution 1: o are experimental points; the calculated error limits are indicated by the dotted line segments.

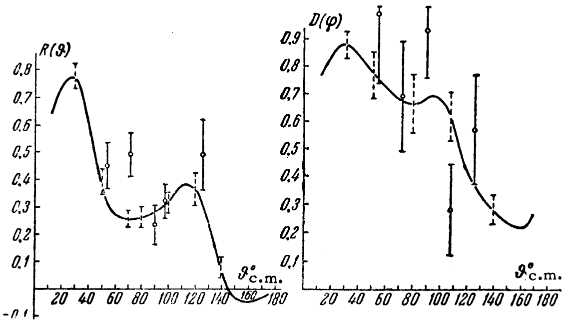

FIG. 2. Angular dependences of the parameters $R(\vartheta)$ and $D(\vartheta)$ according to solution 1: o are the experimental points; the calculated error limits are indicated by the dotted line segments.

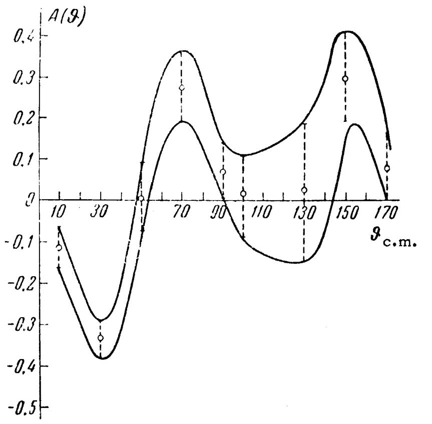

FIG. 3. Predicted values of the parameter $A(\vartheta)$ according to solution 1; the calculated error limits are indicated by the dotted line segments.

After completing the present work, we learned of the article by Hoshizaki and Machida [18], who found solutions for the real elastic p-p scattering phase shifts at 660 MeV by using the average values of the coefficients of absorption $r = \exp(-2\bar{\delta}^I)$ in $^3P_{0,1,2}$ and $^3F_{2,3}$ states calculated on the basis of the Mandelstam model. The authors of [18] did not take into account the contribution of the one-meson diagram and the Coulomb interaction; they did not present error limits for their phase shifts, which hampers a quantitative comparison of the results of both analyses.

A calculation by us of the goodness-of-fit criterion for the solution found in [18] led to the value $\chi^2/\overline{\chi^2} = 3$. Variation of the phases, previously fixed in [18], performed by us with account of the Coulomb interaction and scattering with $l > l_{max} = 4$, significantly decreasing χ^2, sharply modified the real phase shifts, gave larger values of $\bar{\delta}^I(^3F_{2,3})$ and led to negative values of the imaginary parts of the phases $\bar{\delta}(^3P_{0,1})$. The latter contradicts the requirement of unitarity of the S matrix. Setting the phases $\bar{\delta}^I(^3P_{0,1})$ equal to zero would increase χ^2, bringing it up to a value ~ 80.

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Note added in proof (Feb. 11, 1963). The value $f^2 = 0.053$ was used by mistake in the present work in place of $f^2 = 0.080$ (concerning this, see Joint Institute for Nuclear Research preprint, Phase-Shift Analysis of N-N Scattering at 147 MeV, by Kazarinov, Kiselev, and Silin). The revised calculation, performed with $f^2 = 0.080$, led to the disappearance of solution 2 and to negligible changes in the other solutions.

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